The Role of Human Capital in Shaping Immigrant Earnings (Preliminary and Incomplete)

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Abstract

We develop a model that accounts for several stylized facts about the evolution of immigrant earnings. First, it implies that new immigrants earn less than natives with a similar level of schooling but that their income grows faster. Second, it predicts that the earnings gap and the rate at which it closes is a function of both age and country of origin. Finally, it implies that migrants have steeper age-earnings profile than stayers.

We use the model to explore how the nature of migration costs influences the selection of migrants. Finally, we report some quantitative implications from a calibrated version of the model.

1 Introduction

It is a well established fact that earnings of immigrants and natives are not equal even when they have similar levels of schooling. Moreover, the evidence suggests that the dynamics of earnings over the life cycle are different for natives and migrants. We summarize the evidence in the following "facts"[1]:

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[1]In section 3 we provide references and more precise definitions of some terms that can reasonably be viewed as somewhat vague.

- **Migrants vs. Stayers**. Migrants and stayers with the same schooling have different age-earnings profiles. Upon migration, the earnings gap increases with age.

- **Migrants vs. Natives**. Even after controlling for schooling and other observable characteristics, migrants' earnings are initially significantly lower than natives'. Over time, the gap narrows (i.e. there is assimilation). The gap in initial earnings is negatively related to the level of GDP per capita in the country of origin.[2]

This paper describes a version of the human capital model originally developed by Ben Porath (1967) augmented with the decision to migrate and uses it to understand the forces that can account for these observations. Our work can be viewed as a dynamic version —with endogenous schooling and on-the-job accumulation of human capital— of the more standard Roy model that is the workhorse of the migration related literature.

In this version we explore the theoretical implications of the model in the case of unanticipated migration. We show that, at least qualitatively, the predictions of the theory match the evidence. When comparing migrants and stayers the key mechanism is that these two groups face different prices and, hence, they make different decisions in terms of on-the-job training efforts which, in turn, result in a steeper age-earnings profiles for the individuals who migrate to the high wage location.

We then study the implications of the model for the differences between migrants and natives. A key insight is that in the model if two individuals chose to acquire the same level of schooling in different economic environments, then they cannot be identical. Moreover, in a model that allows education to be two dimensional —with a quantity dimension as measured by years of schooling, and a quality dimension that we view as a better measure of human capital— the individual in the low wage location chooses, in equilibrium, a lower quality of schooling and this, in turn, implies that he has lower human capital than a native at the time of migration. These two results explain both the lower initial earnings —because initial human capital is lower— and the steeper age-earnings profiles —because the migrant must have had higher innate ability to choose the same years of schooling in a lower return environment. Moreover, since the "quality" of human capital,

[2]Recent immigrant cohorts to the U.S. do not show the rate of assimilation of earlier cohorts. We discuss possible reasons in section 2.2

holding years of schooling constant, varies positively with the level of output per worker in the country of origin, the model is consistent with the observation that migrants from poorer locations earn less than migrants from higher income regions.

We describe the migration decision and show that, in this model, selection is two-dimensional. Individuals differ in terms of human capital and ability —a measure of the potential gains from on the job training— and depending on the transferability of human capital there cab be positive or negative selection. Our model implies that, when human capital is highly transferable, workers must be positively selected in terms of human capital, while if human capital is not very transferable only the low human capital high innate ability individuals choose to migrate.

We study the impact on earnings of temporary migration. Formally we show that a higher probability of returning to the sending (low wage) country results in flatter age earnings profiles and, hence, in lower assimilation of migrants.

We present some very preliminary results from a calibrated version of the model to understand whether human capital and selection can account for the evidence on the earnings of immigrants. Even though our exercise is very preliminary, we find the results encouraging as the model seems consistent with the data in some dimensions. However, the quantitative exercise also shows some other aspects of the model that need improvement.

2 Theoretical Results

In this section we describe the basic model, characterize its solution, and describe the implications for earnings.

2.1 Unexpected Migration: Individual Decision Problem

Here we present a sketch of the model. A more thorough treatment of the optimization problem can be found in Manuelli and Seshadri (2014). The representative individual maximizes the present discounted value of net income. We assume that each agent lives for T periods and retires at age

$R \leq T$. The maximization problem is

$$\max_{x_s, x_w, x_E, n, s} \int_6^R e^{-r(a-6)}[wh(a)(1-n(a)) \tag{1}$$
$$-(\aleph_{[n(a)=1]} p_s x_s(a) + (1-\aleph_{[n(a)=1]}) p_w x_w(a))] da - p_E x_E$$

subject to

$$\dot{h}(a) = z_h [n(a)h(a)]^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \tag{2}$$

$$n(a) \in [0,1] \text{ or } n(a) \in \{1\} \cup [0, \bar{n}] \qquad a \in [6, R) \tag{3}$$

and

$$h(6) = h_E = h_B x_E^{\upsilon} \tag{4}$$

with h_B given.[3] Here, $\aleph_{[n(a)=1]}$ is an indicator function that takes the value one whenever the individual allocates 100% of his time to schooling. Even though the notation is somewhat cumbersome, it is convenient to allow for the price of the inputs used in the production of human capital to vary depending on whether the individual is in or out of school.

Equations (2) and (4) correspond to the standard human capital accumulation model initially developed by Ben-Porath (1967). This formulation allows for both market goods, $x_j(a)$ $j \in \{s, w\}$, and a fraction $n(a)$ of the individual's human capital, to be inputs in the production of human capital. We assume that in the "schooling period," which we identify with the length of time that the individual optimally chooses $n(a) = 1$, the price of market inputs used in the production of human capital is p_s, while in the working period, the unit price of market inputs that increase human capital is p_w. Investments in early childhood[4], which we denote by x_E (e.g. medical care, nutrition and development of learning skills), determine the level of each individual's human capital at age 6, $h(6)$, or h_E for short.[5] Our formulation captures the idea that nutrition and health care are important

[3] The solution to the income maximization problem is also the solution to a utility maximization problem when the number of children is given, parents have a bequest motive, and bequests are unconstrained. For details, see Manuelli and Seshadri (2014).

[4] The emphasis on early childhood as one of the important determinants of human capital formation follows Carneiro, Cunha and Heckman (2003) who also specify a production technology in which goods and time invested by parents affect children's human capital.

[5] It should be made clear that if market goods ($x_j(a)$ and x_E) are produced using the same technology as the final goods production function, then our formulation implies that the production function for human capital is more labor intensive than the final goods technology.

determinants of early levels of human capital, and those inputs are, basically, market goods.[6]

There are two important features of our formulation. First, we assume that the human capital accumulation technology is the same during the schooling and the training periods. Second, we assume that the market inputs used in the production of human capital —$x_j(a)$, $j \in \{s, w\}$— are privately purchased. In the case of the post-schooling period, this is not controversial. However, this is less so for the schooling period. Here, we take the 'purely private' approach as a first pass.[7] In fact, for our results to go through it suffices that, at the margin, individuals pay for the last unit of market goods allocated to the formation of human capital.

The full solution to the income maximization problem is in Manuelli and Seshadri (2014). The solution to the problem is such that $n(a) = 1$, for $a \leq 6 + s$. Thus, we identify s as *years of schooling.* In what follows, we assume that the economy is at the steady state and, hence, that the relevant prices are fixed and given by the marginal products.[8][9]

We next describe the implications of this simple investment model for the age earnings profiles of "similar individuals," in the sense that they have exactly the same years of schooling, that differ in their destination countries (i.e. we compare migrants with stayers) and in the environment in which they acquired their education (i.e. we compare migrants with natives).

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[6] It is clear that parents' time is also important. However, given exogenous fertility, it seems best to ignore this dimension. For a full discussion see Manuelli and Seshadri (2009). We allow for the possibility that the price of the inputs needed to produce early childhood human capital, p_E in our notation, is not necessarily equal to the price of general consumption.

[7] An alternative explanation is that Tiebout like arguments effectively imply that public expenditures on education play the same role as private expenditures. The truth is probably somewhere in between.

[8] In this version we abstract from financial constraints and assume that financial or family markets do not keep individuals from making efficient investments in human capital. Even though there is some evidence that financial constraints do not seem to be very important in influencing schooling decisions in the U.S. it is less clear that this is a resonable assumption in the case of very poor countries. However, it seems that a natural first step should be to understand the workings of the model in the absence of frictions other than those imposed by the technology.

[9] In addition to the basic model that assumes a smooth transition between school and work, we present quantitative results corresponding to an alternative version in which individuals who work must allocate a minimum number of hours to actually working (as opposed to on-the-job training).

2.1.1 Migrants vs. Stayers

Consider now two individuals in the home country who have completed their desired level of schooling and have exactly the same level of human capital (and equal schooling) at age a_m. We identify the "home" country with a vector of prices: after tax wages, w, and the price of market inputs used in on-the-job training, p_w. As a matter of notation we use * to indicate the same variables in the destination (high wage) country.

Manuelli and Seshadri (2014) show that the amount of human capital accumulated after age a_m —the age of migration— in the destination country is a function of the prices prevailing in the location in which investment takes place. The level of human capital at age a in location (w, p_w) is

$$h(a) = e^{-\delta_h(a-a_m)}h(a_m) + \left(\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}z_h\left(\frac{w}{p_w}\right)^{\gamma_2}\right)^{1/(1-\gamma)} w \int_{a_m}^{a} e^{-\delta_h(a-t)}\left(\frac{m(t, r+\delta_h)}{r+\delta_h}\right)^{\frac{\gamma}{1-\gamma}} dt, \qquad a \geq a_m, \tag{5}$$

where

$$m(a, r+\delta_h) = 1 - e^{-(r+\delta_h)(R-a)},$$

Let $h_m(a)$ and $h_s(a)$ denote the human capital level of the migrant and the stayer respectively. Define the difference in human capital between migrants and stayers by

$$\Delta_h^s(a, a_m; \varepsilon) = h_m(a) - h_s(a).$$

It follows that

$$\Delta_h^s(a, a_m; w^*, p_w^*, w, p_w) = \left[\left(\frac{w^*}{p_w^*}\right)^{\frac{\gamma_2}{1-\gamma}} w^* - \left(\frac{w}{p_w}\right)^{\frac{\gamma_2}{1-\gamma}} w\right] (\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}z_h)^{1/(1-\gamma)} \int_{a_m}^{a} e^{-\delta_h(a-t)}\left(\frac{m(t, r+\delta_h)}{r+\delta_h}\right)^{\frac{\gamma}{1-\gamma}} dt$$

Assuming that the retirement age —R in the model— is the same in the origin and destination countries we find:

1. The larger the difference between $(w^*/p_w^*)^{\frac{\gamma_2}{1-\gamma}} w^*$ and $(w/p_w)^{\frac{\gamma_2}{1-\gamma}} w$, the larger the human capital differentials. Depending both on the the level of the wage rate and the cost of training, the differences need not be proportional to the TFP of the origin and destination countries

2. The function $\Delta_h^s(a, a_m; w^*, p_w^*, w, p_w)$ has the following properties:

   (a) It is increasing in a for $a = a_m$ and decreasing for $a = R$.

   (b) There exists a unique $\hat{a} < R$ (independent of (w^*, p_w^*)) that maximizes $\Delta_h^s(a, a_m; w^*, p_w^*, w, p_w)$ over a.

   (c) It is concave in a whenever it is increasing (and at $\hat{a}$) and possibly convex for $a = R$.

   (d) It is decreasing in a_m.

3. Since the basic Ben-Porath model implies that the level of schooling —given all prices— is increasing in ability, z_h in the model, human capital differences are larger for those with higher levels of schooling.

One important message from these results is that the differences between the levels of human capital of migrants and stayers depend in a complex way on the differences in environments, and that the actual differences are not independent of both age and experience in the destination country.

Earnings of a worker are given by

$$y(a; w, p_w) = wh(a)(1 - n(a)) - p_w x_w(a).$$

In the appendix we show that optimal behavior implies that given the level of human capital at the time of migration, $h(a_m)$, the difference in the earnings of a migrant and a stayer defined as

$$\Delta_y^s(a, a_m; w^*, p_w^*, w, p_w) = [y(a; w^*, p_w^*) - y(a; w, p_w)]$$

is given by

$$\Delta_y^s(a, a_m; w^*, p_w^*, w, p_w) = (w^* - w)e^{-\delta_h(a-a_m)}h(a_m) + (\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}z_h)^{1/(1-\gamma)} \left[\left(\frac{w^*}{p_w^*}\right)^{\frac{\gamma_2}{1-\gamma}} w^* - \left(\frac{w}{p_w}\right)^{\frac{\gamma_2}{1-\gamma}} w\right]\theta(a, a_m), \quad (6)$$

where the function $\theta(a, a_m)$ (defined in the Appendix) is such that

$$\theta(a_m, a_m) < 0, \;\; \theta(R, a_m) > 0, \frac{\partial\theta}{\partial a}(a, a_m) > 0, \;\text{ and }\; \frac{\partial\theta}{\partial a_m}(a, a_m) < 0$$

Equation (6) summarizes the implication of the model for the differences in age earnings profiles between migrants and stayers. These differences have two components. First, the migrant's earnings at any age have to be scaled up by the differences in wage rates. This is the standard implication of exogenous (or learning-by-doing with indivisible labor) human capital models. Second, there is a change in the age earnings profile associated with the response of the migrant to the prices he faces in the destination location. The most interesting implications are:

1. The "scale" estimate overpredicts earnings of the migrant at the time of migration. This follows from the fact that $\theta(a_m, a_m) < 0$. This, in turn, reflects the decision on the part of the migrant to engage in more (relative to the stayer) on activities that result in higher future earnings.

2. Over time, the gap between earnings of the migrant and the stayer increase and, at least close to retirement, the differences exceed the differences in wage rates. Thus, the shape of the age earnings profile of migrants and stayers is different, with migrants exhibiting a steeper age-earnings profile. Measurement of differences among older migrants would tend to overestimate actual productivity differentials.

3. The differences in age-earnings profiles are not independent of the age at migration, a_m. In particular, the older the migrant, the flatter his post-migration age-earnings profile.

Even though human capital differences peak at some pre-retirement age, no such non-monotonicity appears in age earnings profiles.

We find these results interesting because they suggest that evidence on the age-earnings profiles of migrants relative to stayers can be used to evaluate whether a model in which investing in human capital is a "rival" activity (e.g. the Ben-Porath model) is more or less consistent with the data relative to the "complementarity" view of on-the-job training (e.g. the learning-by-doing model). We view the learning-by-doing model as not being able to account for the differences in the shape of the age-earnings profiles.

2.1.2 Migrants vs. Natives

In this section we compare natives and immigrants with the *same level of schooling*. We concentrate on the case in which migration is, as before, completely unexpected and it takes place after the individual has completed his schooling (in the native location). Moreover, we assume that the relevant price differentials are such that the migrant chooses not to go back to school.[10]

Unlike the previous section the level of human capital of the native and the migrant at the age of migration, $h_n(a_m)$ and $h_m(a_m)$ respectively, are not in general equal. Thus, the differences in age earnings profiles are driven by both the differences in initial human capital as well as differences, if any, in the human capital accumulation decisions by the two individuals after age a_m.

Let the difference in income between the two individuals be

$$\Delta_y^n(a, a_m; w^*, p_w^*) = y_m(a; a_m, w^*, p_w^*) - y_n(a, 0; w^*, p_w^*),$$

where $y_j(a, a_m; w^*, p_w^*)$ is earnings of individual $j \in \{n, m\}$ at age a who migrated at age a_m. It follows that

$$\begin{aligned}\Delta_y^n(a, a_m; w^*, p_w^*) &= w^* e^{-\delta_h(a-a_m)} \left(h_m(a_m) - h_n(a_m)\right) + \\ &\quad \left(z_{h,m}^{1/(1-\gamma)} - z_{h,n}^{1/(1-\gamma)}\right) \left(\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\right)^{1/(1-\gamma)} \left(\frac{w^*}{p_w^*}\right)^{\frac{\gamma_2}{1-\gamma}} w^* \theta(a, a_m),\end{aligned}$$

where $z_{h,m}$ ($z_{h,n}$) are the ability levels of a migrant and a native with the same level of schooling.

We summarize the results in the following proposition:

Proposition 1 *Assume that all prices (other than the wage rate) are equal across countries. Let n and m denote variables corresponding to a native and an immigrant to a higher wage location with* ***the same years of schooling****, and let a_m be the age at which m migrates. Assume that $(1-\alpha_1)\upsilon < \alpha_2$ then:*

1. *At the time of migration, the level of human capital of the native exceeds that of the migrant. Formally, $h_n(a_m) > h_m(a_m)$.*
2. *The innate ability of the immigrant is higher than the ability of the native, that is, $z_{h,m} > z_{h,n}$.*

[10] Basically we assume that in the new environment, the optimal solution to the income maximization problem is interior. When ruling out corners we are effectively ruling out not only individuals who transition from being workers in the location of origin to being students in the new location, but also individuals whose first "job" is an unpaid internship.

3. *At the time of migration, the income of the immigrant is lower than the income of the native,*

$$\Delta_y^n(a_m, a_m; w^*, p_w^*) < 0.$$

4. *The income gap between migrant and natives narrow as a function of experience in the destination country,*

$$\frac{\partial \Delta_y^n}{\partial a}(a, a_m; w^*, p_w^*) > 0$$

5. *The income gap between immigrants and natives is inversely related to the wage rate (level of development) of the sending country,*

$$\frac{\partial \Delta_y^n}{\partial w}(a, a_m; w^*, p_w^*) < 0$$

6. *The return to experience in the migrants home location is negative,*

$$\frac{\partial y_m}{\partial a_m}(a; a_m, w^*, p_w^*) < 0$$

   and this gap is increasing in innate ability and the difference in wage rates between the sending and receiving locations.

Proof. [See Appendix] ■

The intuition for the results is as follows. Since the migrant makes his schooling decision in a low wage country he has to have more innate ability (higher z_h) in order to choose to acquire the same number of years of schooling as the rate of return to schooling is lower. However, since the "shadow" unit cost of schooling is a combination of the wage rate and market goods, the relative price of market goods is higher in the sending location. This implies that, optimally, fewer market goods are used in producing human capital. Thus, even though both individuals have spent the same amount of time in school, the migrant has done so in lower quality schools (as measured by the amount of x_s) than the native and, hence, his human capital at the end of the schooling period is lower. Thus, the initial difference in human capital at the time of migration "explains" the lower wage.

Part two of the proposition shows that the migrant has higher innate ability than the native. Thus, when faced with the same relative prices as the native he rationally chooses to invest more in increasing his human capital (he is more efficient at investing). This has two effects. First, it further contributes to the lower initial earnings as the migrant chooses more "investment friendly" employment options. Second, it implies that the higher human capital accumulated by the migrant results in a steeper age-earnings profile and, hence, in the narrowing of the gap.

Finally, the model predicts that the amount of human capital at the age of migration, $h_m(a_m)$, depends positively on the wage rate in the location of origin. Thus, larger wage (or TFP) gaps result in larger differences in earnings upon migration. Moreover, experience in the sending location has a negative impact on the level of income. In section 3 we describe evidence that is consistent with the implications of the model.

Before we continue, it is relatively straightforward to relax the assumption of equal prices for the inputs in the production of human capital in the three stages: early childhood, schooling and on-the-job training. A brief summary of the effects is as follows (throughout we keep the wage differentials constant):

1. The higher the price of early childhood human capital, p_E, in the sending location the higher the gap in human capital levels at the time of migration. Higher p_E also imply that the gap in innate skills, $z_{h,m} - z_{h,n}$ is even larger. This implies larger initial differences and faster catch up rates.

2. Holding p_w/p_s constant, higher levels of p_w imply that the gap between $z_{h,m}$ and $z_{h,n}$ increases.

3. Finally, it is interesting to highlight the role played by early childhood human capital: If all individuals had exactly the same human capital at age 6 (i.e. in our formulation $\upsilon = 0$) then, conditional on choosing to attain a given level of schooling, the level of human capital at age $6 + s$ would be independent of all prices. Thus, even though the qualitative predictions of the model would be the same (it is still the case that $z_{h,m} > z_{h,n}$), we suspect that the model without early childhood human capital will have difficulty matching the evidence.[11]

[11] At this point we find the evidence on the role of early childhood somewhat difficult to evaluate. On the one hand, Heckman and co-authors (reference needed) claim that most of the variance in human capital is explained by variance in early childhood capital. At the

2.1.3 The Migration Decision: Selection

In the migration literature one of the most studied questions relates to the nature of migration and, in particular, whether migrants are positively or negatively selected (see Borjas (1987) and Grogger and Hanson (2011) for somewhat opposing views). In order to define selectivity we study migration from the "w" country to the "w^*" country. We assume that if the individual decides to migrate he incurs two types of costs: He loses a fraction η of his human capital, and he has to pay a fixed cost C. It is straightforward to assume that the fixed cost depends on the wage rates of both the home and the foreign country. The key assumption is that it is not age dependent.[12]

A standard argument shows that the present discounted value of income of an individual of age a who is out of school, has human capital h, and lives in the "w^*" location is

$$V(h, a; z_h, w^*, p_w^*) = V_0(a; z_h, w^*, p_w^*) + V_1(a; w^*)h$$

where the function $V(h, a; z_h, w^*)$ gives the present discounted value of income of an a year old individual who has human capital h, who lives in a country with prices (w^*, p_w^*), and who innate ability z_h.

In the Appendix we show that

$$V_1(a; w^*) = w^* \frac{m(a; r + \delta_h)}{r + \delta_h}, \tag{7}$$

and

$$\begin{aligned} V_0(a; z_h, w^*, p_w^*) &= (1-\gamma)\left(\gamma_1^{\gamma_1}\gamma_2^{\gamma_2} z_h \left(\frac{w^*}{p_w^*}\right)^{\gamma_2}\right)^{\frac{1}{1-\gamma}} \\ &\quad (w^*)^{-\frac{\gamma}{1-\gamma}} \int_a^R e^{-r(t-a)} V_1(t; w^*)^{\frac{1}{1-\gamma}} dt, \end{aligned} \tag{8}$$

The interpretation of these two components of the value function is straightforward. The term $V_1(a; w^*)$ captures the payoff to additional units of human capital. The term $V_0(a; z_h, w^*, p_w^*)$ is the present discounted value of the

other end, Schoellman (2012) finds that the educational attainment of refugees that arrive in the U.S. does not depend on whether the arrival date is right after birth (which implies that they acquire "U.S. early childhood human capital," or at age five which means that they have "foreign" (and presumably lower quality) early childhood human capital.

[12] Note that, implicitly, there is age dependency in our cost function as the human capital cost, $\eta h(a)$, depends on the individual's age.

"profits" associated with on-the-job training. Its value depends —aside from the prices— both on the individual's ability, z_h, his age, a, and the degree of returns to scale implicit in the Ben-Porath technology. If the technology displays constant returns to scale (i.e. $\gamma = 1$) then $V_0(a; z_h, w^*, p_w^*) = 0$.[13]

The average individual chooses to migrate if and only if[14]

$$\underbrace{(w^*(1-\eta) - w)\frac{m(a, r+\delta_h)}{r+\delta_h}h}_{\text{Direct payoff to human capital}} \geq C - \left[\underbrace{V_0(a; z_h, w^*, p_w^*) - V_0(a; z_h, w, p_w)}_{\text{Profits associated with training}}\right] \tag{9}$$

Equation (9) captures the mechanisms at work when making the migration decision. To be consistent with the model, we assume that after the individual leaves school his human capital and his ability to learn on the job are given by

$$h(6+s) = \hat{h}(6+s)(1+\upsilon), \text{ and } z_h = \hat{z}_h(1+\varsigma)^{15}$$

where $\hat{h}(6+s)$ is the level of human capital that the individual chooses and $h(6+s)$ is the effective amount that gets rewarded in the labor market. Similarly, z_h is the ability to learn on the job which we take to be related to the ability to learn while in school. It is natural to assume that υ and ς are zero mean. The presence of shocks creates heterogeneity in the population both in terms of human capital and ability to learn on the job and allows us to discuss selection

There are two cases to consider in analyzing equation (9).

1. **Case I**: $(w^*(1-\eta) - w) > 0$. In this case equation (9) implies that, for each z_h, the set of migrants is the set of individuals who satisfy

$$h \geq \underbrace{A_0(a, \eta, w^*, w)}_{+} - (z_h)^{\frac{1}{1-\gamma}} \underbrace{A_1(a, \eta, w^*, p_w^*, w, p_w)}_{+}. \tag{10}$$

   Not surprisingly, this condition says that there is a trade off (for the marginal individual who is indifferent between migrating or staying)

[13] In this case the model implies, counterfactually, that age earnigs profiles are downward sloping.

[14] In the Appendix we allow for a random shock component to the payoff associated with migrating and discuss how the results get mapped into more standard selection criteria.

[15] Note that so far we have set $h(6+s) = \hat{h}(6+s)$ and $z_h = \hat{z}_h$ to simplify the presentation. It is straightforward to extend the analysis to this random case.

between his human capital and his innate ability: The higher the innate ability the lower the minimum human capital required for migration.

How does the set of potential migrants changes with age and TFP in the country of origin? Age has a positive effect on quality. Formally, the older the worker the boundary function in equation (10) moves to the northeast. The intercept increases and the slope is flatter. Thus, the set of older migrants —holding the cost of migration constant— is "better" in the sense that it excludes some low human capital-low innate ability that would migrate at a younger age.

The effect of country of origin TFP is exactly the opposite. A lower level of TFP —which corresponds to lower w— shifts the boundary described by equation (10) to the southwest and it increases the set of possible migrants by adding some low human capital-low innate ability individuals.

Equation (9) implies that, as $a \to R$, the payoff to human capital and the profits from training converge to zero. Thus, the average relatively older worker will not migrate. Given that human capital is bounded, there is an age beyond which no individual will choose to migrate for economic reasons.

It is simple to translate this criterion in terms of selection based on income at the time of migration. The analogue of equation (10) is the condition that, if given the opportunity to migrate, an individual will choose to migrate if and only if his income $y(a, h; z_h)$ **exceeds** a threshold y_I^L that has the form

$$\begin{aligned} y_I^L(a, z_h, \eta, w^*, p_w^*, w, p_w) &= \underbrace{D_{I,0}^L(a, \eta, w^*, w)}_{+} \\ &\quad -(z_h)^{\frac{1}{1-\gamma}} \underbrace{D_{I,1}^L(a, \eta, w^*, p_w^*, w, p_w)}_{+}. \end{aligned} \tag{11}$$

The impact of the different variables on y_I^L is summarized by

$$y_I^L(\underset{+}{a}, \underset{-}{z_h}, \underset{+}{\eta}, \underset{+}{w^*}, \underset{-}{p_w^*}, \underset{-}{w}, \underset{+}{p_w}).$$

In this case older workers are positively selected in terms of income but individuals with higher innate ability have a lower income threshold.

2. **Case II:** $(w^*(1-\eta)-w) \leq 0$. In this case, human capital is not easily transferable and the pattern of migration is different. The relevant condition is

$$h \leq \underbrace{B_0(a,\eta,w^*,w)}_{-} + (z_h)^{\frac{1}{1-\gamma}} \underbrace{B_1(a,\eta,w^*,p_w^*,w,p_w)}_{+}. \qquad (12)$$

The boundary described by equation (12) implies that there is a minimum level of innate ability, $z_h^L(a,w)$, that is consistent with migration. Low ability individuals will not migrate. However, given that a potential migrant has high innate ability he will only choose to migrate if his human capital is low. Thus, in this case, selection is positive on innate ability and negative on human capital.

As in the previous case, age plays a role: the older the potential migrant the higher the minimum ability (i.e. $z_h^L(a,w)$ is increasing in a). The set of possible migrants also depends on TFP (w) of the country of origin. In this case the lower the level of w the lower the minimum innate ability (i.e. $z_h^L(a,w)$ is increasing in w).

The reason for this outcome is simple: When $w^*(1-\eta) < w$ the effective price of human capital is lower in the destination country. In this case only individuals who have relatively low human capital and can reap high payoffs from training on the job will choose to migrate.

The model implies that in the case of near "North-North" migration (i.e. when $w^* > w$ but $w^*(1-\eta)-w < 0$) then only high skill-low human capital individuals will migrate. As in the previous case, older workers will not migrate.

As in the previous case it is possible to translate the condition in equation (12) into an equivalent condition in terms of income at the time of migration. In this case the migration criteria is to migrate only if $y(a,h;z_h)$ **falls short** of y_{II}^L where

$$\begin{aligned} y_{II}^L(a,z_h,\eta,w^*,p_w^*,w,p_w) &= \underbrace{D_{II,0}^L(a,\eta,w^*,w)}_{-} + \\ &\quad (z_h)^{\frac{1}{1-\gamma}} \underbrace{D_{II,1}^L(a,\eta,w^*,p_w^*,w,p_w)}_{+}. \end{aligned} \qquad (13)$$

Our results imply that selection is effectively three-dimensional and depends on wage differentials, returns to training and age. Second, the model ascribes a very definite role to age, as older worker are "different" in terms both of their human capital and innate ability than their younger counterpart. In terms of income, the model implies that the decision can be summarized by a simple threshold. However, that threshold is dependent on age, the difference in development between the sending and receiving country and innate ability which implies that estimating counterfactual wage distributions (i.e. where in the wage distribution in the country of origin the migrant would have been) does not provide a lot of information on performance after migration.

The model suggests that there is a difference in the type of migrants depending on whether the migration is North-North or South-North. In the first case, migrants are negatively selected with respect to income but high innate ability. In the second case, immigrants are positively selected with respect to income but negatively with respect to innate ability.

2.2 Transitory Migration: Impact on Human Capital Accumulation

In this section we study the human capital accumulation decision of a migrant who is out of school in a country characterized by the vector (w^*, p_w^*) and that with probability λdt will return to country (w, p_w).Note that if we view the * country as the destination high wage location, then λdt captures the probability that an exogenous shock will result in a migrant returning to this home country or of being deported in the case of an illegal immigrant.

The post-return migration present discounted value of income is given by $V(h, a; w, p_w)$. Standard arguments show that the value function is of the form

$$V(h, a; w, p_w) = V_0(a; w, p_w) + V_1(a; w)h,$$

with $V_0(a; w, p_w)$ and $V_1(a; w)$ given by equations (7) and (8) evaluated at (w, p_w).

Let $M(h, a; w^*, p_w^*, w, p_w, \lambda)$ be the present discounted value of income of a worker who resides in the * country and who will return to his home country with probability λdt. The HJB equation for a potential mover satisfies

$$
\begin{aligned}
rM(h, a; w^*, p_w^*, w, p_w, \lambda) = \max_{n,x} \{ w^* h(1-n) - p_w^* x \\
+ \frac{\partial M}{\partial h}(h, a; w^*, p_w^*, w, p_w, \lambda)[z_h (n(a)h(a))^{\gamma_1} x(a)^{\gamma_2} - \delta h(a)] \\
+ \frac{\partial M}{\partial a}(h, a; w^*, p_w^*, w, p_w, \lambda) \\
+ \lambda [V(h, a; w, p_w) - M(h, a; w^*, p_w^*, w, p_w, \lambda)].
\end{aligned}
$$

We conjecture that

$$M(h, a; w^*, p_w^*, w, p_w, \lambda) = M_0(a; w^*, p_w^*, w, p_w, \lambda) + M_1(a; w^*, w, \lambda)h.$$

It follows that

$$M_1(a; w^*, w, \lambda) = (w^* - w)\frac{m(a; r + \delta_h + \lambda)}{r + \delta_h + \lambda} + w\frac{m(a; r + \delta_h)}{r + \delta_h}, \qquad (14)$$

and

$$\frac{\partial M_1}{\partial \lambda}(a; w^*, w, \lambda) < 0.$$

An increase in the probability of moving to a lower wage location reduces the incentives to accumulate human capital in the high wage country. Moreover,

$$
\begin{aligned}
\lim_{\lambda \to 0} M_1(a; w^*, w, \lambda) &= V_1(a; w^*) \\
\text{and} \lim_{\lambda \to \infty} M_1(a; w^*, w, \lambda) &= V_1(a; w),
\end{aligned}
$$

that is, in the case of no return migration ($\lambda = 0$) the value of human capital is given by the prices in the current location while in the case of instantaneous return migration ($\lambda = \infty$) the marginal value of human capital coincides with the value in the home (lower wage) country.

Simple calculations show that $\partial M_1/\partial w > 0$, $\partial^2 M_1/\partial w \partial a < 0$, and $\partial^2 M_1/\partial \lambda \partial a > 0$.

Let $h(a; \lambda)$ be the human capital of a migrant that faces a return probability λdt to a low wage location. In what follows, functions indexed by λ describe the optimal choices of the migrant in the temporary migration case.

Standard calculations show that total investment in human capital defined as

$$k(a; \lambda) = w^* n(a; \lambda) h(a; \lambda) + p_w^* x_w(a; \lambda)$$

is given by

$$k(a;\lambda) = \gamma \left(\gamma_1^{\gamma_1} \gamma_2^{\gamma_2} z_h \left(\frac{w^*}{p_w^*} \right)^{\gamma_2} \right)^{\frac{1}{1-\gamma}} (w^*)^{-\frac{\gamma}{1-\gamma}} M_1(a; w^*, w, \lambda)^{\frac{1}{1-\gamma}}.$$

Higher probability of return implies lower values of $M_1(a; w^*, w, \lambda)$ and, consequently, lower investment in human capital on the part of the temporary migrant. Labor income is given by

$$y(a;\lambda) = w^* h(a;\lambda) - k(a;\lambda) \tag{15}$$

where

$$h(a;\lambda) = h(a_I) e^{-\delta_h (a-a_I)} + \left(\gamma_1^{\gamma_1} \gamma_2^{\gamma_2} z_h \left(\frac{w^*}{p_w^*} \right)^{\gamma_2} \right)^{\frac{1}{1-\gamma}} (w^*)^{-\frac{\gamma}{1-\gamma}} \int_{a_I}^{a} e^{-\delta_h (a-t)} M_1(t; w^*, w, \lambda)^{\frac{\gamma}{1-\gamma}} dt \tag{16}$$

and a_I is an arbitrary initial period. In what follows we take $a_I = a_m$, the age at migration but, in general, it corresponds to the age at which the individual believes he will return with probability λdt

Increases in λ unambiguously lower $h(a;\lambda)$ and this lowers income. However, it also decreases investment in human capital —the second term in equation (15)— and this tends to increase income. As it turns out the strength of these two opposing forces depend on age (given a_m).

The age earnings profile is given by (omitting the arguments of the M_1 function)

$$\frac{\partial y}{\partial a}(a;\lambda) \equiv \dot{y}(a;\lambda) = \left(\gamma_1^{\gamma_1} \gamma_2^{\gamma_2} z_h \left(\frac{w^*}{p_w^*} \right)^{\gamma_2} \right)^{\frac{1}{1-\gamma}} (w^*)^{-\frac{\gamma}{1-\gamma}} M_1^{\frac{\gamma}{1-\gamma}} \left(w^* - \dot{M}_1 \right) - \delta_h h(a;\lambda), \tag{17}$$

where

$$\dot{M}_1(a; w^*, w, \lambda) \equiv \frac{\partial M_1}{\partial a}(a; w^*, w, \lambda)$$

Proposition 2 *Assume that* $\gamma > 1/2$[16]

[16] In our calibration γ is always in this range. Values of γ less thatn 1/2 would imply unrealitically steep age earnings profiles.

1. *There exists an* $\bar{a}$ *such that for all* $a \leq \bar{a}$, $\partial \dot{y}(a;\lambda)/\partial\lambda < 0$.

2. *If* $\delta_h = 0$ *then* $\bar{a} = R$.

Proof. (See Appendix) ■

Proposition 3 *Assume that* $\gamma > 1/2$

1. *There exists an* $\tilde{a}$ *such that for all* $a \leq \tilde{a}$, $\partial \dot{y}(a;\lambda)/\partial w > 0$.

2. *If* $\delta_h = 0$ *then* $\tilde{a} = R$.

Proof. (See Appendix) ■

These two results have implications for the rate of assimilation of immigrants. They imply:

1. The higher the probability of return migration the lower the rate of assimilation.

2. The higher the productivity gap between the current (i.e. destination) country and the sending country the lower the rate of assimilation.

In comparing rates of assimilation across countries these results suggest that if the average immigrant comes from a poorer country and the probability of return migration is high, then the average assimilation rate will be lower. In the quantitative section we explore the how these two factors might have contributed to the lower rate of assimilation observed in recent immigrant cohorts in the U.S. (see Borjas (2014))

2.3 Rate of Return to Schooling [Missing]

In this section we show that the model implies that the standard return to schooling in the destination country is an increasing function of the wage (and hence output per worker) in the country of origin.]

2.4 Expected Migration

In this section we study how the probability of migrating to a higher wage country affects the human capital accumulation decisions of everybody in the sending country. As before, we study the decision of an individual who resides in the (w, p_w) country and that with instantaneous probability υdt can migrate to the (w^*, p_w^*) (high wage) country. Since we only consider individuals who are out of school their migration decision —conditional on getting an "offer"— is the same as above: migrate if

$$\underbrace{(w^*(1-\eta)-w)\frac{m(a,r+\delta_h)}{r+\delta_h}h}_{\text{Direct payoff to human capital}} \geq C - \left[\underbrace{V_0(a;z_h,w^*,p_w^*)-V_0(a;z_h,w,p_w)}_{\text{Profits associated with training}}\right]. \tag{18}$$

Let

$$\begin{aligned} G(h,a,z_h) &= \max\left\{\underbrace{w^*(1-\eta)\frac{m(a,r+\delta_h)}{r+\delta_h}h+V_0(a;z_h,w^*,p_w^*)}_{V^*(a,h)},\right. \\ &\qquad \left.\underbrace{w\frac{m(a,r+\delta_h)}{r+\delta_h}h+V_0(a;z_h,w,p_w)}_{V(a,h)}-C\right\}. \end{aligned}$$

As before, here are two cases to consider

1. **Case I**: $(w^*(1-\eta)-w)>0$. In this case, for each z_h, the set of migrants is the set of individuals who satisfy

$$h \geq \underbrace{A_0(a,\eta,w^*,w)}_{+} - (z_h)^{\frac{1}{1-\gamma}}\underbrace{A_1(a,\eta,w^*,p_w^*,w,p_w)}_{+} \equiv A(a,z_h) \tag{19}$$

2. **Case II:** $(w^*(1-\eta)-w)\leq 0$. In this case, human capital is not easily transferable and the pattern of migration is different. The relevant condition is

$$h \leq \underbrace{B_0(a,\eta,w^*,w)}_{-} + (z_h)^{\frac{1}{1-\gamma}}\underbrace{B_1(a,\eta,w^*,p_w^*,w,p_w)}_{+} \equiv B(a,z_h). \tag{20}$$

For simplicity, we only look at Case 1, since the other is similar. The relevant HJB equations are, for $h \geq A(a, z_h)$

$$rV^H(h,a) = \max_{n,x}\{wh(1-n) - p_w x$$
$$+\frac{\partial V^H}{\partial h}(h,a)[z_h(n(a)h(a))^{\gamma_1}x(a)^{\gamma_2} - \delta h(a)]$$
$$+\frac{\partial V^H}{\partial a}(h,a)$$
$$+\lambda[V^*(a,h) - V^H(h,a)],$$

and for $h \leq A(a, z_h)$

$$rV^L(h,a) = \max_{n,x}\{wh(1-n) - p_w x$$
$$+\frac{\partial V^L}{\partial h}(h,a)[z_h(n(a)h(a))^{\gamma_1}x(a)^{\gamma_2} - \delta h(a)]$$
$$+\frac{\partial V^L}{\partial a}(h,a)$$
$$+\lambda[V(a,h) - V^L(h,a)].$$

Claim: If an individual chooses not to migrate at age a, then he will not migrate at any other age $a' \geq a$.

In this case (if the claim is true) then $V^L(h,a) = V(h,a)$. Thus, finding the function $V^H(h,a)$ is equivalent to solving the PDE

$$rV^H(h,a) = \max_{n,x}\{wh(1-n) - p_w x$$
$$+\frac{\partial V^H}{\partial h}(h,a)[z_h(n(a)h(a))^{\gamma_1}x(a)^{\gamma_2} - \delta h(a)]$$
$$+\frac{\partial V^H}{\partial a}(h,a)$$
$$+\lambda[V^*(a,h) - V^H(h,a)],$$

for

$h \geq A(a, z_h)$ with boundary condition $V^H(h,a) = V(h,a)$ for $h = A(a, z_h)$

2.5 The Impact of Wage Uncertainty

In this section we extend the model to allow for wages that evolve according to the following process

$$dw_t = \mu w_t dt + \sigma w_t dZ_t.$$

The HJB equation for an individual who is out of school (and setting $p_w = 1$) is given by

$$rV(h,a,w) = \max\left\{ w(1-n)h - x + \frac{\partial V}{\partial h}(h,a,w)[z_h(nh)^{\gamma_1}x^{\gamma_2} - \delta_h h] + \frac{\partial V}{\partial a}(h,a,w) + \frac{\partial V}{\partial w}(h,a,w)\mu w + \frac{\partial^2 V}{\partial w^2}(h,a,w)\frac{\sigma^2}{2}w^2 \right\}$$

If we ignore the constraint $n \leq 1$ (should we worry about this? Probably yes) we conjecture that the solution is of the form

$$\begin{aligned} V(h,a,w) &= w\underbrace{\frac{1-e^{-(r-\mu+\delta_h)(R-a)}}{r-\mu+\delta_h}}_{V_0(a)}h + \\ &\quad w^{\frac{1-\gamma_1}{1-\gamma}}\underbrace{(1-\gamma)\left(z_h\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\right)^{\frac{1}{1-\gamma}}\int_a^R e^{-\rho(r,\mu,\sigma)(t-a)}\left(\frac{1-e^{-(r-\mu+\delta_h)(R-t)}}{r-\mu+\delta_h}\right)^{\frac{1}{1-\gamma}}dt}_{V_1(a)} \end{aligned}$$

where

$$\rho(r,\mu,\sigma) = \frac{(1-\gamma_1)(r-\mu) - \gamma_2(r+\frac{\sigma^2}{2})}{1-\gamma}.$$

Thus, the solution is of the form

$$V(h,a,w) = V_0(a)wh + V_1(a)w^{\frac{1-\gamma_1}{1-\gamma}}$$

If the derivation is correct, then it follows that $V_0(a)$ depends on μ but not on σ while the present value of on-the-job learning, $V_1(a)$, increases with σ. Thus,

$$\frac{\partial V}{\partial \sigma}(h,a,w) > 0$$

and hence that regions with more wage variability have higher expected payoff. Note that this higher payoff is not associated with having higher h.

Differences in σ have no impact on the market value of h. The impact of σ works through its impact on human capital accumulation and this depends only on z_h. Thus, high σ regions would attract high z_h (and maybe relatively low h) individuals.

If in the U.S. there was a decrease in the effective σ then this would have changed the pool of migrants toward more low z_h workers and if the reference native group is unchanged, then it would imply less catch-up.

In this setting, the stochastic model for human capital, wage rates and income is given by the following three equations

$$
\begin{aligned}
dh_t(a) &= \left[\frac{1}{V_0(a)}\left(V_0(a)z_h\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\right)^{\frac{1}{1-\gamma}} w^{\frac{\gamma_2}{1-\gamma}} - \delta_h h_t(a)\right] dt, \\
dw_t &= \mu w_t dt + \sigma w_t dZ_t, \\
y_t &= w_t h_t(a) - \gamma\left(V_0(a)z_h\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\right)^{\frac{1}{1-\gamma}} w^{\frac{1-\gamma_1}{1-\gamma}}.
\end{aligned}
$$

Since $dt = da$ and applying Ito's lemma to the equation defining income we get that (need to double check for possible algebraic mistakes)

$$dy_t = \hat{\mu}(h_t(a), w_t, a)dt + \hat{\sigma}(h_t(a), w_t, a)dZ_t$$

where

$$
\begin{aligned}
\hat{\mu}(h, w, a) &= wh(\mu - \delta_h) + w^{\frac{1-\gamma_1}{1-\gamma}}\left(V_0(a)z_h\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\right)^{\frac{1}{1-\gamma}} \times \\
& \left[\frac{1}{V_0(a)}\left(1 - \frac{\gamma}{1-\gamma}\dot{V}_0(a)\right) - \frac{\gamma(1-\gamma_1)}{1-\gamma}\left(\mu + \frac{\gamma_2}{1-\gamma}\frac{\sigma^2}{2}\right)\right]
\end{aligned}
$$

and

$$\hat{\sigma}(h, w, a) = \sigma\left[wh - \frac{\gamma(1-\gamma_1)}{1-\gamma}\left(V_0(a)z_h\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\right)^{\frac{1}{1-\gamma}} w^{\frac{1-\gamma_1}{1-\gamma}}\right].$$

Note that $\hat{\sigma}(h, w, a)$ is convex in w. Thus, the model implies more variability (conditional on h) at high income levels. Higher σ increases both $\hat{\sigma}(h, w, a)$ and $\hat{\mu}(h, w, a)$. To do: Look at the impact of μ (recall that it enters in $V_0(a)$)

3 Confronting the Evidence

In this section we discuss some evidence on the earnings of migrants and use the model to explore if the quantitative predictions match the data.

3.1 Migrants vs. Stayers

Jasso and Rosenzweig (2002) analyze earnings of immigrants in their country of origin and in the U.S. They report differences in wages that correspond to average 'before' and 'after' migration wages. Their findings suggest that, on average, a migrant obtains a PPP adjusted 83% increase over his home country earnings. The average value masks a substantial amount of heterogeneity. For example, Jasso and Rosenzweig report that 24% of the sample earned less in the U.S. than in their country of origin.

Ambrosini and Peri (2012) study the performance of Mexican immigrants to the U.S. and find that the imputed wage premia typically exceeds 400% although adjusting for the fact that actual migrants earned, on average, 23 % less than non-migrants prior to migration should reduce this estimate somewhat. They also find that the migration premium decreases with the level of schooling: individuals with more than 12 years of schooling earn a premium that is about 1/2 of the premium earned by Mexican with less than four years of schooling.[17]. Bauer et. al. find some evidence of steeper age earnings profiles for Portuguese immigrants in Germany relative to stayers (Table 4).

3.2 Migrants vs. Natives

There is a large literature on the economic performance of immigrants. Here we present a sample of the findings. Even though some qualitative features like the existence of an initial wage or income penalty experienced by immigrants and a decrease in the gap with respect to natives as a function of years since migration some results appear to be quite robust, the estimates of the strength of those effects and others show a wide range of values. Below, we selectively describe some of the more recent estimates.

Initial Wage Differentials Borjas (2014) presents estimates of the initial income gaps for different immigrant cohorts to the U.S. The values range from 24% to 33%. Once differences across cohorts in education and age are accounted for the initial income penalties vary from 13% to 16%.

[17] Ambrosini and Peri rely on the ACS with matching occupations between US and Mexican data. The model, does not assume (or imply) that a doctor in Mexico will be a doctor in the U.S. It is possible that after migrating, the Mexican doctor will choose to be a nurse.

Lubotsky (2007) using longitudinal data finds initial gaps close to the estimates in Borjas (2014) for the pre-1980 period. Using an adjustment to the date at which an individual arrived (but not controlling for differences in schooling), he estimates that the penalty for the 1985-89 cohort was 44%. Rho (2013) finds that recent (from the 1995-1999 migration cohort) immigrants to the U.S. who arrived when they were between 31 and 35 years old had income losses relative to natives that vary depending on the level of schooling, ranging from 18% for those with a bachelor's degree to 30% for high school graduates. Older migrants (age 41-45) experience income losses that range from 39% (high school dropouts) to 54 % (graduate degree).

Lemos (2013), using longitudinal data from the U.K. finds that there is a significant gap in initial earnings of immigrants (they earn between 10% and 70% less than natives). Hirsch et. al. (2014) study the economic performance of ethnic Germans that migrated to Germany from Poland, Romania and the Former Soviet Union (FSU) and find that the initial wage penalty ranges from 12% (for earlier cohorts) to 30% (more recent cohorts). Rodriguez-Planas (2012) presents data on initial income gaps for individuals who migrated to Spain (mostly in the 1990s and 2000s). She finds that initial income losses relative to natives range from 24% (high school dropouts) to 41% (college graduates).

In addition to individual characteristics (education, experience, gender, marital status) the literature finds that the country of origin of a migrant has an impact on the initial income gap. Borjas (2014), who studies immigrants to the U.S., indicates that "controlling for the changing national origin mix ... greatly attenuate(s) the shifts in entry wages observed across successive immigrant cohorts." In earlier work Borjas (1994) estimates that the elasticity of earnings of migrants with respect to country of origin GDP is around 0.04, while Borjas (2000, Table 1.6 column 4) finds that the elasticity is around 0.05. Moreover, for the U.S., the income of the average sending country has changed over time. Borjas (1994) reports that the level of GNP per capita in the country of origin of the typical recent immigrant in 1970 was slightly above 50% of the U.S., while in the 1980s (we do not have data for 1990) it had decreased to approximately 39% of U.S. GNP per capita. Borjas (2014) concludes that not all the changes in observed economic penalties can be accounted for by changes in national origins.

Abramitzky et. al. (2013) study assimilation of immigrants to the U.S. early in the 20th Century. They find that migrants from "England, Scotland and Wales held higher paid occupations than U.S. natives upon first arrival

... permanent immigrants from two sending countries that started out with occupation scores below those of natives (Denmark and Portugal) experienced sizable occupation convergence over thirty years." To the extent that England, Scotland and Wales had a level of development that was similar to that of the U.S. around 1900 while Denmark and Portugal's was lower, this is exactly what the selection model implies: catch up by migrants from poor countries, and no differences in earnings in the case of migrants of relatively rich countries.

Assimilation As in the case of initial income gaps, the available estimates of the rate at which the gap closes cover a fairly large range. Borjas (2014) reports that initial, 10 year, growth rate of earnings of immigrants to the U.S. relative to natives for the pre 1980 cohorts to be between 12% and 14%. However, the estimate falls short of 9% for the 1985-1989 arrivals and is zero for the 1995-1999 immigrant cohort. Lubotsky (2007) using longitudinal data that matches the 1990 and 1991 SIPP data with the 1994 March Supplement to the CPS and earnings shows that selection biases the results from repeated cross sections. In particular, he finds that "immigrant earnings grow by about 10-15 percentage points more over their first 20 years in the United States than the earnings growth experienced by natives." For individuals who have been in the U.S. for more than 30 years the decreases in the gap is 28.4%. Rho (2013) finds catch-up rates between 8% and 10% for individuals with at most a high school education but small (or even negative) catch up rates for migrants with higher level of schooling

Lemos (2013) analyzes the income of immigrants to the U.K. and states that in reference to the earnings gap between immigrants and natives "After 10 years, this gap narrows to around zero...After 20 years ...immigrants earn roughly between 0% and 30% more than natives. After 30 years the gap is positive and larger." Hirsch et. al. (2014) estimate that after 9-11 years ethnic Germans who migrated to Germany had closed the gap with natives by about 7.2% and this increases to 9% after 15 years. Rodriguez-Planas (2012) using repeated cross sections for immigrants to Spain estimates significantly higher catch up rates. Her estimates for the 10 year rate of assimilation ranges from 12% (high school dropouts) to 27% (college graduates). Izquierdo et. al. (2009) using longitudinal data for immigrants to Spain from outside the EU-15 estimates a decrease in the wage gap of around 15% in the first 5-6 years, but a slower rate of assimilation as a function of experience as the decrease in the gap is 20% after 20 years.

The Effect of Experience in the Sending Location Several studies have estimated the impact on a migrants income of additional experience in the country of origin. Lubotsky (2007) reports that the penalty for 6-10 years of foreign experience is 7.6%, and for 11-15 years 21.3%. Goldmann et. al. (2011) study the returns to different forms of human capital to immigrants to Canada and find that the "rate of return to pre-immigration labour market experience is negative and statistically significant." Hirsch et. al. (2014) estimate that the income penalty associated to ten years of foreign experience is 13.4% and that 20 years of experience in the home country reduces initial income by almost 24%. Rodriguez-Planas (2012) finds that the decrease in relative income for 10-14 years of experience abroad ranges from 12.3% (high school dropouts) to 20.3% (college graduates).

Rate of Return to Schooling Chiswick and Miller (2008) find, using census data, that the rate of return to schooling in the U.S. is lower for immigrants. On average, they find that the rate of return foreign schooling earns about 50% of the return that natives enjoy. Chiswick and Miller argue that the rate of return to "required education" —by this they mean the return to individuals whose educational level matches the usual educational requirements for their occupations— is very similar between natives and immigrants. However, they find that the rate of return to "overeducation" is lower for immigrants and that the fraction of immigrants that are overeducated is larger than the corresponding value for natives. This finding is consistent with a model in which the true occupational requirement is given by human capital and not schooling. If that were the case, the model in this paper would imply that immigrants from low income countries are in the overeducated category.

Schoellman (2012) estimates rates of return to schooling for immigrants in the U.S. He finds that the rate of return is positively correlated with the GDP per capita of the country of origin. Using his estimates the semi-log elasticity of returns with respect to output per worker is 0.015. This implies that an immigrant from a country with output per worker is one fifth of the U.S. has a rate of return on schooling that is about 2.4% lower. Moreover, the returns to schooling for an individual of the "poor" country (i.e. 20% of U.S. output per worker) is 1.3% higher than the return corresponding to an immigrant from the "median" country (50% of U.S. output per worker). Li and Sweetman (2014) study the effect of educational quality and output per worker in the country of origin on the return to schooling for immigrants to Canada. They find that the relationship GDP per capita in the country of origin and the return to schooling is positive. For example, using their estimates in Table 4 (Column 4) the additional return to a year of schooling for an immigrant whose GDP of origin increases from 20% to 50% of the U.S. is about 0.6% which is half the size of the estimates implied by the Schoellman paper.

4 Model: Quantitative Results (preliminary)

In this section we report some very preliminary results about the implications of the model for the age-earnings profiles of migrants, stayers and natives.

4.1 Calibration

As a first pass we used the parameters in Manuelli, Seshadri and Shin (2013) to pin down the shape of the human capital production function. In particular, we choose $\gamma_1 = 0.571$ and $\gamma_2 = 0.289$ to match the ratio of earnings at age 49–51 to earnings at age 24–26 in NLSY79 (2.33) and the ratio of earnings at age 64 relative to age 50 (0.92). The early childhood human capital production parameter is $\nu = 0.50$, which replicates pre-primary education expenditure relative to GDP in the US (0.01). We assume that $\delta_h = 0.01$ and that the relevant interest rate is $r = 0.04$. These parameters are held constant across individuals and countries.

The most important difference across countries is in rental rates of human capital (w), which is assumed to derive from differences in aggregate total factor productivity. We normalize $w = 1$ for the U.S. and lowers it for poorer countries. We set the relative prices of all inputs to one, the price of consumption.

Within a country, individuals are different in their learning ability z_h, which leads to differences in years of schooling. In each country, we consider those who have exactly 12 and 16 years of schooling. Holding other things equal, individuals from poorer (i.e., lower w) countries must have higher z_h to choose the same years of schooling as those from richer countries.

In the quantitative results we assume that all individuals who are not in school can allocate at most a fraction $\bar{n}$ of their time to training. We set $\bar{n} = 0.5$. This constraint applies to all agents.

To illustrate the implications of the model we consider the case of a migrant from a country that has output per worker that is 50% of the U.S. (e.g., Greece, Mauritius, Portugal) and another from a country with output per worker equal to one-fifth of the U.S. (e.g., Ecuador, Jordan, Romania, Thailand).

4.2 Migrants vs. Natives

In this section we present some preliminary results of the implications of the model for the evidence on the economic performance of immigrants.

Initial Gap and Assimilation: Younger and Older Migrants In Table 1 we display the results for migrants with high school and college education who migrate at age 26. We report the gap relative to natives with the same level of schooling and age for the initial period (defined as the first three years after migration), after 15 and 25 years in the country of destination

Table 1: Relative Income $[a_m = 26]$

Income	$y = .50 * US$		$y = .20 * US$	
	$s = 12$	$s = 16$	$s = 12$	$s = 16$
Initial	50%	45%	31%	36%
+15	86%	62%	68%	70%
+25	90%	64%	83%	85%

There several interesting results:

1. The initial income gap is large. Even though it is within the range of estimates, it is a the high end (of the gap).

2. The size of the gap increases as the GDP of the country of origin decreases. The model implies that the impact of "educational quality" —which in this model is completely colinear with output per worker in the sending country— is significant.

3. The model predicts fairly high levels of assimilation. For example a college graduate coming from a "middle class" country earns, after

25 years, a level of income that is only 10% lower than that of the native. As in the case of the initial gap, these estimates are within the range but are close to the high end. The model implies that the rate of assimilation is higher the lower the income level of the country of origin.

4. The difference gap, defined as the gap of college graduates relative to the gap corresponding to high school graduates is not monotonic as a function of GDP in the country of origin. While college graduates from middle income countries have a larger gap relative to high school graduates from the same country, the reverse is true for migrants from the poor country.

As shown in Table 2 the age at migration influences the income gap.

Table 2: Relative Income (%) $[a_m = 36]$

Income	$y = .50 * US$		$y = .20 * US$	
	$s = 12$	$s = 16$	$s = 12$	$s = 16$
Initial	69%	67%	35%	31%
+15	81%	83%	65%	68%
+25	82%	83%	67%	68%

The model predicts smaller initial income gaps for older migrants and slower rates of assimilation. The latter is driven by the shorter horizon that older migrants have in the destination country and the consequent limits on increasing their human capital. The results suggest that the age distribution of migrants can have a large impact on the estimates of the rate of assimilation. For example, comparing a college graduate form the poor country around age 60 who migrated at age 26 with a similar individual who migrated at age 36, the latter has significant lower income (68% of a native vs. 87%).

The Return to Experience in the Country of Origin In Table 3 we show the returns to delayed migration for two individuals at age 50 who migrated at age 26 and 36.

Table 3: Income Gain

$y = .50 * US$		$y = .20 * US$	
$s = 12$	$s = 16$	$s = 12$	$s = 16$
9%	11%	18%	17%

Thus, for a college graduate from a poor country the penalty for delayed migration is 17%. The reason why the gains from early migration are higher for individuals from low income countries is that they have higher innate ability and, hence, they can increase their human capital are a higher rate than those who migrate from a higher income country.

The Rate of Return to Schooling The theoretical model predicts that the return to schooling is increasing in the level of development of the sending country. Since schooling is endogenous, it is not obvious what is meant — in the context of the model— by the "rate of return on schooling." We consider two extreme views. First, we derive what we label the **endogenous** rate of return. To compute this rate we start with the level of ability (z_h) that an individual from a given country requires to choose exactly s years of schooling. We then find a new level of ability, z_h', such that the optimal choice of schooling in $s+1$ years. We do the same for natives and we compare for several ages the difference in returns to schooling (measured as difference in log earnings for each age-experience level) between natives and migrants.

The **exogenous** rate of return —-a notion that is more consistent with the implicit assumptions in most empirical work— simply views the additional year of schooling as an accident. That is, an individual plans to go to school for s period but accidentally ends up with $s + 1$ periods. Of course, in this case, the rate of return must be lower since the individual finds himself with too much human capital relative to his (private) optimum. His optimal strategy from then on is to reduce investment so that his actual human capital adjusts to his desired level.

The results for both concepts and for immigrants from a poor country are in Table 4.

Table 4: Rate of Return on Schooling (%)

Category	$s = 12$			$s = 16$		
	$a_m = 26$	$a_m = 36$	$a_m = 46$	$a_m = 26$	$a_m = 36$	$a_m = 46$
Endogenous-Natives	7.8	8.7	8.9	10.1	11.8	12.1
Endogenous-Immigrants	7.6	8.7	8.8	9.5	11.4	11.8
Exogenous-Natives	2.6	2.06	1.9	2.7	2.1	1.9
Exogenous-Immigrants	2.2	1.7	1.7	2.5	1.9	1.8

Even though the theory implies that the rate of return to schooling should be lower for immigrants, the size of the difference is minuscule. Effectively, for these parameter values, the model implies no differences.[18]

Temporary Migrants In this section we report the relative income of an immigrant from the poor country that migrate at age 26 or 36 (in parenthesis in Tables 5 and 6) as a function of the expected duration of stay. For comparison, we also include the relative income of a permanent migrant. We show two expected duration of the stay in the destination country: 2 and 8 years.

Table 5: Income Relative to Natives

Age (income)	$s = 12$		
	Permanent	2 Years	8 Years
Initial	31% - (35%)	75% - (58%)	72% - (57%)
+15	68% - (65%)	44% - (48%)	47%- (50%)
+25	83% - (67%)	40% - (48%)	44% - (50%)

Table 6: Income Relative to Natives

Age (income)	$s = 16$		
	Permanent	2 Years	8 Years
Initial	36% - (31%)	89% - (60%)	83% - (58%)
+15	70% - (68%)	42% - (47%)	43%- (50%)
+25	85% - (68%)	37% - (47%)	39% - (50%)

In all cases the results show that the probability of migrating severely reduces the incentives to accumulate human capital and hence there is negative assimilation. Moreover, it appears that the elasticity with respect to expected duration (conditional on staying of course) is pretty low.

4.3 Migrants vs. Stayers: Model Predictions

Using the above parameterization the predictions of the model for the relative income of migrants and stayers are summarized in Tables 7 and 8.

[18] If income includes a non-cognitive component that is similar for individuals with the same level of education, this will tend to lower all the rates of return but more so those of immigrants.

Table 7: Income Ratio: Migrants vs. Stayers (“ L”)

Category	$s = 12$		$s = 16$	
	$a_m = 26$	$a_m = 46$	$a_m = 26$	$a_m = 46$
Lifetime	2.43	1.94	2.42	1.95
First 5 years	1.11	1.85	1.26	1.84
Last 5 years	3.07	1.98	3.11	2.00

Table 8: Income Ratio: Migrants vs. Stayers (“M”)

Category	$s = 12$		$s = 16$	
	$a_m = 26$	$a_m = 46$	$a_m = 26$	$a_m = 46$
Lifetime	1.44	1.32	1.44	1.32
First 5 years	0.95	1.22	0.93	1.31
Last 5 years	1.60	1.41	1.62	1.33

The quantitative results show that migrants’ age-earnings profiles relative to stayers are significantly steeper and that the degree of steepness depends on the quality of education (as proxied by the GDP of the country of origin) and the age at migration: older migrants from higher income countries have fairly flat relative age-earnings profiles.

5 Conclusion [missing]

6 Appendix

Migrants vs. Stayers It is possible to show that earnings at age a, given that human capital at age a_m is $h(a_m)$ is given by

$$y(a,w,p_w) = we^{-\delta_h(a-a_m)}h(a_m) + \left(\gamma_1^{\gamma_1}\gamma_2^{\gamma_2} z_h \left(\frac{w}{p_w}\right)^{\gamma_2}\right)^{1/(1-\gamma)} w$$

$$\underbrace{\left[\int_{a_m}^{a} e^{-\delta_h(a-t)}\left(\frac{m(t,r+\delta_h)}{r+\delta_h}\right)^{\frac{\gamma}{1-\gamma}} dt - \gamma w \left(\frac{m(a,r+\delta_h)}{r+\delta_h}\right)^{\frac{\gamma}{1-\gamma}}\right]}_{\equiv\theta(a,a_m)}$$

Thus,

Let $\theta(a,a_m)$ be given by

$$\theta(a,a_m) = \int_{a_m}^{a} e^{-\delta_h(a-t)}\left(\frac{m(t,r+\delta_h)}{r+\delta_h}\right)^{\frac{\gamma}{1-\gamma}} dt - \gamma \left(\frac{m(a,r+\delta_h)}{r+\delta_h}\right)^{\frac{\gamma}{1-\gamma}}.$$

which implies that

$$y(a,w^*,p_w^*) - y(a,w,p_w) = (w^*-w)e^{-\delta_h(a-a_m)}h(a_m) +$$
$$\left(\gamma_1^{\gamma_1}\gamma_2^{\gamma_2} z_h\right)^{1/(1-\gamma)}\left[\left(\frac{w^*}{p_w^*}\right)^{\frac{\gamma_2}{1-\gamma}} w^* - \left(\frac{w}{p_w}\right)^{\frac{\gamma_2}{1-\gamma}} w\right]\theta(a,a_m).$$

Then, simple calculations show that

$$\theta(a_m,a_m) < 0,\ \theta(R,a_m) > 0, \frac{\partial\theta}{\partial a}(a,a_m) > 0 \text{ and } \frac{\partial\theta}{\partial a_m}(a,a_m) < 0$$

Migrants vs. Natives **Proof of Proposition 1.** Manuelli and Seshadri (2014) show that, during the schooling period, the optimal investment in market goods, $x_s(t)$, is given by

$$x_s(t) = x_s e^{g(t-6)},\ \text{where } g = \frac{(r+(1-\gamma_1)\delta_h)}{1-\gamma_2} \text{with } x_s(6) = x_s.$$

Given this level of optimal investment, the ODE

$$\dot{h}(t) = z_h h(t)^{\gamma_1} x(t)^{\gamma_2} - \delta_h h(t)$$

has a solution given by

$$h(t) = e^{-\delta_h(t-6)} \left[h_B^{1-\gamma_1} x_E^{\upsilon(1-\gamma_1} + (1-\gamma_1) z_h x^{\gamma_2} \left(\frac{e^{g^*(t-6)} - 1}{g^*} \right) \right]^{\frac{1}{1-\gamma_1}},$$

where

$$g^* = \frac{\gamma_2 r + (1-\gamma_1)\delta_h}{1-\gamma_2}.$$

The human capital at the end of the (endogenous) schooling period it is given then by

$$H(x_E, x, s) \equiv e^{-\delta_h s} \left[h_B^{1-\gamma_1} x_E^{\upsilon(1-\gamma_1} + (1-\gamma_1) z_h x^{\gamma_2} \left(\frac{e^{g^* s} - 1}{g^*} \right) \right]^{\frac{1}{1-\gamma_1}} \tag{21}$$

The value of such a stock of human capital is

$$V_0(6+s; z_h, w, p_w) + V_1(a; w) H(x_E, x, s).$$

The cost of attaining that payoff is

$$e^{rs} p_E x_E + p_s \int_6^{6+s} e^{r(6+s-t)} x_s e^{g(t-6)} dt.$$

The individual decision problem is given by

$$\max_{x_E, x, s} V_0(6+s; z_h, w, p_w) + V_1(a; w) H(x_E, x, s) - \left(e^{rs} p_E x_E + p_s \int_6^{6+s} e^{r(6+s-t)} x_s e^{g(t-6)} dt. \right). \tag{P1}$$

Manuelli and Seshadri (2014) show that, at the optimal schooling length s, the level of human capital must satisfy

$$h(6+s) = \gamma_1 \left[z_h \left(\frac{w}{p_w} \right)^{\gamma_2} (\gamma_1^{\gamma_1} \gamma_2^{\gamma_2}) \left(\frac{m(6+s; r+\delta_h)}{r+\delta_h} \right) \right]^{\frac{1}{1-\gamma}} \tag{22}$$

Solving the maximization problem (P1) with respect to (x_E, x), and imposing equation (22), implies that the following restriction

$$A(s) \left(\frac{w}{p_E} \right)^{\frac{\upsilon(1-\gamma_1)}{1-\upsilon(1-\gamma_1)}} \left(z_h \left(\frac{w}{p_w} \right)^{\gamma_2} \right)^{-\frac{(1-\upsilon)(1-\gamma_1)}{(1-\upsilon(1-\gamma_1))(1-\gamma)}} + B(s) \left(\frac{p_w}{p_s} \right)^{\frac{\gamma_2}{1-\gamma_2}} = C(s), \tag{23}$$

where $A(s), B(s)$ and $C(s)$ depend on the level of schooling but not on prices or ability, z_h.

Note that if $\upsilon = 0$ —this corresponds to equal early childhood human capital in all countries— equation (23) implies that, given a level of schooling,

$$z_h \left(\frac{w}{p_w}\right)^{\gamma_2} = \text{constant}.$$

In this case individuals in high w/p_w countries must have low z_h. However, given s, equation (22) implies that the level of human capital is exactly the same independently of where the individual was born.

If, as it is the case in our calibration, $(1-\alpha_1)\upsilon < \alpha_2$, then higher w requires lower z_h, but results in higher $z_h(w/p_w)^{\gamma_2}$. Thus, an individual who attains a given level s of schooling in a high w country must have lower innate ability than an individual who has the same schooling in a low w country. However, as equation (22) shows the individual in the high wage country will have a higher level of human capital after he leaves school due to better quality of schools.

Since investment in human capital in the post-schooling period satisfies

$$x(a) = \frac{\gamma_2}{\gamma_1} n(a)h(a)$$

and

$$n(a)h(a) = \gamma_1 \left[z_h \left(\frac{w}{p_w}\right)^{\gamma_2} (\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}) \left(\frac{m(a; r+\delta_h)}{r+\delta_h}\right)\right]^{\frac{1}{1-\gamma}}$$

and since the individual in the low wage country has a lower $z_h(w/p_w)^{\gamma_2}$ the differences in human capital at the end of schooling become larger as time goes by. Thus, for any a_m, if the migrant comes from a country with wage $w < w^*$ then $h_m(a_m) < h_n(a_m)$ and $z_{h,m} > z_{h,n}$.

Since p_w and p_s are both market prices relative to aggregate consumption there is no clear prior about the direction in which their ratio moves as a function of TFP. If health care —a large component of early childhood human capital— is more expensive (relative to aggregate consumption) in less developed countries then this reinforces the effects found for w.

The rest of the proposition follows from the expression for the difference in income given by

$$\Delta_y^n(a, a_m; w^*, p_w^*) = w^* e^{-\delta_h(a-a_m)}[h_m(a_m) - h_n(a_m)] +$$
$$[z_{h,m}^{1/(1-\gamma)} - z_{h,n}^{1/(1-\gamma)}] (\gamma_1^{\gamma_1}\gamma_2^{\gamma_2})^{1/(1-\gamma)} \left(\frac{w^*}{p_w^*}\right)^{\frac{\gamma_2}{1-\gamma}} w^* \theta(a, a_m).$$

As argued above, if $w < w^*$ then $h_m(a_m) < h_n(a_m)$, and the gap increases as w decreases. Moreover, the gap narrows as $(\partial\theta/\partial a)(a, a_m) > 0$ and it increases with a_m as the initial human capital gap is larger and the gains from training on the job smaller. The impact of the prices in the sending country (w, p_E, p_s, p_w) all follow from their impact on $h_m(a_m)$ as given by equation (23) ■

Temporary Migrants **Proof of Proposition 2.** Since

$$\frac{\partial \dot{y}}{\partial \lambda}(a;\lambda) = \left(\gamma_1^{\gamma_1}\gamma_2^{\gamma_2} z_h \left(\frac{w^*}{p_w^*}\right)^{\gamma_2}\right)^{\frac{1}{1-\gamma}} (w^*)^{-\frac{\gamma}{1-\gamma}} \frac{\gamma}{1-\gamma}$$
$$\left[M_1^{\frac{2\gamma-1}{1-\gamma}} \frac{\partial M_1}{\partial \lambda}(w^* - \frac{\gamma}{1-\gamma}\dot{M}_1) - M_1^{\frac{\gamma}{1-\gamma}} \frac{\partial \dot{M}_1}{\partial \lambda}\right.$$
$$\left. - w^* \delta_h \int_{a_m}^{a} e^{-\delta_h(a-t)} M_1(t; w^*, w, \lambda)^{\frac{2\gamma-1}{1-\gamma}} \frac{\partial M_1}{\partial \lambda}(t; w^*, w, \lambda) dt\right]$$

Let

$$K(a;\delta_h) \equiv \left[M_1^{\frac{2\gamma-1}{1-\gamma}} \frac{\partial M_1}{\partial \lambda}(w^* - \frac{\gamma}{1-\gamma}\dot{M}_1) - M_1^{\frac{\gamma}{1-\gamma}} \frac{\partial \dot{M}_1}{\partial \lambda}\right.$$
$$\left. - w^* \delta_h \int_{a_m}^{a} e^{-\delta_h(a-t)} M_1(t; w^*, w, \lambda)^{\frac{2\gamma-1}{1-\gamma}} \frac{\partial M_1}{\partial \lambda}(t; w^*, w, \lambda) dt\right].$$

Since

$$\frac{\partial M_1}{\partial \lambda} < 0, \frac{\partial \dot{M}_1}{\partial \lambda} > 0 \text{ and } \dot{M}_1 < 0,$$

it follows that $K(a_m;\delta_h) < 0$ and $K(R;\delta_h) > 0$. Moreover, $K(a;0) < 0$ for all a. ■

Proof of Proposition 3. Since

$$\frac{\partial \dot{y}}{\partial w}(a;\lambda) = \left(\gamma_1^{\gamma_1}\gamma_2^{\gamma_2} z_h \left(\frac{w^*}{p_w^*}\right)^{\gamma_2}\right)^{\frac{1}{1-\gamma}} (w^*)^{-\frac{\gamma}{1-\gamma}} \frac{\gamma}{1-\gamma}$$
$$\left[M_1^{\frac{2\gamma-1}{1-\gamma}} \frac{\partial M_1}{\partial w}(w^* - \frac{\gamma}{1-\gamma}\dot{M}_1)\right.$$
$$\left. -w^*\delta_h \int_{a_m}^{a} e^{-\delta_h(a-t)} M_1(t;w^*,w,\lambda)^{\frac{2\gamma-1}{1-\gamma}} \frac{\partial M_1}{\partial w}(t;w^*,w,\lambda)dt\right]$$

Let

$$L(a;\delta_h) \equiv \left[M_1^{\frac{2\gamma-1}{1-\gamma}} \frac{\partial M_1}{\partial w}(w^* - \frac{\gamma}{1-\gamma}\dot{M}_1)\right.$$
$$\left. -w^*\delta_h \int_{a_m}^{a} e^{-\delta_h(a-t)} M_1(t;w^*,w,\lambda)^{\frac{2\gamma-1}{1-\gamma}} \frac{\partial M_1}{\partial w}(t;w^*,w,\lambda)dt\right].$$

Since$\partial M_1/\partial w > 0$, it follows that $L(a_m;\delta_h) > 0$ and $L(R;\delta_h) < 0$. Moreover, $L(a;0) > 0$ for all a. ■

More Standard Selection Measures The migration literature defines selection in terms of fractions of individuals in different categories as function of location. If we add to the migration decision a preference shock that has an extreme value distribution, it is possible to translate our results into more standard measures. We assume that when the option to migrate materializes, the payoffs from migrating and staying also include a random component that captures the monetary equivalent of other considerations associated with migrating. Moreover, assume that these random components —independent across individuals— have an extreme value distribution and that their difference, which we denote ε, has a logistic distribution. The individual will choose to migrate if

$$V((1-\eta)h, a; z_h, w^*, p_w^*) - C + \varepsilon \geq V(h, a; z_h, w, p_w).$$

To simplify notation, let

$$\Lambda(h, a; z_h, w^*, p_w^*, w, p_w) = V((1-\eta)h, a; z_h, w^*, p_w^*) - C - V(h, a; z_h, w, p_w).$$

In this case, the fraction of age a individuals with innate ability z_h who chose to migrate is given by

$$P[\varepsilon \geq -\Lambda(h, a; z_h, w^*, p_w^*, w, p_w)]$$

Let $N^m(a, z_h)$ be the number of individuals with innate ability z_h who migrate from "w" to "w^*" at age a, and let $N^s(a, z_h)$ be the number of stayers with the same ability level. Given the distributional assumptions, it follows that

$$\ln\left(\frac{N^m(a, z_h)}{N^s(a, z_h)}\right) = \Lambda(h, a; z_h, w^*, p_w^*, w, p_w).$$

Since the function $\Lambda(h, a; z_h, w^*, p_w^*, w, p_w)$ is increasing in w^* the model predicts that there will be positive sorting: Countries with a higher return to human capita (higher w^* in this model) will attract more immigrants. This effect is reinforced if w^*/p_w^* is higher in the destination country, that is, if training on the job is relatively cheaper.

Consider now two types of individuals indexed by their innate ability levels, $z_h > z_h'$. It follows (see Manuelli and Seshadri (2014) for details) that $h(a; z_h) > h(a; z_h')$. We refer to the z_h individual as the high skill person, and the z_h' as the low skill worker.

The migration literature defines positive selection as a situation in which the skill composition of the migrants is more tilted towards the high skilled than that of the stayers. Formally, migrants are positively selected if

$$\ln\left(\frac{N^m(a, z_h)}{N^m(a, z_h')}\right) > \ln\left(\frac{N^s(a, z_h)}{N^s(a, z_h')}\right)$$

Since this condition is equivalent to

$$\ln\left(\frac{N^m(a, z_h)}{N^s(a, z_h)}\right) > \ln\left(\frac{N^m(a, z_h')}{N^s(a, z_h')}\right)$$

it is satisfied if and only if

$$\Lambda(h, a; z_h, w^*, p_w^*, w, p_w) > \Lambda(h, a; z_h', w^*, p_w^*, w, p_w)$$

or, equivalently,

$$(w^*(1-\eta) - w)\frac{m(a, r+\delta_h)}{r+\delta_h}[h(a; z_h) - h(a; z_h')] + (1-\gamma)\left(\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\right)^{\frac{1}{1-\gamma}} \tag{24}$$

$$\left(\left(\frac{w^*}{p_w^*}\right)^{\frac{\gamma_2}{1-\gamma}} w^* - \left(\frac{w}{p_w}\right)^{\frac{\gamma_2}{1-\gamma}} w\right)\left(\int_a^R e^{-r(t-a)}\left(\frac{m(t, r+\delta_h)}{r+\delta_h}\right)^{\frac{\gamma}{1-\gamma}} dt\right)\left[(z_h)^{\frac{1}{1-\gamma}} - (z_h')^{\frac{1}{1-\gamma}}\right] > 0.$$

There are two cases to consider:

1. **Case I:** $(w^*(1-\eta)-w)>0$. In this case the model implies that selection is positive. Thus, when the variable component of the cost of migrating is small (i.e. η is small), selection is positive.

2. **Case II**: $(w^*(1-\eta)-w)\leq 0$. In this case, for migration to be feasible it must be the case that

$$\left(\frac{w^*}{p_w^*}\right)^{\frac{\gamma_2}{1-\gamma}} w^* - \left(\frac{w}{p_w}\right)^{\frac{\gamma_2}{1-\gamma}} w > 0$$

The Impact of Uncertainty In this section we describe the impact that different level of uncertainty about the future evolution of wage rates in the two countries has upon the migration decision. To this end we assume that the stochastic process for the wage rate satisfies

$$dw_t = \mu w_t dt + \sigma w_t dW_t,$$

and we take p_w constant. Assume that $r>\mu$. We conjecture that the value function for an individual who is out of school and who has no restrictions on $n(a)$ (that is, we do not even impose that $n(a)\leq 1$) is given by

$$V(h,w,a) = V_0(a;\mu,\sigma) w^{\frac{1-\gamma_1}{1-\gamma}} + wV_1(a;\mu)h,$$

where

$$V_1(a;\mu) = \frac{m(t, r-\mu+\delta_h)}{r-\mu+\delta_h}$$

and

$$V_0(a;\mu,\sigma) = z_h^{1/(1-\gamma)} \underbrace{(1-\gamma)\left(\gamma_1^{\gamma_1}\left(\frac{\gamma_2}{p_w}\right)^{\gamma_2} z_h\right)^{1/(1-\gamma)}}_{K} \underbrace{\int_a^R e^{-\rho(r,\mu,\sigma)(t-a)}\left(\frac{m(t,r-\mu+\delta_h)}{r-\mu+\delta_h}\right)^{\frac{1}{1-\gamma}} dt}_{\hat{V}(a;\mu,\sigma)}$$

and

$$\rho(r,\mu,\sigma) = r - \frac{1-\gamma_1}{1-\gamma}\mu - \frac{1-\gamma_1}{1-\gamma}\frac{\gamma_2}{1-\gamma}\frac{\sigma^2}{2}$$

It follows that

$$\frac{\partial V_0}{\partial \sigma}(a;\mu,\sigma) > 0.$$

Thus, if we consider migrating from country (w, μ, σ) to (w^*, μ, σ^*) then the migration criterion is (in case I, the normal case)

$$h(a) \geq \frac{C}{(w^*(1-\eta) - w)V_1(a;\mu)} - z_h^{1/(1-\gamma)} \frac{K}{(w^*(1-\eta) - w)V_1(a;\mu)} \left[(w^*)^{\frac{1-\gamma_1}{1-\gamma}} \hat{V}(a;\mu,\sigma^*) - (w)^{\frac{1-\gamma_1}{1-\gamma}} \hat{V}(\right.$$

It follows that if $\sigma^* > \sigma$ this implies that the term

$$\left[(w^*)^{\frac{1-\gamma_1}{1-\gamma}} \hat{V}(a;\mu,\sigma^*) - (w)^{\frac{1-\gamma_1}{1-\gamma}} \hat{V}(a;\mu,\sigma) \right]$$

is larger and the negatively sloped boundary in (z_h, h) becomes steeper. Thus, there is a set of individuals who migrate if $\sigma^* > \sigma$ that would not have migrated if $\sigma^* = \sigma$. Thus, in this sense the model implies that higher variance has a negative impact on selection. Question: Is this what Borjas' model implies?

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