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ANGULAR MOMENTUM EFFECTS
IN THE COMPOUND-STATISTICAL MODEL FOR NUCLEAR REACTIONS

(II). The importance of the Fermi gas model parameters
on calculated excitation functions

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Abstract: The Monte Carlo technique has been employed in calculations of excitation functions in the framework of the evaporation model with detailed dependence of the emission probability on angular momentum. Comparison of calculated excitation functions in the ^{107}Ag+^{4}He ion system with experiment is made. The dependence of the calculated excitation functions on the Fermi gas model parameters for the level density expression is discussed. The effect of gamma-ray competition for de-excitation on the calculated excitation functions is examined. Values of the level density parameter a in the range $\approx A/12.5$ to $\frac{1}{15}A$ and of moments of inertia of the rigid body provide satisfactory agreement with experiment. A significant fraction of de-excitation by gamma-ray emission needs to be assumed in order to obtain better agreement with experiment.

1. Introduction

In a previous communication [1]), a formalism was presented for the evaporation model as derived from the compound-statistical theory for nuclear reactions. This formalism is suitable for computation of cross sections for reactions involving the evaporation of several particles and contains explicitly the dependence of the emission probability on angular momentum.

A sounder basis can be given to the evaporation model if good agreement can be obtained between theoretical calculations and experiment on all measured quantities in reactions thought to proceed via formation of a compound nucleus and induced on the same target. Such measured quantities are excitation functions, particle-evaporation spectra, isomer yield ratios, recoil ranges and angular distributions of either the emitted particles or the recoiling residual nuclei.

Calculations of these quantities should be made with a consistent set of model-dependent parameters. Following this approach we have chosen to examine in detail one reaction system, namely, ^{107}Ag+^{4}He, for which excitation function data [2]), isomer yield ratios [3]) and some evaporation spectra [4]) are available.

In this paper we compare calculated excitation functions on the reactions induced

by ^{4}He ions on ^{107}Ag with experiment. The effect on calculated excitation functions of the model parameters, namely, the level density parameter and the moment of inertia is discussed in some detail. A comparison of isomer ratios calculated with the same model parameters with experiment is presented in the following paper.

2. Calculations and comparison with experiment

A survey of the values for the model-dependent quantities in the reaction system ^{107}Ag$+^4$He from previous analyses indicates the following. The first parameter is the so-called level-density parameter a which appears in the expression for the density of nuclear levels as derived from the Fermi gas model [1]. The value of a obtained from analysis of the data using a formalism which does not include the dependence on angular momentum is given [2] by $\approx \frac{1}{27}A$, which is lower than predicted by the Fermi gas model [1]. Analysis of the ^{107}Ag(α, n) reaction, however, by Esterlund and Pate [5] using a formalism taking the angular momentum effects into account in an approximate way gave $a = \frac{1}{8}A$ if the moment of inertia was reduced to 0.4 of the rigid body. A value of $\frac{1}{56}A$ for a was obtained by Hurwitz et al. [6] in an attempt to reproduce the proton evaporation spectrum obtained by Eisberg et al. [4] in a 40 MeV ^{4}He-ion bombardment of ^{107}Ag. A value of $\frac{1}{4}A$ was obtained by Thomson [7] from his inelastic neutron data on indium. Analysis of isomer ratios in the same system by Bishop et al. [3] gave a value of $\frac{1}{7}A$ for a if the moment of inertia was taken as $\frac{3}{4}$ of the rigid-body value. Such discrepancies in the a-values can be found in analyses of other reaction systems as well.

The second parameter is the value for the moment of inertia which enters in the dependence of the level density on angular momentum [1]. According to the Fermi-gas model this quantity should have the rigid-body value. On the other hand, in the superconductor model for nuclear level densities [8,9] the moment of inertia is an increasing fraction of the rigid-body value with increasing energy up to about 16 MeV of excitation where the Fermi gas value of $\mathscr{I}/\mathscr{I}_r = 1$ is predicted. Esterlund and Pate [5] have observed a systematic decrease of $\mathscr{I}/\mathscr{I}_r$ with increasing atomic number when they took angular momentum effects partially into account and attempted to fit excitation functions of the type (α, n). Values of $\mathscr{I}/\mathscr{I}_r$ as low as 0.35 were obtained. Dudey and Sugihara [10] find a systematic decrease of $\mathscr{I}/\mathscr{I}_r$ with decreasing average excitation energy in calculations of isomer yield ratios.

It can be seen that the latter effect is implicitly included, at least in part, in eq. (24) of ref. [1] for the density of levels, since reduction of $\approx 20\%$ in an effective moment of inertia with decreasing excitation can be obtained from eq. (25) of ref. [1].

The third important factor that should be considered is the magnitude of de-excitation by gamma rays in competition with particle evaporation. The effect of this on excitation functions was considered by Grover [11] and discussed by Sarantites and Pate [1].

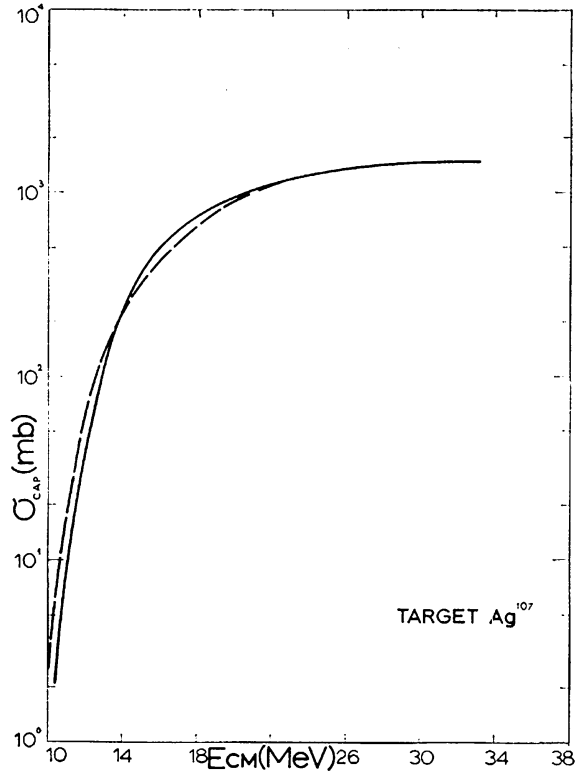In the present calculations the optical-model transmission coefficients used give

Fig. 1. Total reaction cross section for ^{107}Ag$+^4$He ions. The dashed curve represents the sum of the experimental cross sections. The solid line gives the reaction cross section calculated via the optical model.
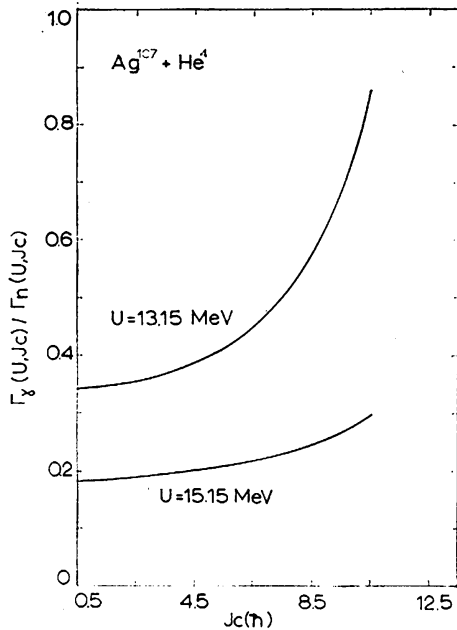

Fig. 2. Ratios of the emission functions $\Gamma_\gamma(U, J_c)/\Gamma_n(U, J_c)$ in the ^{111}In compound nucleus for $U = 13.15$ and 15.15 MeV, as a function of J_c. The parameters employed provide excitation functions in agreement with experiment.

a total reaction cross section (solid line) shown in fig. 1 in comparison with the experimental curve obtained from the data of Fukushima *et al.* [2]) (broken line) corrected for the non-measurable (α, p) reaction. The agreement is satisfactory, but the reported excitation functions were normalized to the experimental total reaction cross section to minimize discrepancies introduced by the optical model.

Using parameters, to be described shortly, we have calculated the emission functions $\Gamma_x(U, J_c)$ for protons, neutrons and gamma rays. The ratio $\Gamma_\gamma(U, J_c)/\Gamma_n(U, J_c)$ in the ^{111}In compound nucleus is given in fig. 2 as a function of J_c for two excitation energies 13.15 and 15.15 MeV.
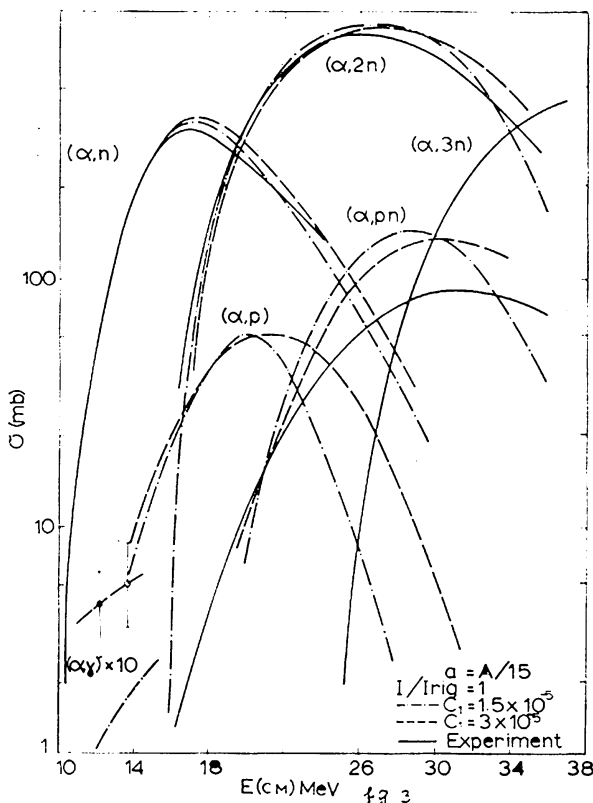

Fig. 3. Comparison of calculated excitation functions in ^{107}Ag + ^{4}He ions with experiment. The solid lines represent the data of Fukushima *et al.* The broken lines give the calculated excitation functions with $a \approx 1/15A$, $\mathscr{I}/\mathscr{I}_r = 1$ and $c_1 = 3 \times 10^{-5} \cdot \text{erg}^{-4} \text{sec}^{-1}$. The dot-and-dash lines give the effect of a reduction of c_1 to $1.5 \times 10^{-5} \text{erg}^{-4} \cdot \text{sec}^{-1}$.

The ratio Γ_γ/Γ_n increases more and more with increasing J_c because the rotational energy cut-off affects much more strongly $\Gamma_\gamma(U, J_c)$ than $\Gamma_n(U, J_c)$. For higher excitation energies the ratio $\Gamma_\gamma(U, J_c)/\Gamma_n(U, J_c)$ is appreciably smaller and only at considerably higher J_c values does gamma-ray emission become a competitor. For the reaction system under discussion, practically at all energies of interest, neutron emission dominates in the first chance evaporation. This is in agreement with the fact that the (α, n) and $(\alpha, 2n)$ reactions are the predominant ones as seen from the solid curves

of fig. 3 which represent the best lines drawn through the data of Fukushima *et al.* [2]). Further, the (α, pn) cross section is lower than the (α, 2n) cross section by an order of magnitude, and one might expect the emission of the neutron to precede that of the proton. A detailed examination of our cascade data indicates that in the majority of the cascades proton emission precedes neutron emission. This may be rationalized in terms of the Coulomb barrier penetration for the proton since proton emission would be inhibited at low excitation present in second chance emissions. Thus the (α, pn) reaction is not likely to affect the (α, 2n) yield. The fraction of emissions leading to the (α, n) product, however, is influenced by several factors.
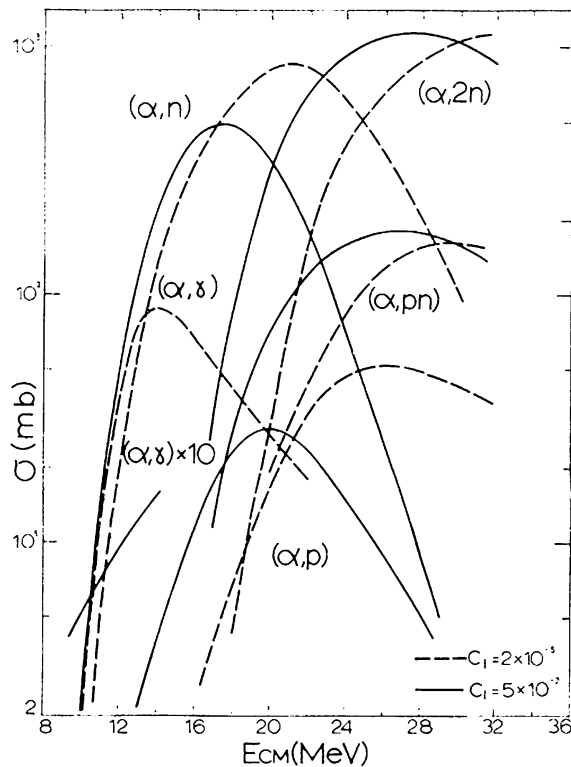

Fig. 4. The effect of the level density parameter a on calculated excitation functions in ^{107}Ag$+^4$He ions. The dashed curve corresponds to $a = A/6.7$ MeV^{-1} and $c_1 = 2 \times 10^{-5}$ erg^{-4} sec^{-1} to be compared with the dashed curves of fig. 3 with $a = 1/15A$ MeV^{-1} and $c_1 = 3 \times 10^{-5}$ erg^{-4} sec^{-1}. The solid lines correspond to $a = A/6.7$ MeV^{-1} and $c_1 = 5 \times 10^{-7}$ erg$^{-4} \cdot$ sec^{-1}. In the former case gamma-ray competition was highly overestimated.

For excitation energies below the (α, 2n) reaction threshold, the competing reactions (α, p) and (α, γ) are small so that the magnitude and shape of the (α, n) excitation function are in the present case practically independent of these competing modes. With increasing excitation, however, the competing modes influence the magnitude of the (α, n) cross section. For excitations higher than ≈ 22 MeV, the major contribution to the (α, n) cross section comes from cascades where after the first neutron emission there is still enough energy to emit a second neutron [12]). The majority of

such first evaporations is followed by gamma-ray emission and only a small fraction proceeds by emission of a proton instead. In what follows a is calculated using eq. (17) of ref. [1]) with $r_0 = 1.17$ fm, and $\mathscr{I}/\mathscr{I}_r$ taken as unity.

In fig. 3 we show the calculated excitation functions using an a-value which corresponds to about $\frac{1}{15}A$ ($r_0 = 1.17$ fm) and two values 3×10^{-5} and 1.5×10^{-5} erg$^{-4} \cdot$ sec^{-1} for the parameter c_1, which determines the strength of the assumed electric dipole emission [1]). The results are represented by the dashed and the dot-and-dash curves, respectively. The values for the separation energies and the pairing energies used are those of Hillman [13,14]) except that the neutron and proton separation energies S_n and S_p for ^{110}Cd were slightly modified in order to reproduce better the (α, pn) cross section. Table 1 gives the values used. The two data points of Bishop

TABLE 1

Particle separation energies and condensation energies used in the calculation of the cross sections for ^{107}Ag+^{4}He ions (energies in MeV)

Emitting nucleus	S_n	S_p	S_α	δ (Hillman)
^{111}In	9.963	4.834	1.935	1.22
^{110}In	7.722	4.953	1.655	0
^{109}In	10.343	6.710		1.09
^{108}In		4.622		0
^{110}Cd	10.400	8.600	2.644	2.55
^{109}Cd	7.199	8.186	2.493	1.33
^{108}Cd	10.536	8.115		2.43
^{107}Cd		7.094		1.33
^{109}Ag	9.302	6.710		1.22
^{108}Ag	7.128	6.375		0
^{107}Ag	9.514	5.544		1.09
^{106}Ag		5.960		0
^{108}Pd	8.967	9.882		2.62
^{107}Pd	6.297	9.214		1.40
^{106}Pd	9.931	9.339		2.49
^{105}Pd				1.40

et al. [3]) for the (α, γ) reaction and the calculated cross sections are shown in fig. 3 in a tenfold scale. The overall agreement with experiment is quite satisfactory in view of the limitations introduced by the requirement to fit all the measured excitation functions. It seems that the value of 1.5×10^{-5} erg$^{-4} \cdot$ sec^{-1} for c_1 gives a better fit to the (α, n) reaction but it underestimates the (α, γ) cross section by a factor of three, yet this is only within two standard deviations assigned to the experimental points. It is further seen that this two-fold increase in the gamma-ray emission strength results in an increase of the (α, n) cross section at the expense of the $(\alpha, 2n)$ as expected. However, this effect is small in the (α, n) reaction but the (α, p) cross section was more strongly affected.

We now proceed to discuss the effect of the level-density parameter a on the cal-

culated excitation functions. As mentioned earlier, the value of a used so far is given by eq. (17) of ref. [1]) with $r_0 = 1.17$ fm and is approximately equal to $\frac{1}{15}A$ MeV^{-1}. In another calculation the value of $a = A/6.7$ MeV^{-1} was used. This value corresponds to a nuclear radius parameter $r_0 = 1.64$ fm. The value of c_1 used was 2×10^{-5} erg$^{-4} \cdot$ sec^{-1}. The results are shown in fig. 4 as dashed curves. It is interesting to observe how much the cross section for (α, n) is overestimated over the $(\alpha, 2n)$ reaction. The cross section for the (α, γ) reaction is now overestimated by approximately two orders of magnitude, although the same c_1 value was used. The decreasing part of the (α, n) curve has now a rather high slope. A reduction in the value for c_1 to 5×10^{-7} erg$^{-4} \cdot$ sec^{-1} with $a = A/6.7$ reproduces the (α, γ) cross section and gives the excitation functions shown in fig. 4 (solid lines). Now both the (α, n) and $(\alpha, 2n)$ cross sections are considerably closer to experiment. It is interesting to notice that in the case of a equal to $A/6.7$ there is an apparent shift in the (α, n) and $(\alpha, 2n)$ thresholds to higher energies associated with a rather drastic increase in c_1. For small changes, however, in c_1 near the value that reproduces the (α, γ) cross section the position of the thresholds is not strongly dependent on the c_1 value. It is obvious that as one lowers the assumed gamma competition its effect becomes increasingly less significant. These calculations agree with the result obtained by Grover [11]) that there are pairs of values of a and c_1 which give excitation functions close to the experimental value. There is however evidence from the present data that the slope of the tail of the (α, n) reaction depends upon the value of a, even when gamma-ray competition is taken into account. The cross section for the (α, n) reaction decreases less with increasing energy for smaller a-values. The same effect has of course been observed in calculations that do not include angular momentum effects. The values of a however, for which we obtain satisfactory agreement with experiment are in the range $A/12.5$ to $\frac{1}{15}A$ which correspond to values of r_0 in the range 1.22–1.17 fm. These a-values are considerably higher than those obtained in the non-angular momentum dependent analyses and are in agreement with the predictions of the Fermi gas model. The associated values for c_1 are 5×10^{-5} and 3×10^{-5} erg$^{-4} \cdot$ sec^{-1} and the magnitude of the gamma competition is significant, ranging from a few percent up to $> 90\%$ depending on J_c for excitations of a few MeV above the particle threshold.

We shall now discuss the effect of the assumed value for the moment of inertia on the excitation functions. The values of a and c_1 used in the calculations to be discussed next were again $\frac{1}{15}A$ MeV^{-1} and 3×10^{-5} erg$^{-4} \cdot$ sec^{-1}, respectively. These values not only give a good agreement with the experimental excitation functions but also reproduce better the experimental proton-evaporation spectrum as determined by Eisberg et al. [4]). We have performed a calculation where we reduced the value of $\mathscr{I}/\mathscr{I}_r$ from unity to 0.6 for all compound nuclei involved. The results are shown in fig. 5 as broken lines. The solid lines in this figure correspond to $\mathscr{I}/\mathscr{I}_r = 1$, $a = \frac{1}{15}A$ and $c_1 = 3 \times 10^{-5}$ erg$^{-4} \cdot$ sec^{-1}. It is apparent that the (α, n) cross section is now increased at the expense of the $(\alpha, 2n)$ but the slope at the tail of the (α, n) has not

changed. In addition the (α, p) cross section is now decreased in the rising with energy part and the (α, pn) has increased somewhat. The overall agreement with experiment in this case of $\mathscr{I}/\mathscr{I}_r = 0.6$ is worse. In another calculation we reduced the value of $\mathscr{I}/\mathscr{I}_r$ to 0.5 for ^{110}In only. This nucleide is the product of the $(\alpha, 2n)$ reaction and it was expected that this would increase the yield of the (α, n) tail because of the decrease of the number of high angular momentum levels available for the second neutron evaporation. The results are presented as the dot-and-dash lines in fig. 5 and as it is seen the (α, n) reaction has now a higher yield at higher energies. The $(\alpha, 2n)$ reaction
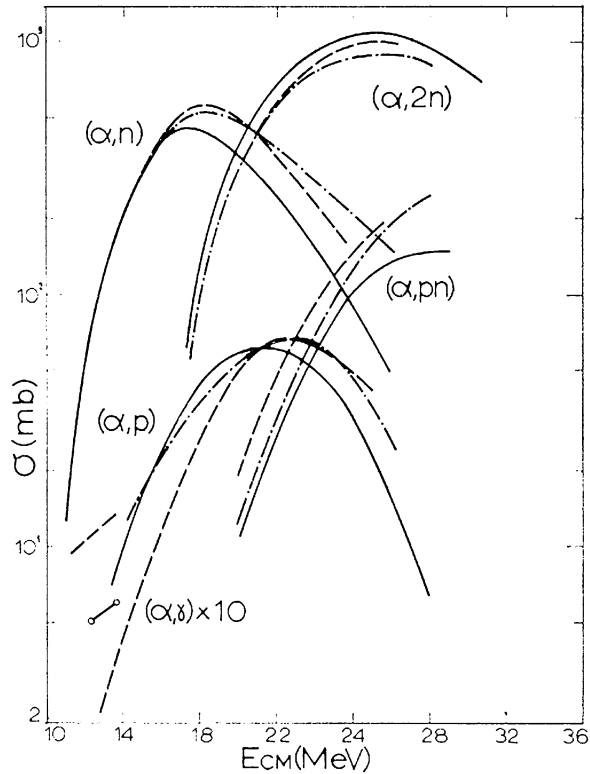

Fig. 5. The effect of the moment of inertia parameter $\mathscr{I}/\mathscr{I}_r$ on calculated excitation functions in ^{107}Ag$+^4$He ions. The solid curves correspond to $a = \frac{1}{15}A$, $c_1 = 3 \times 10^{-6}$ erg$^{-4} \cdot$ sec^{-1} and $\mathscr{I}/\mathscr{I}_r=1$. The dashed curves were obtained when $\mathscr{I}/\mathscr{I}_r$ was reduced to 0.6 for all compound nuclei involved. The dot-and-dash curves were obtained when $\mathscr{I}/\mathscr{I}_r$ was reduced to 0.5 for only ^{110}In, the $(\alpha, 2n)$ product nucleus.

has decreased at all energies and the (α, p) to (α, pn) have both increased at high energies. A major part of latter changes may be due to readjustments in the magnitude of the major reactions. Thus a reduction in the $(\alpha, 2n)$ cross section may raise the (α, p) or (α, pn) when one normalizes to the same total reaction cross section. It is worth observing that the (α, n) cross section is increased also near the $(\alpha, 2n)$ threshold when the moment of inertia is decreased which indicates that not only the emission to the relatively high angular momentum states is affected by this cut-off but also to the lower ones, so that the overall emission function is affected by this as well.

Finally, we have performed a calculation where the gamma emission was taken to be entirely of the electric quadrupole type. No significant difference over the results represented by the solid lines in fig. 5 were observed.

In conclusion it appears that in the framework of this formalism, satisfactory agreement between calculated excitation functions and experiment is obtained with model parameters as predicted by the Fermi gas model. The parameter values $a \approx \frac{1}{15}A$ and $c_1 = 3 \times 10^{-5} \cdot \mathrm{erg}^{-4} \mathrm{sec}^{-1}$ with $\mathscr{I}/\mathscr{I}_r = 1$ gave good agreement with experiment. Reduction of the moment of inertia below the rigid-body value was not found necessary. The superconductor model for the density of levels in the nucleus is more realistic and the dependence of $\mathscr{I}/\mathscr{I}_r$ on excitation energy may provide slightly better agreement with experiment. When isomer yield ratios and particle evaporation spectra are evaluated using the parameters above, reasonable agreement with experiment is observed. Further calculations on other reaction systems where these three types of measurements are available need to be performed in order to generalize the above conclusions as applicable to all reactions proceeding via the compound nucleus mechanism.

References

1) D. G. Sarantites and B. D. Pate, Nuclear Physics **A93** (1967) 545
2) S. Fukushima et al., Nuclear Physics **41** (1963) 275
3) C. T. Bishop, J. R. Huizenga and J. P. Hummel, Phys. Rev. **135** (1964) B401
4) R. Eisberg, G. Igo and H. Wegner, Phys. Rev. **100** (1955) 1309
5) R. A. Esterlund and B. D. Pate, Nuclear Physics **69** (1965) 401
6) C. Hurwitz, S. J. Spencer, R. A. Esterlund, B. D. Pate and J. B. Reynolds, Nuclear Physics **54** (1964) 65
7) D. B. Thomson, Phys. Rev. **129** (1963) 1649
8) D. W. Lang, Nuclear Physics **42** (1963) 353
9) H. K. Vonach, R. Vandenbosch and J. R. Huizenga, Nuclear Physics **60** (1964) 70
10) N. D. Dudey and T. T. Sugihara, Phys. Rev. **139** (1965) B896
11) J. R. Grover, Phys. Rev. **127** (1962) 2142
12) D. G. Sarantites, Nuclear Physics **A93** (1967) 576
13) M. Hillman, Brookhaven National Laboratory Report BNL-846 (T-333) unpublished
14) M. Hillman, private communication (1965)