

# Prediction of Well Levels in the Alluvial Aquifer along the Lower Missouri River

by Robert E. Criss<sup>1</sup> and Everett M. Criss<sup>2</sup>

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## Abstract

Temporal variations in the head of wells in the alluvial aquifer along the lower Missouri River are accurately simulated by summation of linear differential terms involving daily variations in river stage and effective precipitation. Scaling parameters were optimized using a fourth order Adams-Bashforth-Moulton method, providing predictions for head that are typically accurate within  $\pm 1.5$  feet (0.5 m) over intervals of 1 to 15 years. Parameter magnitudes represent the product of realistic aquifer properties and geometric factors.

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## Introduction

Readily available electronic databases of hydrological and meteorological parameters provide an important means to evaluate and test hydrologic assumptions and models. In the view adopted here, the accuracy with which detailed observational data are simulated is the unbiased arbiter of model effectiveness. An important axiom involves “parsimony,” in particular, that simple models involving the fewest free parameters are preferred over more complex models that offer little or no improvement in accuracy. This paper combines detailed meteorological and hydrological databases to test the assertion (Emmett and Jeffery 1968) that groundwater head in the large alluvial aquifer along the lower Missouri River is primarily controlled by river stage and precipitation. Specifically, we evaluate whether the head responds in a linear manner to perturbations in these

alleged master variables. We conclude affirmatively, as we have identified a simple differential algorithm that quantifies and compares those dependencies, explains their magnitudes, and successfully predicts the temporal variations in head from the independent observations.

## Regional Hydrogeologic Setting

The lower Missouri River has a normal width of about 1300 feet (400 m), and meanders within a broad, two-mile (3 km) wide floodplain bounded by steep bluffs that can be more than 200 feet (60 m) high (Figure 1). This floodplain is largely constituted of farmland and wetland, underlain by a sand and gravel aquifer that extends from bluff to bluff. This clastic aquifer is typically 80 to 120 feet thick (24 to 36 m), mostly represented by an upward-fining sequence bounded by Paleozoic bedrock whose permeability is lower by several orders of magnitude (Emmett and Jeffery 1968; Grannemann and Sharp 1979). The aquifer supplies hundreds of irrigation wells and large well fields that supply Kansas City, Independence, Columbia, St. Charles, and many other municipalities in Missouri and Kansas (Kelly 1996). Well yields of 100 to 3000 gpm (0.006 to 0.2 m<sup>3</sup>/s) are common (Miller and Appel 1997).

Regional groundwater flow in the alluvial aquifer is predominantly downstream and toward the river, but in detail is geographically and temporally complex (Grannemann and Sharp 1979). The head in the alluvial aquifer is primarily controlled by river stage and precipitation

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<sup>1</sup>Corresponding author: Department of Earth and Planetary Sciences, Washington University, One Brookings Drive, Saint Louis, MO 63130-4899; (314) 935-7441; fax: (314) 935-7361; criss@wustl.edu

<sup>2</sup>Department of Mechanical and Aerospace Engineering, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0085; (858) 534-4282; fax: (858) 534-6373; ecrist@ucsd.edu

Received May 2011, accepted September 2011.

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doi: 10.1111/j.1745-6584.2011.00877.x



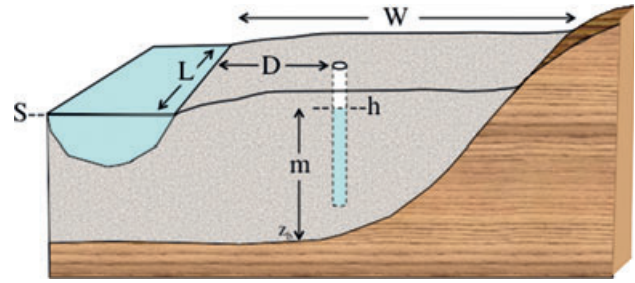
**Figure 1.** Satellite photo of the Jefferson City area, showing the Missouri River and its floodplain to the north, and the fortunate proximity of monitoring well 383550092094201 and its coincident well log (014453; circle), NOAA weather station 234271 (thick cross); and USGS gauging station 06910450 (large triangle).

(Emmett and Jeffery 1968). Groundwater flow may be directed either toward or away from the river, discharging the aquifer during low stages and increasing bank storage during high stages (Sjodin et al. 2001). Available data show that, approximately 95% of the time, the water table in the highly permeable aquifer adjusts to remain within  $\pm 10$  feet (3 m) of the fluctuating river level, but during times of rapid river rise, the stage can be 25 feet (8 m) or more above than the head.

## Theoretical Methods

Many authors have examined the theoretical response of the head of unconfined aquifers to water level changes of a proximal body of surface water. The classical problem is analogous to commonly studied configurations in heat flow and involves the response of groundwater to an abrupt change in the level of the surface water, which is thereafter held constant (Marino 1973). Another example considers how groundwater responds to sinusoidal variations in the level of the surface water body, as could occur in a coastal aquifer influenced by tides (Shultz and Ruppel 2002). Analytical models for aquifers along streams (Moench and Barlow 2000) involve numerous free parameters and simplifying assumptions, and because they ignore the direct addition of precipitation to aquifers, their utility has not been demonstrated over long intervals nor at large distances from the channel. Thus, while useful for their intended applications, the boundary conditions governing such situations cannot realistically represent the large and chaotic variations in river stage and precipitation that affect alluvial aquifers.

A direct way to consider the latter problem is from a flux equation. Assuming linearity and unconfined



**Figure 2.** Sketch showing the meaning of geographic variables used in the equations.

conditions, the gradient in head is the difference between the elevations of the river ( $S$ ) and the water table ( $h$ ) divided by the distance ( $D$ ) of an observation well from the river. According to Darcy's law, the groundwater flow for an arbitrary length ( $L$ ) along the river should be equal to this gradient multiplied by the hydraulic conductivity ( $K$ ) and the thickness of the saturated aquifer ( $h - z_b$ ), where  $z_b$  is the bedrock elevation (Figure 2). Under unconfined, quasi-steady conditions, material balance requires that this flow will be proportional to the rate of change in head  $dh/dt$  multiplied by the floodplain area, here approximated as  $WL$ , where  $W$  is the floodplain width, also multiplied by the specific yield, symbolically represented as the relevant porosity ( $\phi$ ) that contributes to changes in storage. An additional term accounts for changes in head due to the fraction ( $f$ ) of precipitation ( $P$ ) that deeply infiltrates the floodplain, multiplied by the floodplain area. Thus, the simplified volumetric flux equation becomes

$$\phi WL \frac{dh}{dt} = K(h - z_b)L \frac{(S - h)}{D} + WLfP \quad (1a)$$

The left side of Equation 1a is the rate of change of storage, while the first term on the right is the Darcian term, and the second term on the right accounts for the precipitation added. All three terms have unit of  $\text{ft}^3/\text{d}$ . An important simplification we test is whether the saturated thickness  $h - z_b$  can be treated as a simple constant  $m$ , giving:

$$\phi WL \frac{dh}{dt} = KmL \frac{(S - h)}{D} + WLfP \quad (1b)$$

Real situations are slightly more complicated. For example, river stages are measured by a network of long-term gauging stations, but the closest station to any particular well is generally located a considerable distance upstream or downstream, so a correction term must be added to the quantity  $S - h$ . Another correction is to change the actual precipitation record to an effective precipitation record to account for evapotranspiration losses, which can be approximated by the method of Criss and Winston (2008). One way to account for these effects is to introduce correction terms to express the right-hand side of Equation 1b in terms of deviations from long-term average values for head, stage, and effective precipitation.

Finally, an additional term can be included to account for the difference in initial and final head.

Considering these correction terms, the volumetric flux (Equation 1b) can be rewritten:

$$\Delta h = a(S - h - d) + b(P_{\text{eff}} - e) + (h_f - h_i)/T \quad (2)$$

where  $a$ ,  $b$ ,  $d$ , and  $e$  are either constants or average values,  $P_{\text{eff}}$  and  $S-h$  represent daily values,  $h_f$  and  $h_i$  indicate the final and initial groundwater levels, respectively, and  $T$  is the total length of the record. By summing the quantities  $(h_f-h_i)/T$ ,  $-ad$ , and  $-be$  into a single constant  $c$ , Equation 2 can be simplified to:

$$\Delta h = a(S - h) + b(P_{\text{eff}}) + c \quad (3)$$

where  $\Delta h$  is the predicted daily change in head,  $S-h$  is the daily value for the actual river stage minus the simulated groundwater head,  $P_{\text{eff}}$  is the daily effective precipitation reported as ft/d, and  $c$  is the combined additive term that includes many different contributions. Parameters  $a$ ,  $b$ , and  $c$  can be determined from observational data, as detailed in the following.

## Practical Methods

Fitting of these free parameters to detailed time-series data is a multistep process. First, a continuous data set consisting of daily river levels, groundwater levels, and effective precipitation must be assembled. Missing river data were estimated by simple linear interpolation; this uncontrived procedure proved to be sufficiently realistic, in part because missing data are few. Second, the three parameters must be estimated to sufficient accuracy to allow numerical integration. Once this is done, Equation 3 must be integrated forward in time. Finally, the predicted heads need to be compared to the actual well values. This last step can then be used to evaluate and refine our parameter estimates.

Initial parameter fitting was done by combining least-squares parameter estimates with forward Euler numerical integration. This rudimentary approach met with limited success; however, this algorithm proved to be too unstable for most of our sites.

We overcame this issue by replacing our forward Euler integrator with one utilizing the Adams-Bashforth-Moulton fourth order method (hereafter, "ABM"). This multistep method achieves forward integration with the following equations, respectively known as the "predictor" and "corrector." Modifying the generic ABM formulation (Kreyszig 1999) to fit our case gives:

$$h_{n+1}^* = h_n + \frac{\Delta t}{24}(55h'_n - 59h'_{n-1} + 37h'_{n-2} - 9h'_{n-3}) \quad (4a)$$

$$h_{n+1} = h_n + \frac{\Delta t}{24}(9h_{n+1}^* + 19h'_n - 5h'_{n-1} + h'_{n-2}) \quad (4b)$$

where  $h_n$  denotes the height of the water table at step  $n$ , \* indicates an initial predicted value,  $\Delta t$  represents the step size (one day), and  $h'$  is the derivative of the water table height given by Equation 3. This method was selected due to its increased stability when compared with the forward Euler method and the Runge-Kutta method (Hoffman 2001). Furthermore, the consistent step size in our data sets, along with preliminary data points used to begin integration, eliminate most disadvantages of multistep methods such as this one (Carnahan et al. 1969).

To utilize these equations to predict groundwater level, we obtained preliminary estimates for our parameters, referred to here as  $\tilde{a}$ ,  $\tilde{b}$ , and  $\tilde{c}$ . Values for  $\tilde{a}$  and  $\tilde{b}$  were calculated using least-squares fits to relate  $\Delta h$  to  $S-h$  and  $P_{\text{eff}}$ , respectively. Of all the parameters,  $\tilde{a}$  is the simplest to estimate, since our data set is easily reduced to only include values with no effective precipitation. These selected data provide an estimate for  $\tilde{a}$  using the equation:

$$\Delta \underline{h} \approx \tilde{a}(S - h) \quad (5)$$

where  $\Delta \underline{h}$  is the actual change in groundwater level when no effective precipitation occurred, and the other values are as before indicated. Least-squares fitting is then used to find  $\tilde{a}$ .

Calculation of  $\tilde{b}$  can be performed in a similar manner; however, it is not possible to discount data with a nonzero ( $S-h$ ). Instead, the estimate  $\tilde{a}$  is placed into Equation 3, resulting in:

$$\Delta h \approx \tilde{a}(S - h) + \tilde{b}P_{\text{eff}} \quad (6)$$

Least-squares fitting is then used to find  $\tilde{b}$ . Once this is complete,  $\tilde{a}$  and  $\tilde{b}$  can be multiplied with long-term average values  $\bar{S} - \bar{h}$  and  $\bar{P}_{\text{eff}}$ , respectively, and combined with the difference between the initial and final groundwater levels over total time  $(h_f-h_i)/T$  to find  $\tilde{c}$ :

$$\tilde{c} \approx -\tilde{a}(\bar{S} - \bar{h}) - \tilde{b}\bar{P}_{\text{eff}} + (h_f - h_i)/T \quad (7)$$

Parameter estimation with this method was sufficiently accurate to allow numerical integration of Equation 3 utilizing Equation 4, producing a predicted well function. This function uses only the complete river stage and precipitation record along with actual head data for the initial four days; the subsequent head values were entirely predicted from Equations 3 and 4.

To refine our parameter estimates, we combined our ABM integrator with an adaptive direct searching scheme. The least-squares fitting outlined in the preceding sections served as a basis for this search. An initial  $11 \times 11 \times 11$  search space was constructed around these points, assuming that  $\tilde{a}$  was accurate to  $\pm 0.01/\text{d}$ ,  $\tilde{b}$  to  $\pm 6$  (unitless) and  $\tilde{c}$  to  $\pm 0.25 \text{ ft/d}$  ( $\pm 0.076 \text{ m/d}$ ). These values result in fast convergence over most data sets and correspond roughly to the expected difference between our preliminary estimates and final values. It is not essential that our final values be inside this search space, however, convergence will be faster if they are. Once a search space

is established, the program will evaluate every different parameter combination in this space by producing a unique predictive curve for the well level using these test parameters. The quality of each parameter set was evaluated by summing the square of the differences between the predicted values and the actual well data. Once the best set was determined, it was used to further optimize our parameters. This was done by centering a new search space on the best values and decreasing its size by a factor of 10. The program will collapse the search space a total of five times before completion. If the optimum parameter combination was on an edge of the search space, the search space was moved in that direction. This helps insure that the optimum parameter combination is actually inside the area examined. Although the actual well levels were used to improve our parameter picks, the predicted equation only uses four initial data points to begin integration.

It is of utmost importance that the differences between the actual well levels and the predicted well levels are not caused by integration errors. Were this the case, optimization using the above mentioned methodology would be ineffective since the best parameter set would be unknown. To prevent this, our ABM integrator was set to have a maximum approximate computational error ( $\Delta h - \Delta h^*$ ) of .01 feet, which is two orders of magnitude less than our average error.

#### Example: Jefferson City

A groundwater well near Jefferson City, Missouri, located in the floodplain about 4800 feet north (1.5 km)

of the Missouri River, provides a useful test of this model (Table 1; Figure 1). Nearly continuous daily records are available at Jefferson City for groundwater head in this well since 1956 and for precipitation since 1890. River stage measurements at Jefferson City were initiated late in 1994, but were recorded intermittently until 1996, when nearly complete and reliable data became available. In particular, Jefferson City has a NOAA meteorological station, and a USGS gauging station is on the Missouri River only 1600 feet (500 m) upstream of the point on the river nearest the well (Table 1; Figure 1). Figure 3 shows the interval 1996 to 2011 over which high quality daily data are available for all these quantities and illustrates their interrelationships.

Figure 4 shows the comparison of observed and predicted heads for the Jefferson City well over this extended interval, the latter utilizing the ABM method with our optimized values  $a = 0.00882/d$ ,  $b = 4.5$  (unitless), and  $c = -0.023 \text{ ft/d}$  ( $-0.0070 \text{ m/d}$ ; Table 2). The average absolute deviation between the actual and predicted head is only 0.61 feet (15 cm). The greatest deviation between these quantities is only 2.8 feet (85 cm) and occurred during flooding; factors that could contribute to head under-prediction during flooding include the increase in saturated thickness (non-linearity under extreme conditions), overland flow and downward percolation, and in cases where the gauging station is far from the observation well, inaccurate correction of river stage (non-constancy of the additive correction “ $d$ ” in Equation 2 at high and low stages).

A graph of the calculated vs. the actual head over the 1996 to 2011 interval has a linear correlation coefficient

**Table 1**  
**Data Sources and Well Characteristics**

Well Name, (RM), and USGS Site #	NOAA Ppt Sta.	Gauging Station (RM)	Well Depth (feet)	Distance to River (feet)	Collar Elevation, MSL (feet)
Nebraska City (571.6) 404629095500501	255810	06807000 (561.9)	48	450	905.36
Atherton 8 (343.3) 391354094161901	234359	06893000 (366.1)		1000	712.93
Eagle Bluffs (175.9) 385156092263202	231791	06909000 (197.1)	90	4200	566.67
Jefferson City (143.6) 383550092094201	234271	06910450 (143.9)	95	4800	548.3
Hermann 4 (96.4) 384258091243001	233793	06934500 (97.9)	80.1	1800	508.69
Hermann 7 (94.9) 384232091225101	233793	06934500 (97.9)	36.9	1000	505.15
Columbia Bottoms (2.5) 384849090092001	237455	Hartford, IL (~0)	104	4500	422
Mazomanie, WI (~81) 431312089475301	477576	05407000 (43.5 Wisc)	146	4820	740
Grand Island, NE (70.0) 405227098165601	253395	06770500 (70.0 Platte)	30	60	1835

Note: Data from USGS 2011a, 2011b; NOAA 2011. Data for Hartford, IL gauging station from USACE 2010. RM, Missouri River Mile; Wisc, Wisconsin River; Platte, Platte River.

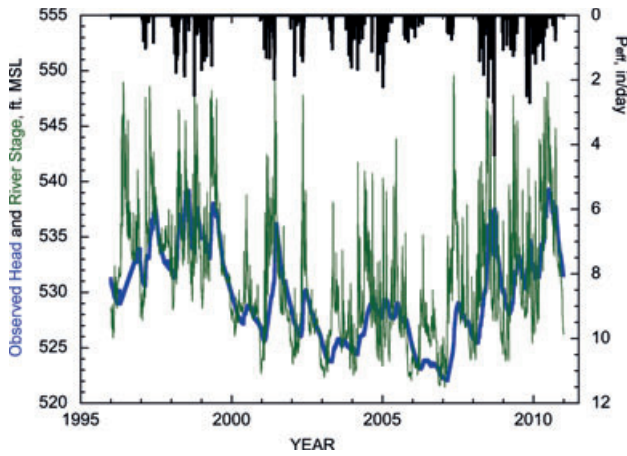


Figure 3. Available daily data illustrating the interrelationships between the observed well head (blue), river stage (green), and effective precipitation (black bars, right scale) for the Jefferson City area. Elevations are in feet relative to MSL (mean sea level).

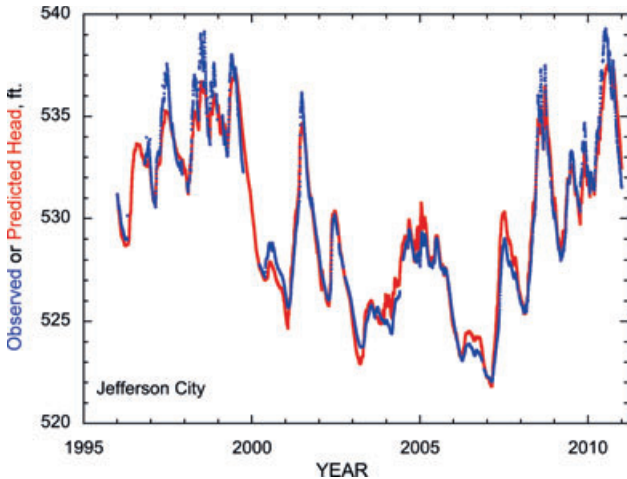


Figure 4. Predicted (red line) and observed (blue points) groundwater heads in the Jefferson City alluvial well plotted against time for the interval 1996 to 2011.

( $R^2$ ) value of 0.969 and a slope of 0.92, although to the eye the trend differs little from a 1:1 line (Figure 5). Slopes are very flat, and regressions are poor on graphs where the difference between the predicted and actual head is plotted against stage (slope =  $-0.023$ ;  $R^2 = 0.029$ ); head (slope =  $-0.066$ ;  $R^2 = 0.126$ ); stage minus head (slope =  $0.026$ ;  $R^2 = 0.018$ ); or year (slope =  $0.029$ ;  $R^2 = 0.026$ ).

#### Additional Examples

Multiyear records are available for several additional wells in the alluvial aquifer along the lower Missouri River (Table 1); for comparison, examples for the Wisconsin and Platte Rivers are also included. Table 2 provides fitting coefficients for these wells as determined by our previously outlined methods. The results show reasonably

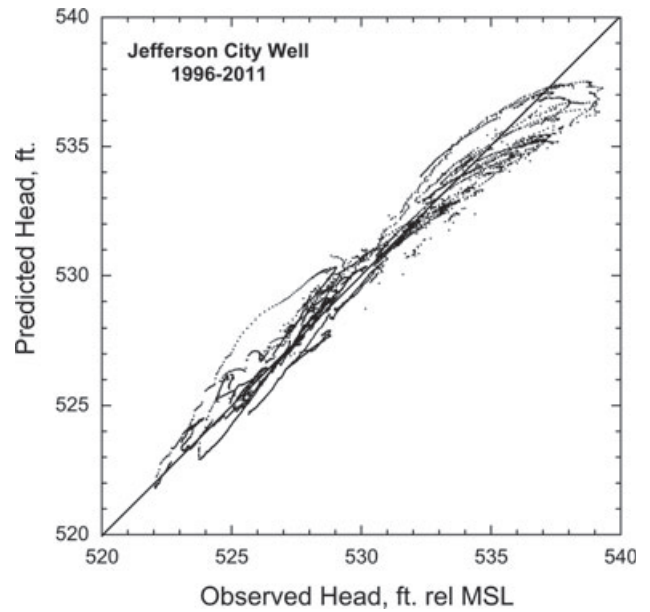


Figure 5. Predicted vs. observed groundwater heads in the Jefferson City alluvial well for the interval 1996 to 2011, compared to a 1:1 line.

**Table 2**  
Numerical Coefficients for Alluvial Wells  
(Equation 3)

	"a" (per day)	"b" (unitless)	"c" (ft/d)	$h$ error <sup>1</sup> (feet)
Nebraska City	0.38	1.4	3.78	0.42
Atherton 8	0.11	5.7	-2.09	1.15
Eagle Bluffs	0.031	10.5	-0.60	1.11
Jefferson City	0.0088	4.5	-0.02	0.61
Hermann 4	0.057	6.9	-0.02	1.08
Hermann 7	0.052	7.3	-0.11	1.10
Columbia Bottoms	0.021	9.8	-0.04	0.62
Mazomanie, WI	0.020	3.5	1.19	0.43
Grand Island, NE	0.49	2.3	-3.31	0.09

<sup>1</sup> Average absolute difference between predicted and observed heads.

coherent values for these parameters, whose functional dependencies are explained in the following. The results also show that the groundwater levels can be predicted to about  $\pm 1.2$  feet ( $\pm 0.3$  m) or better.

Particularly, good fits are realized at Jefferson City and Grand Island, where river gauging stations are located very close to the observation wells. The likely explanation is that, for cases where the gauging station is located a considerable distance upstream or downstream of the well, a constant was added to the measured stage to correct for its location. Because the slope of the river surface can depend on the flow conditions, this correction is not strictly accurate.

## Interpretation of the Numerical Constants

Comparison of Equations 1 through 3 provides the following interpretations for the coefficients  $a$ ,  $b$ , and  $c$ :

$$a = Km/\phi WD \quad (8a)$$

$$b = f/\phi \quad (8b)$$

$$c = -a(\bar{S} - \bar{h}) - b\bar{P}_{\text{eff}} + (h_f - h_i)/T \quad (8c)$$

Parameter  $a$  should be positive as it represents the product of several positive quantities. This expectation is realized for all cases (Table 2). Approximate values for  $a$  can be estimated from Equation 8a, as follows. The thickness ( $m$ ) of the alluvial aquifer along the Missouri River is generally close to 100 feet (30 m). Slug tests for alluvial wells in this aquifer suggest that hydraulic conductivity ( $K$ ) is about 750 ft/d (230 m/s; NRC 2010), and normal values of porosity ( $\phi$ ) are  $0.25 \pm 0.15$  for alluvial sediments. Taking the Jefferson City well as an example, the distance ( $D$ ) of the well from the river is 4800 feet (1460 m), and the width ( $W$ ) of the floodplain on the north side of the river in this area is 8000 feet (2440 m; cf. Figure 1; Table 1). Using these values and the assumed value for porosity, the estimated value of  $a$  given by Equation 8a is about 0.008/d, close to our previously calculated value (Table 2). Using the same approach, values for the other wells listed in Table 2 can be estimated to within about a factor of two from our optimized values using the data in Table 1. Constant  $a$  shows a good inverse correlation with the distance of each well from the river, as predicted by Equation 8a.

Values for constant  $b$  should similarly be positive, as this constant depends on the quotient of the positive quantities  $f$ , the fraction of precipitation which infiltrates the floodplain and  $\phi$ , the relevant porosity. This expectation is realized in all cases. Additionally, if it is assumed that surface runoff is low for the floodplain so that most of the effective precipitation recharges the groundwater system, then  $f \sim I$  and values for  $b$  are seen to approximate the inverse porosity. The values for  $b$  in Table 2 provide reasonable estimates for  $\phi$ .

Parameter  $c$ , as calculated from Equation 8c using the specified average data and the  $a$  and  $b$  values from our ABM based algorithm (Table 2), is in all cases well within  $\pm 0.04$  ft/d of the tabulated value that was determined by treating  $c$  as a free parameter. In effect, this result indicates that the model represented by Equation 3 has two free parameters, not three. Detailed interpretation of parameter  $c$  is of little value as it contains the separate contributions embodied in parameters  $a$  and  $b$ . Values depend on many factors but are typically negative when the river gauging stations are located upstream of the particular well, and positive when the gauging station is downstream. The largest absolute values for  $c$  are seen for wells located farthest (>10 miles; or >16 km) from the relevant river gauges, except for Grand Island where a significant survey discrepancy appears to exist between the gauge zero and the reported elevation of the well collar.

Note that Equations 8a through 8c provide the means to independently estimate the coefficients for any real or hypothetical well, permitting detailed predictions of groundwater head from a daily record of precipitation and river stage. In particular, any of several simple iterative algorithms can calculate  $\Delta h$  values using Equation 3, and use them to create a predictive series of head values.

## Evaluation of Linearity Hypothesis

Comparison of the actual and predicted heads is satisfactory and very good for Jefferson City and Grand Island, where the river gauging and precipitation stations are proximal to the observation wells. The accurate predictions (Figure 5; Table 2) strongly support the reasonableness of the linearity approximation.

The assumption that the thickness of the saturated aquifer is effectively constant is commonly used to simplify equations of unconfined flow, a familiar case represented by the linearized Boussinesq equation. The standard justification is that this approximation is reasonable when the saturated thickness is great compared to the variations in the head. While this explanation is useful, a much stronger justification may be made for typical alluvial aquifers, which like the lower Missouri River are dominantly characterized by fining upward sedimentary sequences. The key transport property relevant to water balance, representing the numerator of Equation 8a, is the aquifer transmissivity  $T$ , typically considered to be the product of the hydraulic conductivity  $K$  and the saturated thickness  $m$ . In fact, because both  $K$  and  $m$  are not simple constants but depend on the groundwater level  $h$ , a more realistic representation of  $T$  would be

$$T = \int_{z_b}^h K dz \quad (9)$$

In a typical upward-fining sequence, the values for  $K$  in the lowermost coarse gravels will be much higher than values in the overlying sand-silt sequences, which in turn will be much higher than the values in the overlying facies such as overbank deposits that include abundant fine silts and clays. Because the changes in head necessarily occur in the uppermost parts of the aquifer, the value of the transmissivity integral is predominantly governed by the perpetually saturated, coarse-grained lower parts of the aquifer, and the total range of variation of  $T$  is much lower than would be expected from changes in  $h$  in a medium of constant  $K$ .

## Conclusion

Groundwater levels in the alluvial aquifer along the lower Missouri River are primarily governed by river stage and secondarily by the delivery of meteoric precipitation. The response of groundwater head to the comparatively chaotic variations of stage and rainfall can be accurately predicted with a linear differential model evaluated with the ABM method. The model is effectively

based on two free parameters, whose magnitudes were determined by an adaptive searching scheme of thousands of possible combinations, whose effectiveness was then evaluated by comparison of the predicted heads to the actual well record. The magnitudes of these parameters can be independently explained in terms of a combination of realistic aquifer properties and geometric factors.

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