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DIRECTIONAL-CORRELATION ATTENUATION FACTORS FOR Ge(Li) DETECTORS: A COMPARISON OF EXPERIMENTAL AND CALCULATED VALUES*

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Refinements in measuring technique are utilized to improve a previously described method for experimentally determining the directional-correlation attenuation factors of Ge(Li) γ-ray detectors. Correction factors for a five-sided true coaxial 29 cm^3 Ge(Li) detector and a five-sided trapezoidal 25 cm^3 Ge(Li) detector are determined using this method and agree well with recently available computer calculations for these values. Various sources of error inherent in the method are discussed in some detail.

In an earlier publication[1]) we presented a method for experimentally measuring the directional-correlation attenuation factors of Ge(Li) detectors. Briefly, this method utilizes absorbers which are designed to attenuate the γ-flux from a point source in a cylindrically symmetrical manner and such that the radial γ-flux is proportional to $\cos^{2m}\beta$. The absorbers are readily used to simulate the integrals involved in the correction factors Q_{2n}, or

$$Q_{2n} = \int_0^\alpha \varepsilon(\beta) P_{2n}(\cos\beta)\mathrm{d}(\cos\beta) \Big/ \int_0^\alpha \varepsilon(\beta)\mathrm{d}(\cos\beta)$$
$$= \sum_{m=0}^{n} L_{2m}^{2n} \int_0^\alpha \varepsilon(\beta)\cos^{2m}\beta\,\mathrm{d}(\cos\beta) \Big/ \int_0^\alpha \varepsilon(\beta)\mathrm{d}(\cos\beta)$$
$$= \sum_{m=0}^{n} L_{2m}^{2n} R_{2m}, \qquad (1)$$

where $\varepsilon(\beta)$ is the detector efficiency for γ-rays approaching the detector at an off-axis angle β, the L_{2m}^{2n} are the appropriate $\cos^{2m}\beta$ coefficients of the even Legendre polynomials $P_{2n}(\cos\beta)$, and the integral limits include the active volume of the detector. The values of R_{2m} are experimentally measured as the ratio of the γ-flux detected with and without the attenuation by the "$\cos^{2m}\beta$" absorber, and the Q_{2n} are calculated from the linear combination of the R_{2m} given in eq. (1).

Recently Camp and Van Lehn[2]) have developed a method for predicting the correction factors by calculating the integrals of eq. (1)

$$J_k = \int_0^\alpha \varepsilon(\beta) P_k(\cos\beta)\mathrm{d}(\cos\beta), \qquad (2)$$

in which they approximate $\varepsilon(\beta)$ by

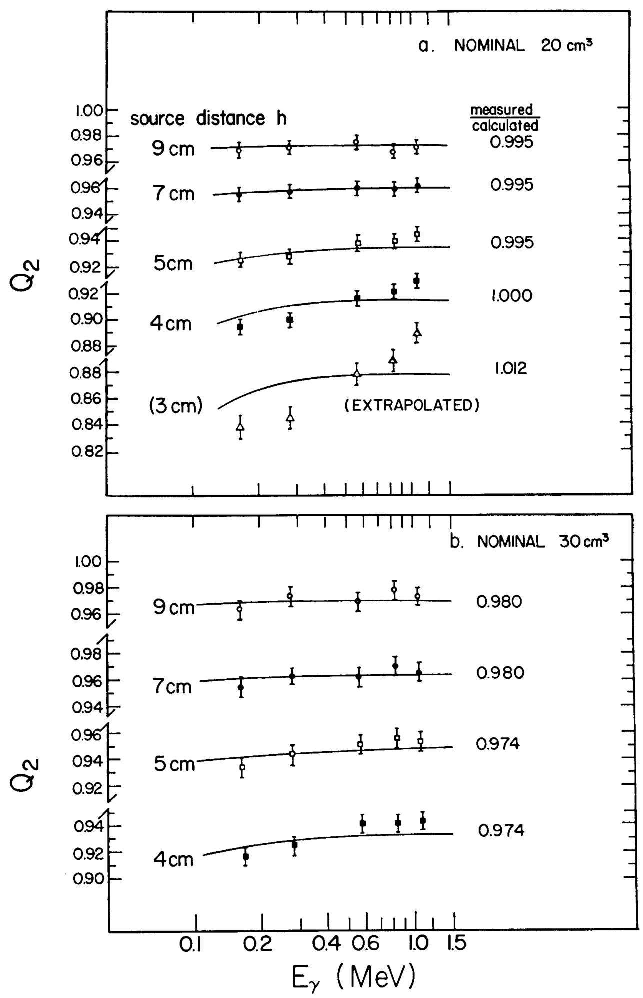

Fig. 1. A comparison of directional attenuation factors obtained from the experimental method of Winn and Sarantites[1]) and the calculation method of Camp and Van Lehn[2]). In the figure, the individual points are from ref.[1]) and the smooth curves [renormalized according to the factor "measured calculated"] are from ref. [2]) and ref. [6]).

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TABLE 1
Comparison of experimental results from ref. [1]) with calculated results from ref. [2]).

Detector dimensions[a]	h (cm)	$\overline{\Delta Q_2}$ (%)	$\lvert\delta\overline{Q_2}\rvert$[d] (%)	$\lvert\Delta Q_2\rvert$ (%)	$\overline{\Delta Q_4}$ (%)	$\lvert\delta\overline{Q_4}\rvert$[d] (%)	$\lvert\Delta Q_4\rvert$ (%)
Nominal 20 cm³ Ge(Li) detector, 5-sided, trapezoidal[b]	9.0	−0.5	good	0.5	−2.7	good	2.7
	7.0	−0.5	good	0.5	−2.3	good	2.3
	5.0	−0.5	fair	0.5	−1.9	fair	1.9
	4.0	0.0	$\leqq$ 1.7	$\leqq$ 1.7	−2.1	$\leqq$ 6.9	$\leqq$ 4.8
	3.0	1.2	$\leqq$ 3.4	$\leqq$ 4.6	−2.6	$\leqq$ 18	$\leqq$ 15
Nominal 30 cm³ Ge(Li) detector, 5-sided, cylindrical[c]	9.0	−2.0	good	2.0	−5.2	good	5.2
	7.0	−2.0	good	2.0	−7.4	good	7.4
	5.0	−2.6	fair	2.6	−8.9	fair	8.9
	4.0	−2.6	fair	2.6	−9.4	fair	9.4

[a] For both detectors the dimensions were chosen to match those given by the manufacturer. For more details concerning these detector parameters, see ref. [1]) and footnotes b and c below.
[b] Detailed dimensions of this detector for calculations of ref. [6]): frontal area 9.011 cm², radial depletion depth 11.00 mm, axial depletion depth 13.20 mm, length 28.00 mm, volume 24.93 cm³, n-region 0.60 mm, height 28.00 mm, width 38.00 mm, base 33.00 mm.
[c] Detailed dimensions of this detector for calculations of ref. [6]): frontal area 4.909 cm², radial depletion depth 8.00 mm, axial depletion depth 8.00 mm, volume 21.658 cm³, n-region 0.50 mm, diameter 26.00 mm, length 50.00 mm.
[d] "Good" means $|\delta Q_{2n}| \leqq 1$ standard deviation; "fair" means $\delta Q_{2n} \leqq 1.5$ standard deviation.

$$\varepsilon(\beta, E_\gamma) = 1 - \exp[-\tau(E_\gamma)\rho x(\beta)]\,, \tag{3}$$

where $x(\beta)$ is the thickness of the active detector traversed by γ-rays at angle β, ρ is the density of germanium, and $\tau(E_\gamma)$ is an average full-energy peak "cross section" for a given detector. The values of $\tau(E_\gamma)$ were determined or interpolated from the full-energy peak efficiencies of nine different Ge(Li) detectors[3]). This method of measuring $Q_{2n} = J_{2n}/J_0$ was tested in a few cases by using Monte Carlo calculations[2]) which incorporate techniques of Yates[4]) and Fry et al.[5]).

Before examining the limitations of the above methods, it is worthwhile to compare their predictions for the Q_{2n} values of specific Ge(Li) detectors. Such a comparison for a "nominal" 20 cm³ and a "nominal" 30 cm³ detector is given in fig. 1 where the experimental points are from ref. [1]), and the smooth curves are calculated by Camp and Van Lehn[6]). [In ref. [1]) a somewhat different curvature is apparent in the smooth curves drawn through these same points; however, the calculated curves in fig. 1 fit these points well enough to indicate that the data are not sufficient to determine much about the finer details of the curves —except that they must be monotonically increasing with E_γ.] To facilitate the comparison, the calculated curves have been normalized so they best fit the experimental points; this provides an examination of both the average agreement (denoted by "measured/calculated" in the figure) and the shape agreement (seen from the deviation of the points from the curve). In table 1 we have listed the average deviation ($\overline{\Delta Q_{2n}}$), the shape deviation ($\delta\overline{Q_{2n}}$), and the total deviation ($\Delta Q_{2n} = \overline{\Delta Q_{2n}} + \delta\overline{Q_{2n}}$) of both detectors for various source distances h. The deviations of the Q_4 values (not plotted) are always larger than those of the Q_2 values; however, this is to be expected as the values of Q_4 are more sensitive to the techniques used in either method[1,2]).

Although the overall agreement between the calculated and measured Q_{2n} values is fairly reasonable, it is useful to examine the regions where the agreement is worst. In general the agreement for both detectors becomes poorer as the source is moved closer, because both methods are more sensitive to errors at these

TABLE 2
Δ_{2n} dependence on α for various detectors in ref. [2]).

α	$\Delta_2(E_\gamma = 300$ keV) (%)	$\Delta_4(E_\gamma = 300$ keV) (%)	Detector
44°	2.2	8.9	1[a]
33°	1.6	6.4	2[b]
26°	0.56	2.25	1
23.5°	0.40	1.6	3[c]
16°	0.25	1.0	1
8°	0.04	0.16	1

[a] 6.6 cm² × 1.2 cm planar.
[b] 4.0 cm in diameter 4.5 cm long 5-sided cylinder with depletion depth 13 mm.
[c] 2.6 cm in diameter 4.42 cm long coaxial with depletion depth 9 mm.

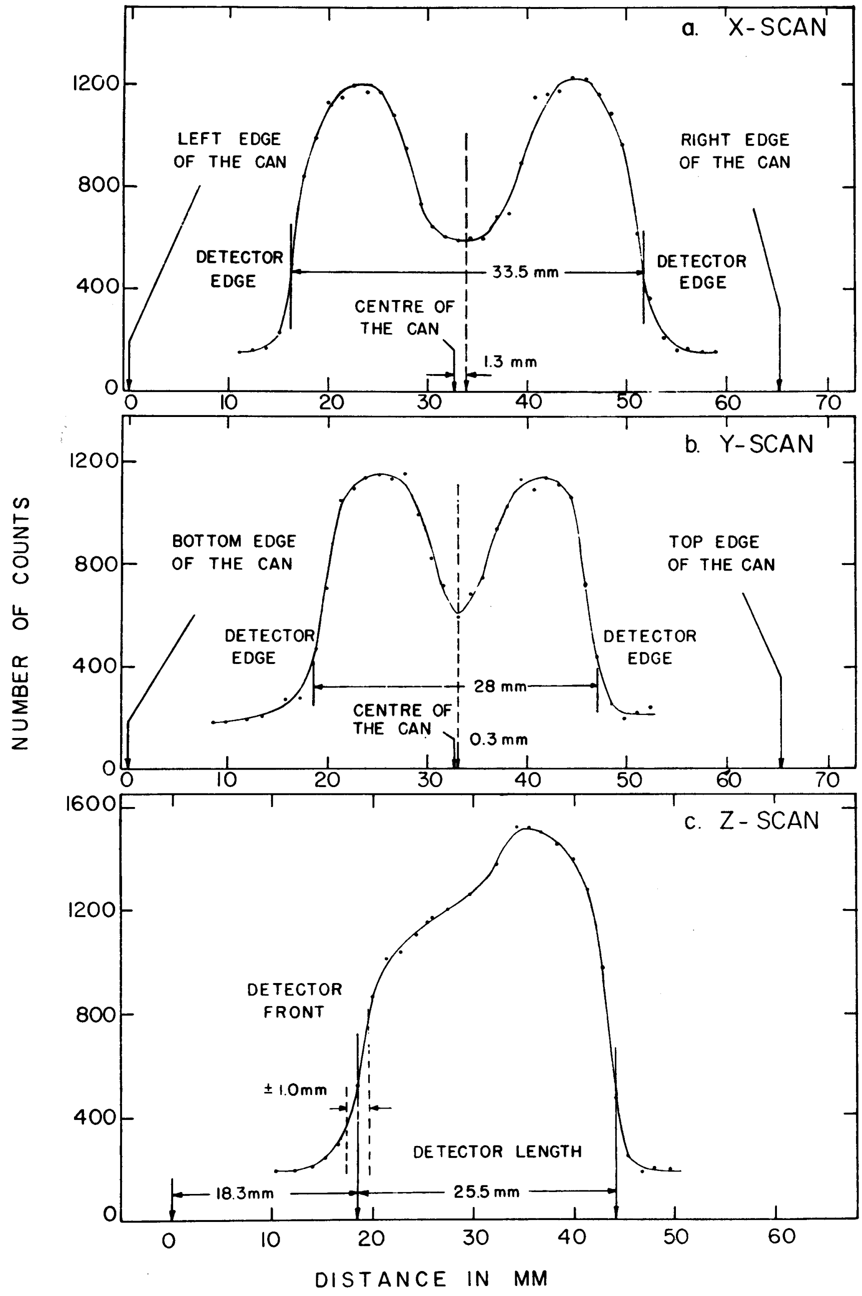

Fig. 2. Collimated beam scans of a 5-sided trapezoidal 25 cm^3 Ge(Li) detector.

source distances. In the calculation method[2]), the approximation for $\varepsilon(\beta)$ in eq. (3) is less rigorous at closer distances because the full-energy peak "cross section" $\tau(E_\gamma)$ should have a more general functional form $\tau(E_\gamma,\beta)$ to account for the decreasing amount of full-energy dissipation near the detector edges. As evidence of this limitation, the Monte Carlo calculations disagree with the results obtained using eq. (3) for close source distances[2]). As a guide to how much this effect should affect the values of table 1, we consider the discrepancy Δ_{2n} between the Monte Carlo calculations and the $\varepsilon(\beta,E_\gamma)$ calculations as a function of the half-angle α subtended from the source to the detector. Using the information of ref. [2]) we construct table 2, from which Δ_4 ($\approx 4\Delta_2$) has been estimated as 3.2% ($= 4\times 0.8\%$) for the 20 cm^3 detector at $h = 3$ cm,

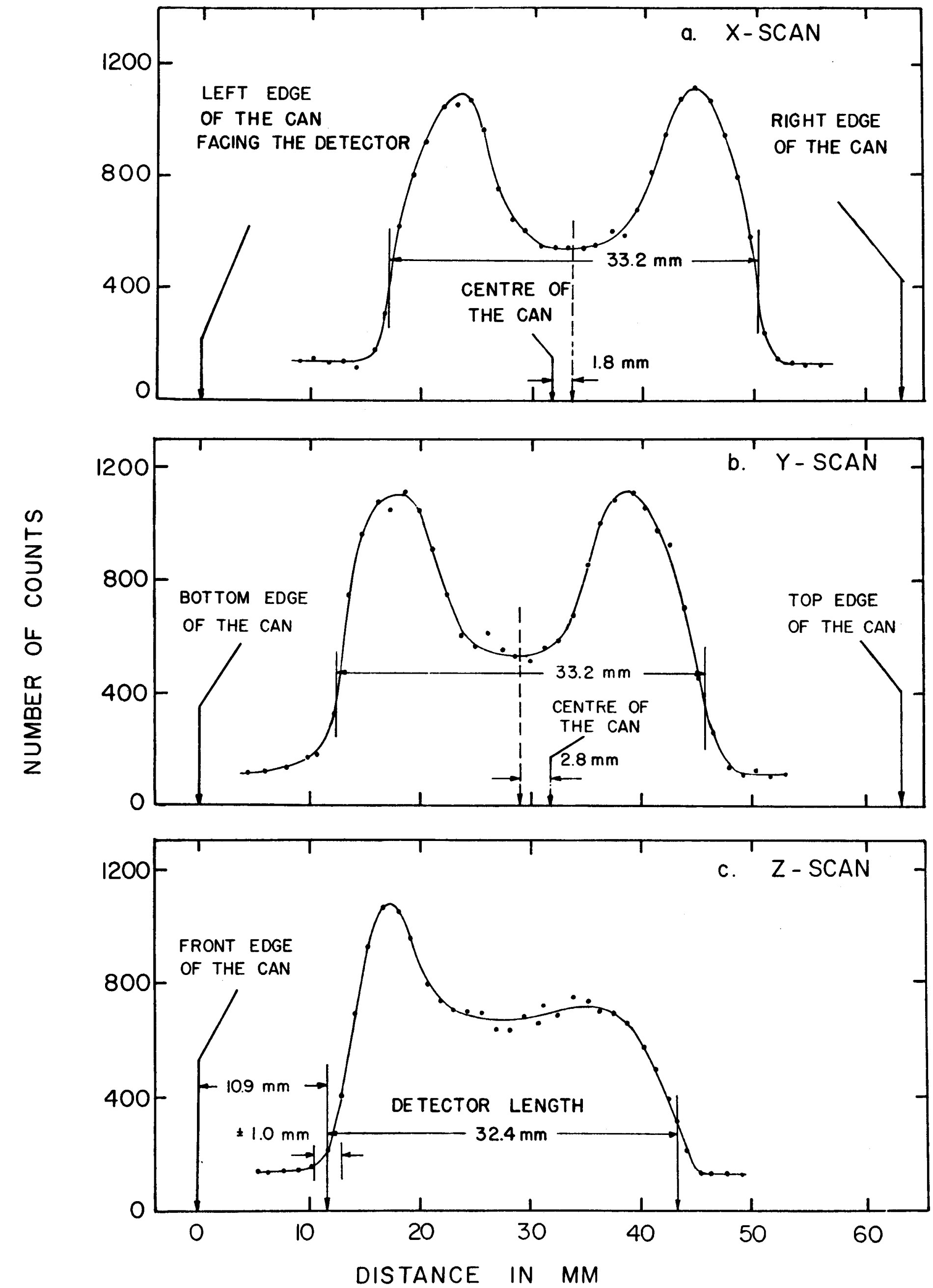

Fig. 3. Collimated beam scans of a 5-sided cylindrical 29 cm^3 Ge(Li) detector.

and 1.2% (=4×0.3%) for the 30 cm^3 detector at $h = 4$ cm. Errors of this size are somewhat small with respect to the discrepancies for these distances listed in table 1.

Actually the only severe disagreement $\overline{\delta Q_{2n}}$ between the experimental and calculated predictions at close distance occurs for the results of the 20 cm^3 Ge(Li) detector, where the values at 3 cm have been obtained by extrapolation. This is probably due to the extrapolation procedure used in ref. [1]): first the relation $R^2 \approx R_4$ was used to obtain better values $\bar{R}_2$ (and $\bar{R}_4$), after which these $\bar{R}_{2n}$ values were used to fit the straight line

$$\arccos R_{2n}(\alpha) = a\alpha + b = \text{fitted}\ (\arccos \bar{R}_{2n}), \quad (4)$$

where the $R_{2n}(\alpha)$ were calculated having solved for a

and b. Although the values of $R_{2n}(\alpha)$ so obtained were within the experimental error of the directly measured R_{2n}, the slight systematic deviations which are not serious for large source distances are greatly magnified upon extrapolation to close distances, and accordingly the Q_{2n} values obtained from $R_{2n}(\alpha)$ are subject to similar effects.

Another area where the disagreement in the two methods is apparent is seen in the average deviation $(\overline{\Delta Q_{2n}})$ for the 30 cm^3 Ge(Li) detector. This kind of deviation is most likely to occur as a result of incorrect information concerning the detector size and its location relative to the source. For example, if the absorber is not aligned properly with respect to the detector axis the Q_{2n} values measured will be low[1]). Also, if the distance of the detector from the front of the can is

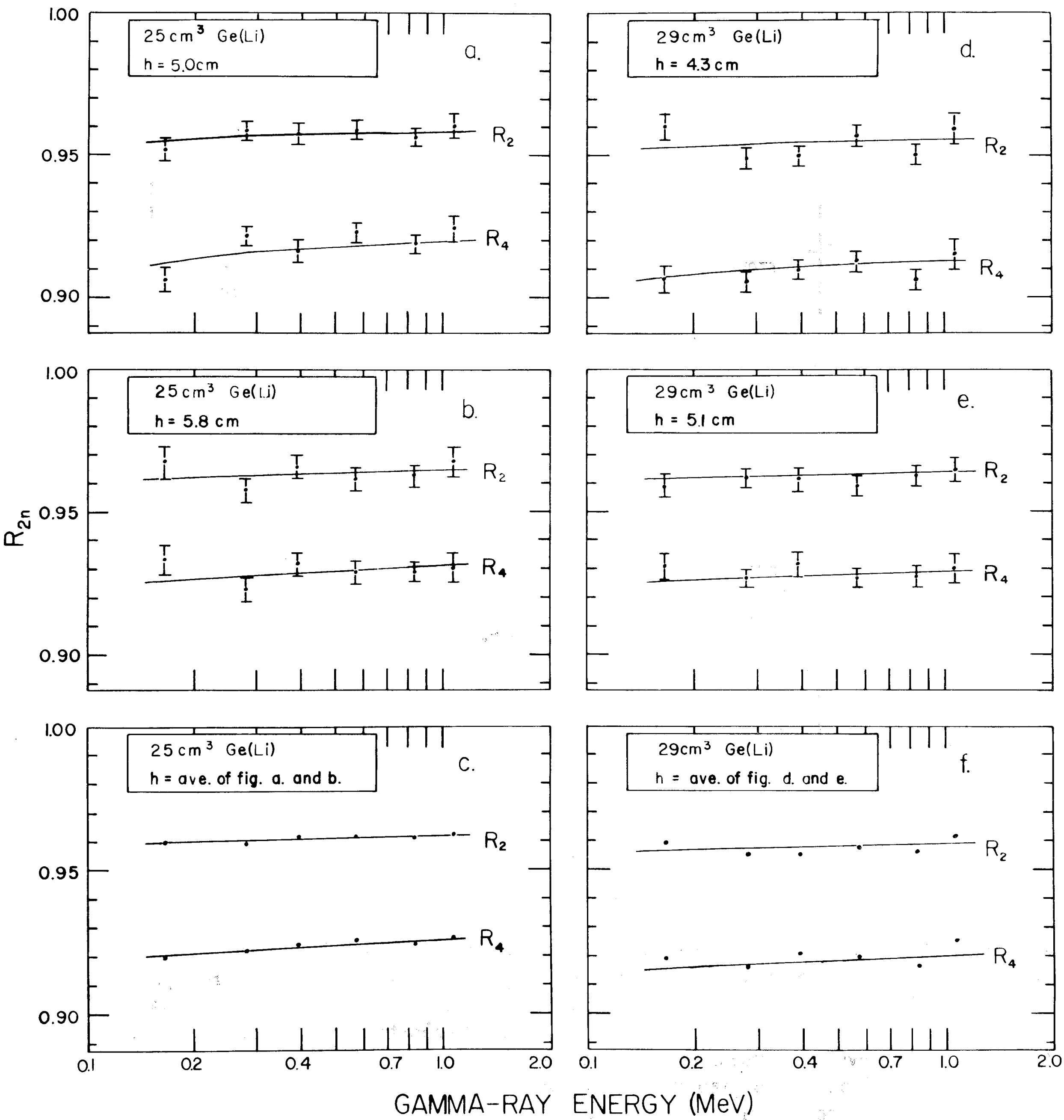

Fig. 4. Experimental values of R_{2n} measured for the 25 cm^3 and 29 cm^3 Ge(Li) detectors.

incorrect, the computer calculations cannot be expected to reproduce the experimental results. As a last example, one might suspect that source extension effects[1]) may have caused the noticeable $\overline{\Delta Q}_{2n}$ discrepancy in the 30 cm^3 detector; however, the fact that $\overline{\Delta Q}_{2n}$ is small for the 20 cm^3 detector results does not support this argument.

Having examined the limitations of both methods, it was desirable to see whether the experimental method could be improved to eliminate the discrepancies discussed above. Accordingly, the Q_{2n} values for a 25 cm^3 Ge(Li) and a 29 cm^3 Ge(Li) detector were measured utilizing the following improvements in the procedure of ref. [1]).

1. Previous to the actual Q_{2n} measurements, each detector was scanned with a collimated ^{133}Ba source to determine the center axis of the detector and the distance from the detector to the front of the can. The collimator consisted of a 5.0 cm lead cube with a 2 mm diameter hole drilled through its center. The source-collimator was mounted on a calibrated table which allowed positioning in the X, Y, and Z directions (see figs. 2 and 3) to be accurate to ± 0.25 mm. The results of the scans for the 25 cm^3 Ge(Li) and 29 cm^3 Ge(Li) detectors are given in figs. 2 and 3, respectively.

2. A more accurately constructed mount was made for the source and absorber and was equipped with a Starret dial gauge capable of measuring the source-absorber distance to an accuracy of ± 0.05 mm.

3. A number of sources were remounted on paper dots of 0.75 mm radius to insure that source extension effects would be negligible[1]).

4. The analysis was performed in a more reliable manner. Instead of applying eq. (4) and the relation $R_2^2 = R_4$ to the experimental points as described earlier, the experimental R_{2n} values were plotted and the curves were fitted subject to the condition that $R_2^2 = R_4$. As the present measurements were taken at only two distances for each detector, eq. (4) could not be utilized in the analysis; instead the results for the two distances were aveiaged to define better the shape of the curves. The experimental values of R_{2n} and the fitted curves are given in fig. 4. From these curves values of Q_{2n} were calculated and are plotted in fig. 5, along with the corresponding calculated curves of Camp and Van Lehn[2,6]).

A comparison of the results of the two methods for each detector is given in tables 3 and 4. It should be mentioned that for the 29 cm^3 Ge(Li) cylindrical detector we have used the calculated results for a trapezoidal detector with the same frontal area. As Camp and Van Lehn have shown that this is a very good approximation for $\alpha < 30°$, this treatment is quite satisfactory for the 29 cm^3 detector in these measurements as $\alpha < 20°$. From the results in the table, it appears that the discrepancy ΔQ_{2n} is negligible, indicating the general validity of the experimental method and thus suggesting that there were no undue errors in the earlier measuring technique. Evidently the assumptions concerning the "nominal" 30 cm^3 detector size or its location in the can were in error. The calculated volume, supposedly 30 cm^3 from the manufacturer's listing, is 21.7 cm^3 when calculated using the parameters supplied by the manufacturer. The smaller volume was used in the calculated Q_{2n} values, which were high compared to the experimental values. If a larger volume were used the Q_{2n} values would be lowered; thus, there is reason to suspect that the calculation utilized incorrect parameters and that the experimentally measured Q_{2n}'s would agree with these values had the correct parameters been used. Although part of the discrepancy could also be due to incorrect information concerning the location of the detector in the can, the discrepancy $\overline{\Delta Q}_{2n}$ for the nominal 30 cm^3 Ge(Li) is much too large to be accounted for by loca-

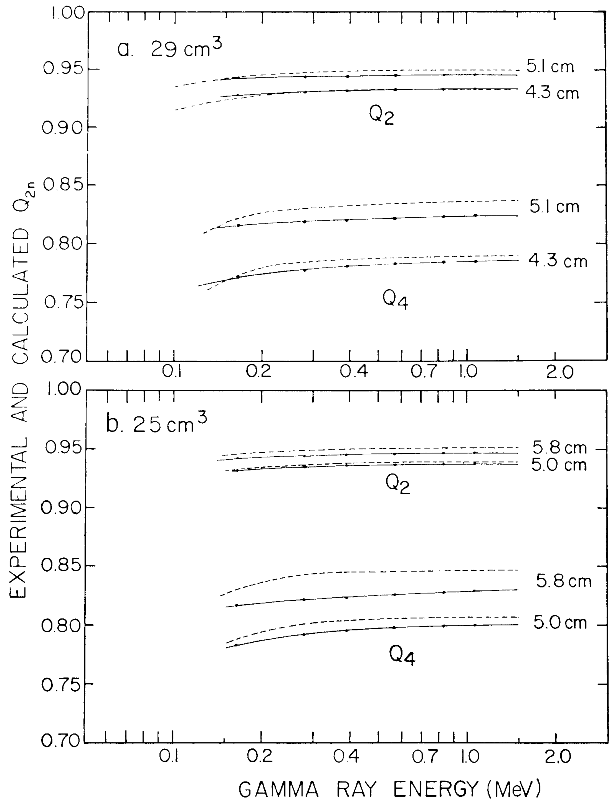

Fig. 5. Comparison of Q_{2n} from present measurements (solid curves) and those of Camp and Van Lehn (dashed curves).

TABLE 3
Comparison of experimental and calculated results from the present measurements.

Ge(Li) detector dimensions[a]	h (cm)	Observed discrepancies $\overline{\Delta Q_2}$ (%)	$\lvert\overline{\delta Q_2}\rvert$[d]	ΔQ_2 (%)	$\overline{\Delta Q_4}$ (%)	$\lvert\overline{\delta Q_4}\rvert$[d]	ΔQ_4 (%)
25 cm^3, 5-sided trapezoidal[b]	5.8	0.5	good	0.5	2.2	good	2.2
	5.0	0.2	good	0.2	1.7	good	1.7
29 cm^3, 5-sided cylindrical[c]	5.1	0.4	good	0.4	1.6	good	1.6
	4.3	0.1	good	0.1	0.7	good	0.7

[a] Same as those given by the manufacturer, except as indicated.
[b] Detailed dimensions for this detector used in obtaining values from ref. 6: frontal area 9.011 cm^2, radial depletion depth 11.00 mm, axial depletion depth 11.00 mm, length 28.00 mm, volume 24.93 cm^3, n-region 0.60 mm, height 28.00 mm, width 38.00 mm, base 33.00 mm.
[c] Detailed dimensions for this detector used in obtaining values from ref. 2: frontal area 8.522 cm^2, radial depletion depth 11.00 mm, axial depletion depth 13.20 mm, length 35.00 mm, volume 28.933 cm^3, n-region 0.60 mm, height 28.00 mm, width 36.00 mm, base 30.00 mm, diameter 32.80 mm (actual 33.20 mm).
[d] "Good" means $\lvert\overline{\delta Q_{2n}}\rvert \leqq 1$ standard deviation.

tion inaccuracies, as is exemplified by the dQ_2/dh dependence displayed in fig. 1. Thus, it is appreciated that not only are the detector scans of figs. 2 and 3 useful in determining the location of the detector in the can, but they also serve to check the size parameters of the detector; however, these parameters are not important unless one wishes to compare experimental Q_{2n} values with calculated ones.

Although the results of fig. 5 do not provide a good test for comparing the methods at close distance, it is interesting to note that in all cases the calculated Q_{2n} values appear to decrease faster than the measured Q_{2n} values as closer distances are approached. This may be an indication that the edge effects due to the approximation $\tau(E,\beta) \approx \tau(E)$ are being noticed.

TABLE 4
Summary of expected errors.

	ΔQ_2 (%)	ΔQ_4 (%)
Experimental method		
Statistical	0.5	1.7
Positioning h ($\pm$ 1 mm)	$\leqq$ 0.5	$\leqq$ 1.0
Source-absorber alignment	$\leqq$ 0.3	$\leqq$ 0.6
Total experimental error	$\leqq$ 0.77	$\leqq$ 2.1
Calculation		
Ge(Li) detector diameter error ($\pm$ 0.5 mm)	$\leqq$ 0.2	$\leqq$ 0.5
Other dimensions	small	small
Δ_{2n} (from table 2)	$\leqq$ 0.3	$\leqq$ 1.3
Total error on calculated values	$\leqq$ 0.36	$\leqq$ 1.3
Discrepancy allowed between experimental method and calculation	$\leqq$ 0.9	$\leqq$ 2.4

In conclusion, it seems apparent that no significant disagreement is observed between the two methods for moderate source distances ($\alpha < 25°$), a result which is in contrast to that implied by Camp and Van Lehn[2]). On the other hand, the difficulties which arise at close distances warrant some caution. In any event, for a typical γ-γ directional-correlation experiment, source distances much closer than those treated here are not likely in terms of the physical sizes of the detector (and its can) when coupled with lead shielding to prevent crystal-to-crystal scattering; therefore, the need for improving either method substantially for measurements at closer distances may not be serious.

We wish to thank Drs. Camp and Van Lehn for calculating the correction factors for our Ge(Li) detectors and for making their preliminary report[2]) available prior to formal publication.

References

[1]) W. G. Winn and D. G. Sarantites, Nucl. Instr. and Meth. **66** (1968) 61.
[2]) D. C. Camp and A. L. Van Lehn, Finite solid-angle corrections for Ge(Li) detectors, UCRL-71378;
D. C. Camp and A. V. Van Lehn, Nucl. Instr. and Meth. **76** (1969) 192.
[3]) J. E. Cline, IEEE Trans. Nucl. Sci. NS-**15**, no. 3 (1968) 198.
[4]) M. J. L. Yates, *Alpha-, beta- and gamma-ray spectroscopy* **2** (ed. K. Siegbahn; North-Holland Publ. Co., Amsterdam, 1965) p. 1691.
[5]) E. S. Fry, T. M. Palms and R. B. Day, Los Alamos Scientific Laboratory Report LA-3456 (1966).
[6]) D. C. Camp and A. L. Van Lehn, private communication (1969).