Technology Advancement and Growth

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A. Introduction

Technological under-achievement is a major barrier to economic development. A more thorough study of technology advancement helps understand its consequences for long-run sustained growth and short-run creative destruction.

Recent studies of R&D and technological choice consider imperfect market structures that permit rents for invention (Shell 1966):

- **Monopoly – *Aghion-Howitt (1992)*, Thesmar-Thoenig (2000)**
- **Monopolistic Competition – Grossman-Helpman (1991), *Romer (1990)*, Peng-Thisse-Wang (2003)**

Technology transfer via imitation and adoption also plays crucial roles, particularly in developing countries. In this case, the distance-to-frontier is important as well, as discussed in Acemoglu-Aghion-Zilibotti (2006) and Wang (in progress).

Technologies are often firm specific and depend on establishment ages. This leads to the birth of the organizational capital literature:

- **classic: Lucas (1978), Prescott-Visscher (1980), Jovanovic (1982)**
- **extensions: *Atkenson-Kehoe* (*2005*, 2007), *Samaniego (2006)*, Burstein & Monge-Naranjo (2009)**

B. Causes and Consequences of Technological Advancements

- **Innovation: Aghion-Howitt (1992), Grossman-Helpman (1991) and Stokey (1995) stress that successful innovations from R&D advance technology**
- **Imitation: Rustichini-Schmitz (1991) emphasizes that imitation plays an important role particularly to less-developed countries – in their model, the optimal policy is to subsidize equally imitation and innovation**
- **Technology adoption: for countries with lower level of R&D, technology may be adopted rather than invented, but adoption may have barriers caused by:**
    1. **adoption inefficiency: Parente-Prescott (1994)**
    2. **incumbent blocks: Parente-Prescott (1999)**
    3. **match frictions: Chen-Mo-Wang (2002), Laing-Palivos-Wang (2002)**
- **Should we subsidize R&D and other technology-advancing activities? Boldrin-Levine (2004) stress that competitive markets may work**
- **Are we missing additional endogenous components of TFP? The answer is most definitely positive: Caselli (2005), *Wang-Wong-Yip (2018)***
- **Are we concerned by machines replacing workers leading to higher unemployment (Keynes 1930)? *Acemoglu-Restrepo* (*2018*, 2019)**
- **What is the role of intangibles? *Crouzet-Eberly-Eisfeldt-Papanikolaou (2022)***
- **Misallocation of talent in innovation: *Celik (2023)***

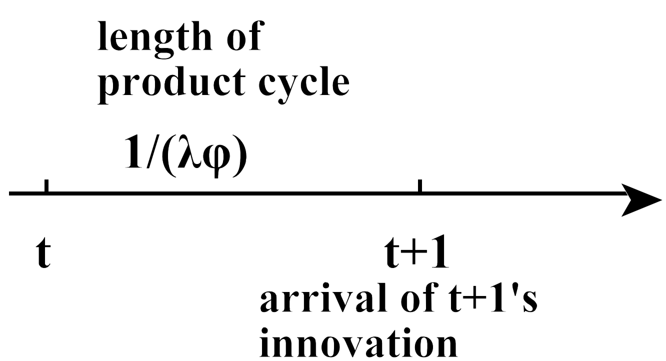
- **C. R&D, Monopoly Rent and Growth: Aghion-Howitt (1992)**

- **The related literature is summarized as follows:**
  - **Classic: Arrow (1962) and Shell (1967), emphasizing on the role played by monopoly rent in promoting inventive activity**
  - **Vertical product innovation and endogenous growth:**
    - **Stokey (1988) – innovation as a by-product of LBD**
    - **Segerstrom-Anant-Dinopoulos (1990) – monopolistic competition, intersectoral trade-off, deterministic arrival**
    - **Grossman-Helpman (1991) – monopolistic competition, intertemporal trade-off, deterministic arrival**
  - **Aghion-Howitt (1992) – monopoly, intertemporal trade-off, random arrival**
- **Key:** $\boldsymbol{R\&D_t} = \boldsymbol{f}(\boldsymbol{R\&D^e_{t+1}}),\ \boldsymbol{f}'<0$**, where there are two important effects:**

  - **Creative destruction effect:** $\boldsymbol{R\&D^e_{t+1}}\uparrow \Rightarrow$ monopoly rent $\downarrow \Rightarrow \boldsymbol{R\&D_t}\downarrow$
  - **GE wage effect:** $\boldsymbol{R\&D^e_{t+1}}\uparrow \Rightarrow \boldsymbol{L^d_{t+1}}(\text{skilled})\uparrow \Rightarrow \boldsymbol{W_{t+1}}\uparrow \Rightarrow \text{rent}\downarrow \Rightarrow \boldsymbol{R\&D_t}\downarrow$

1. The Model

- **Labor (M, N and R are all exogenous; leisure is inelastic):**
  - **unskilled (M)**
  - **skilled (N) = manufacturing (L) + R&D (n)**
  - **researcher (R)**
- **Goods:**
  - **final good:** $y = AF(x)$
  - **intermediate good:** $x = L = N - n$
- **Arrival of Innovation:** $\lambda\,\phi(n, R)$, ϕ:CRS, $\phi(0, R) = 0$ **(Poisson process)**



- **Productivity:** $A_t = A_o \gamma^t$, $\gamma > 1$
- **Monopolist's Behavior: *ex post* monopoly over the final good market facing consumers with constant marginal utility**
  - **Profit:** $\Pi_t = (P_t - W_t)x_t = A_t[F'(x_t) - w_t]x_t$
  - **MR :** $\dfrac{d\ [F'x]}{dx} = F' + xF'' = \tilde{w}(x_t)$
  - **FOC:** $w_t = \tilde{w}(x_t)$ or $x_t = \tilde{x}(w_t)$

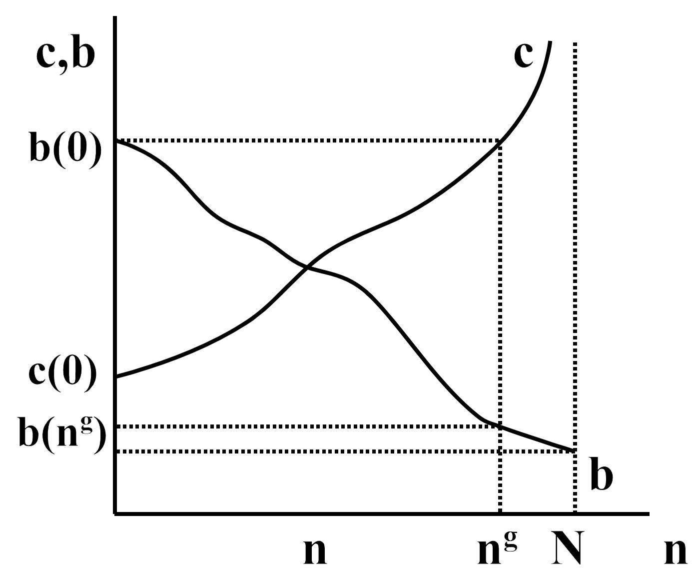
    $\Rightarrow \Pi_t = A_t \tilde{\pi}(w_t)$
  - **Assumption (A1):** $\tilde{w}'(x) < 0$, $\lim_{x\to 0} \tilde{w}(x) = \infty$, $\lim_{x\to\infty} \tilde{w}(x) = 0$**, which ensures invertbility and profit as an increasing function of *x***
- **Innovator's Behavior: ex ante perfectly competitive; ex post monopoly**
  - **optimization:**

$$\max_{n} \lambda\varphi(n_t)V_{t+1} - W_t n_t - W_t^R R$$

where $\varphi(n_t) = \phi(n_t, R)$, **with** $V_{t+1} = \dfrac{\Pi_{t+1}}{r + \lambda\varphi(n_{t+1})}$ **taken as given**

- FOC: $$n_t\left[\lambda\varphi'(n_t)\frac{\Pi_{t+1}}{r+\lambda\varphi(n_{t+1})}-W_t\right]=0$$ **([.] = 0 if n > 0)**
  - **Arrow replacement effect: incumbent's net profit = $v_{t+1}-v_t < V_{t+1}$ = new entrant's net profit**
  - **intertemporal spillover effect: R&D increase A_t permanently, but gains rent only over (t, t+1)**

2. Equilibrium:



- **MC ≥ MB:**

$$c(n_t)=\frac{\tilde{w}(N-n_t)}{\lambda\varphi'(n_t)}\geq\frac{\gamma\tilde{\pi}[\tilde{w}(N-n_{t+1})]}{\gamma+\lambda\varphi(n_{t+1})}=b(n_{t+1})$$

  - **no- growth equilibrium:** $b(0)\leq c(0)\Rightarrow n=0\Rightarrow\lambda\phi=0$
  - **2-Cycle (Sarkovskii) on** $\{o,n^g\}$
  - **positive-growth equilibrium:** $b(n^g)<c(0)<b(0)\Rightarrow n>0\Rightarrow\lambda\phi>0$

- **Characterization of the positive-growth equilibrium**
  - **R&D:** $\underset{\;\;-\;\;+\;\;+\;\;+\;\;+}{n = n(r,\ \gamma,\ N,\ \lambda,\ 1-\alpha)}$, **where** $1-\alpha = \dfrac{P - MC}{P}$ **(mark-up)**
  - **mean growth:** $\theta = \lambda\varphi(n)\ln\gamma$
  - **length of product cycle:** $l = 1/(\lambda\varphi(n))$
  - $\lambda\uparrow \Rightarrow$ creative destruction$(\theta\uparrow,\ l\downarrow)$

3. Welfare

- **Social inefficiency:**
  - **replacement (business stealing):** $n > n^*$
  - **intertemporal spillover:** $n < n^*$
  - **appropriability:** $n < n^*$
  - **monopoly distortion:** $n > n^*$
- **R&D subsidy need not be welfare-improving**

D. R&D and Horizontal Innovation: Romer (1990)

- **Labor allocation: L_1 for production and L_2 for R&D, with $L_1 + L_2 = L$**
- **Final good production (numeraire):**
  - **perfectly competitive**
  - **produced with labor and a basket of M intermediate goods x_i**
    - **the larger M is, the more sophisticated the production line is**
    - **the sophistication of the production line can growth, depending on R&D labor:** $\dot{M}/M = \lambda L_2$
  - **production function:** $\mathbf{Y = L_1^{1-\alpha} \int_0^M x_i^{\alpha} di}$
  - **labor demand:** $MPL_1 = (1-\alpha) L_1^{-\alpha} M x^{\alpha} = w$ **(*ex post* symmetry x_i=x)**
- **Intermediate goods production:**
  - **monopolistically competitive**
  - **total cost = x, MC = 1**
  - **marginal revenue:**

$$MR = \frac{d\left(p_x x\right)}{dx} = \frac{d\left[\alpha \left(L_1\right)^{1-\alpha} x^{\alpha}\right]}{dx} = \alpha^2 \left(L_1\right)^{1-\alpha} x^{\alpha-1} = \alpha p_x$$

  - **MR = MC => $p_x = 1/\alpha$**

- o **intermediate good supply:**

  $$x = \alpha^{\frac{2}{1-\alpha}} L_1$$

- o **maximized profit (earned forever with new entry):**

  $$\Pi = p_x x - x = (p_x - 1)\, x = \frac{1-\alpha}{\alpha} x = \eta x$$

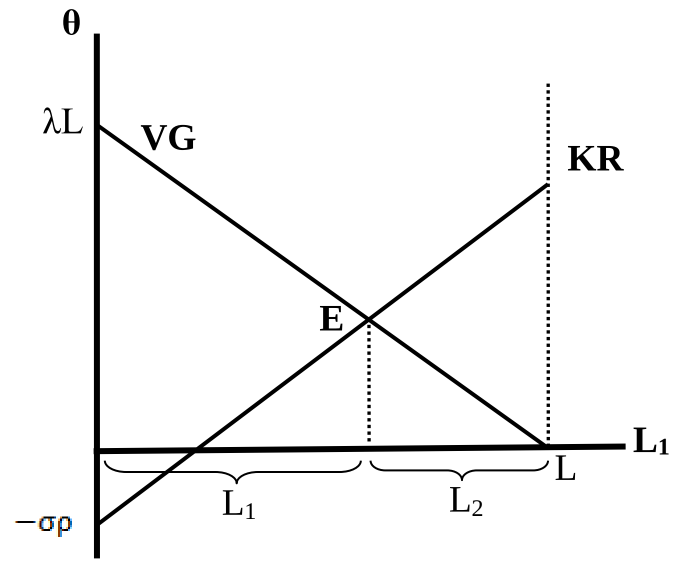
  where the monopoly rent is measured by the markup $\eta = (1-\alpha)/\alpha$

- **R&D decision facing a discounting rate r_D:**
  - o **innovator's profit:** $\Pi_R = (\Pi / r_D)\dot{M} - wL_2 = (\Pi / r_D)\lambda L_2 M - wL_2$
  - o **labor demand:** $MPL_2 = (\Pi / r_D)\lambda M = w$
- **The two labor demand conditions together with intermediate good supply and maximized intermediate firm profit yield:**
  - o **market discount rate:** $r_D = \alpha\lambda L_1$
  - o **so the rate of return to R&D per unit of M can be expressed as:**

    $r = \alpha\lambda L_1$ **(RD)**

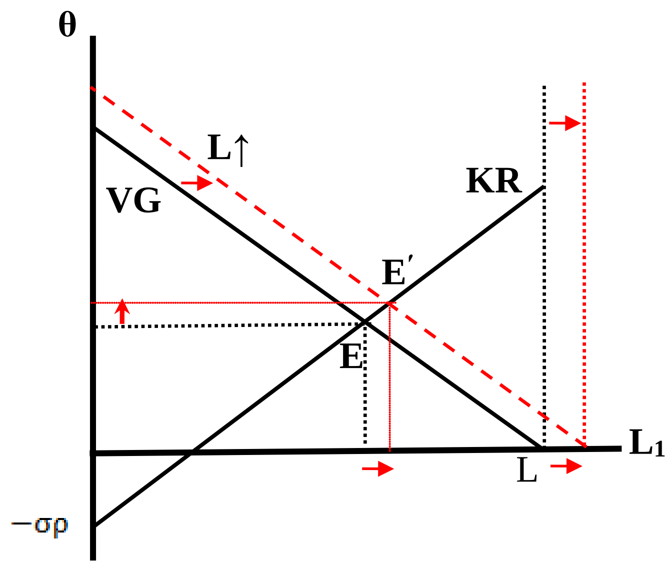
    - **higher λ raises R&D efficiency => higher return to R&D**
    - **higher α lowers intermediate firm's markup => to restore free entry requires higher return to R&D**

- **Intermediate varieties growth (VG):** $\theta = \frac{\dot{M}}{M} = \lambda L_2 = \lambda(L - L_1)$
  - **downward sloping in L_1**
  - **higher λ improves R&D efficiency and thus raises intermediate varieties growth**
- **Keynes-Ramsey (KR) using (RD):** $\theta = \sigma(r - \rho) = \sigma(\alpha\lambda L_1 - \rho)$
  - **upward sloping in L_1**
  - **higher λ improves R&D efficiency and enhances output growth**
  - **higher α lowers intermediate firm's markup and raises the return to R&D**



- **Main findings:**
  - **higher R&D productivity λ encourages R&D, reallocates labor away from production and raises economic growth (both VG and KR rotate up)**

- **higher α lowers the markup, raises the return to R&D, reallocates labor away from production and raises economic growth (KR rotates up):**
  - **this is not entirely intuitive, due mainly to the free entry condition under monopolistic competition that leads to negative relationship between the markup and the rate of return to R&D**



  - **while a more sophisticated model of monopolistic competition can fix this problem, one may simply resource to Aghion-Howitt's monopoly setup**
- **larger employment size (L↑) raises production labor, R&D labor as well as growth: thus, there is a scale effect, that is, larger countries grow faster, which is unfortunately unrealistic, as pointed out by Jones (1995) – one may fix this problem by a concave transformation of CES aggregator or by shifting to a perfectly competitive setting, such as in Wang (in progress)**

E. Technology Gap and R&D: Wang (in progress)

- **Innovation versus implementation:**
  - **the leading-edge frontier technology: $\bar{A}$, growing on the *quality ladder* at rate $\gamma(n)$, depending on R&D effort n**
  - **fraction of sectors on the frontier (innovating sectors): η**
  - **fraction of sectors below the frontier (implementing sectors): $(1-\eta)$**
  - **technology gap: $\bar{A} - A$, with its effect on technical progress depending on implementation effort m – it is convenient to denote the technology gap ratio as $(\bar{A}-A)/A = a > 0$**
- **Technology advancement:**

$$\dot{A}/A = \eta\gamma(n) + (1-\eta)\psi(m)(\bar{A} - A)/A$$

  - **$\gamma = \gamma_0 n$ captures the frontier technology expansion rate**
  - **$\psi = \psi_0 m^b$ captures the imitation technology, where the effectiveness of imitation depends on a**
- **The fraction η is given exogenously in the benchmark setting. In a more general setup, $\eta = \eta_0 N^{1-\beta}$ and $\gamma = \gamma_0 n^{\beta}$, where the society's innovation effort N is regarded as given by individuals and N = n in equilibrium**
- **Effective labor: L=A(1-n-m)**

- **Goods production (sector 1):** $F(K,L) = K^{1-\beta}(A(1-n-m))^{\beta}$
- **Capital accumulation (budget constraint):**

  $\dot{K} = K^{1-\beta}(A(1-n-m))^{\beta} - \delta K - c$, with $K(0) = K_0 > 0$
- **Optimization:**

  $$\max \quad U = \int_0^{\infty} \frac{c^{1-\sigma^{-1}} - 1}{1-\sigma^{-1}} e^{-\rho t} dt$$

  s.t. $\dot{K} = K^{1-\beta}(A(1-n-m))^{\beta} - \delta K - c$, $K(0) = K_0 > 0$

  $\dot{A} = \eta\gamma(n)A + (1-\eta)\psi(m)(\bar{A} - A)$, $A(0) = A_0 > 0$

  where $\bar{A}$ is taken as given by individuals, $\dot{\bar{A}}/\bar{A} = \gamma$, $(\bar{A}-A)/A = a > 0$
- **First-order conditions (w.r.t. c, n and m):**

  $c^{-1/\sigma} = \lambda$

  $MPn_1 = MPn_2$ or $\lambda\beta\left(\frac{K}{A(1-n-m)}\right)^{1-\beta} = \mu\eta\gamma_0$

  $MPm_1 = MPm_2$ or $\lambda\beta\left(\frac{K}{A(1-n-m)}\right)^{1-\beta} = \mu(1-\eta)b\psi_0 m^{b-1}a$

- **Euler equations (w.r.t. K and A):**

$$\dot{\lambda} = \lambda\left[\rho + \delta - (1-\beta)\left(\frac{K}{A(1-n-m)}\right)^{-\beta}\right]$$

$$\dot{\mu} = \mu(\rho - \eta\gamma_0 n + (1-\eta)\psi_0 m^b) - \lambda\left[\beta(1-n-m)\left(\frac{K}{A(1-n-m)}\right)^{1-\beta}\right]$$

- **TVCs:** $\lim_{t\to\infty}\lambda K e^{-\rho t} = 0$, $\lim_{t\to\infty}\mu A e^{-\rho t} = 0$
- **From the FOCs w.r.t. n and m,** $\eta\gamma_0 = (1-\eta)b\psi_0 m^{b-1}a$, **or,**

$$m(a) = \left(b\frac{1-\eta}{\eta}\frac{\psi_0}{\gamma_0}a\right)^{1/(1-b)}$$

yielding a positive relationship between m and a, $m_a > 0$, depending:
  - **negatively on frontier technology growth γ_0**
  - **positively on implementation efficiency ψ_0**
  - **negatively on the fraction of sectors on the frontier η**
- **From the two evolution equations, c, K, A, $\bar{A}$ and Y must all grow at rate $\theta = \gamma = \gamma_0 n$, along the BGP; thus, $n = \theta / \gamma_0$**

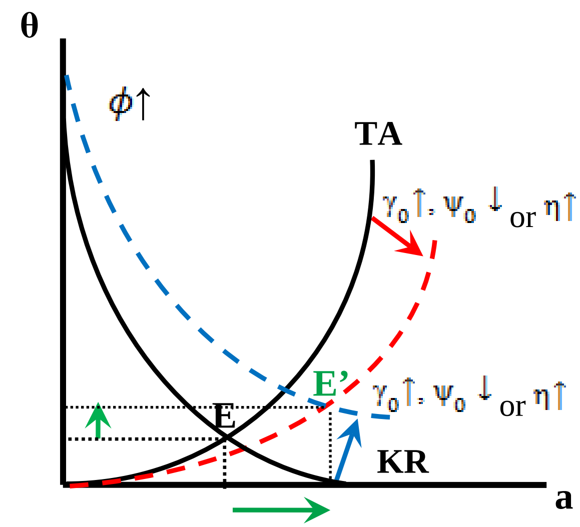
- **Manipulating first-order conditions and Euler equations give two Keynes-Ramesy equations:**
  - $\theta = \frac{\dot{c}}{c} = -\sigma\frac{\dot{\lambda}}{\lambda} = \sigma\left[(1-\beta)\left(k/(1-n-m)\right)^{-\beta} - (\rho+\delta)\right]$
    - **solving BGP effective capital k = K/A as a function of (a, θ) in a recursive manner:**

      $$k = \left(\frac{1-\beta}{\rho+\delta+\sigma^{-1}\theta}\right)^{1/\beta}\left[1-\eta\gamma_0 - \left(b\frac{1-\eta}{\eta}\frac{\psi_0}{\gamma_0}a\right)^{1/(1-b)}\right]$$
    - **k depending negatively on technology gap ratio and growth (via r)**
  - $\theta = \sigma\left\{\eta\gamma_0 - \rho - (1+ba)\left[(1-\eta)\psi_0\left(ba/(\eta\gamma_0)\right)^b\right]^{1/(1-b)}\right\}$
    - **yielding a KR locus in (a, θ) space, which entails a negative relationship between the technology gap ratio and growth**
    - **imitation lowers growth by reducing firms' incentive to innovate**
    - **for a given technology gap a, higher γ_0, lower ψ_0 or more frontier sectors η will raise growth θ (i.e., KR locus shifts upward)**

- **The second constraint yields the technology advancement rule (TA):**

$$\theta = \frac{1}{1-\eta\gamma_0}\left[(1-\eta)\psi_0\left(\frac{b}{\eta\gamma_0}\right)^b a\right]^{1/(1-b)}$$

  - **yielding a positive relationship between technology gap and growth**
  - **imitation is productive when an economy is far below the frontier**
  - **when $b > \eta\gamma_0$, for a given θ, higher frontier technology growth γ_0, lower implementation efficiency ψ_0 or fewer frontier sectors η will enlarge the technology gap ratio a (i.e., TA locus shifts rightward)**
- **Main findings:**
  - **Both innovation and imitation are valuable: the larger the technology gap ratio a, the more valuable implementation effort m is**
  - **Higher frontier growth γ_0, lower implementation efficiency ψ_0 or a larger fraction of frontier sectors η will promotes economic growth but widens the technology gap ratio**



F. Organizational Capital: Atkeson and Kehoe (2005)

- **Organizational capital is an important part of intangible capital**
- **Organizational capital can be tied to the life cycle of a plant:**
  - **variable profit of a plant of age s:** $d_s = \max_l f_s(l) - wl$
  - **cost of the fixed factor: w_m**
  - **organization rent:** $d_s - w_m$
  - **free entry condition:** $\sum_{s=0}^{N} \left(\frac{1}{1+i}\right)^s (d_s - w_m) = 0$
  - **cross-section aggregate organization rent:** $\pi = \sum_{s=0}^{N} (d_s - w_m)$
  - **if MPL rises with plant age (learning by doing), then older plants will be larger and hire more labor than younger ones**
  - **thus, organizational capital is summarized by the plant-specific productivity (f_s') as well as the age of the plant (s)**
  - **letting variable profit to grow at a constant rate $\gamma > 1$ (i.e., $d_s = \gamma^s d_0$), we can then use free entry condition to obtain:** $w_m = d_0 \frac{\sum_{s=0}^{N} [\gamma/(1+i)]^s}{\sum_{s=0}^{N} [1/(1+i)]^s}$
  - **thus,** $\pi = d_0(N+1) \sum_{s=0}^{N} \gamma^s \omega_s$**, where** $\omega_s = \frac{1}{N+1} - \frac{[1/(1+i)]^s}{\sum_{s=0}^{N} [1/(1+i)]^s}$

1. The Basic Model

- **Preference: U =** $\sum_{t=0}^{\infty} \beta^t \log(c_t)$
- **Budget constraint:** $\sum_{t=0}^{\infty} p_t c_t \leq \sum_{t=0}^{\infty} p_t(w_t + w_{mt}) + k_0 + a_0$
- **Production:** $y = zA^{1-\nu}F(k,\ l)^{\nu}$
    - **F is CRTS**
    - **z = aggregate technology**
    - **v = span of control parameter determining the return to scale (Lucas 1978)**
- **Organization capital (A, s): a plant with organization capital (A, s) at t has stochastic organization capital (Aε, s+1) at t+1**
- **Time-to-build: a plant built in t-1 can start operating in t**
- **Frontier knowledge: productivity τ_t, adopted by all new plants, implying a new plant built in t-1 will have organization capital (τ_t, 0) at t**
- **Plant optimization:** $\max_{k,l} z_t A^{1-\nu}F(k,\ l)^{\nu} - r_t k - w_t l - w_{mt}$
    - **variable profit:** $d_t(A) = z_t A^{1-\nu}F(k_t(A),\ l_t(A))^{\nu} - r_t k_t(A) - w_t l_t(A)$
    - **fixed cost of hiring a manager (one per plant, fixed supply): w_m**
    - **Bellman:** $V_t(A,\ s) = \max\left[0,\ d_t(A) - w_{mt} + \frac{p_{t+1}}{p_t}\int_{\epsilon} V_{t+1}(A\epsilon,\ s+1)\pi_{s+1}(d\epsilon)\right]$

- **Plant operating decision $x_t(A, s)$ (=1 if operating, =0 otherwise)**
- **Plant establishment decision, determined by the value of a new plan:** $V_t^o = -w_{mt} + \frac{p_{t+1}}{p_t} V_{t+1}(\tau_{t+1}, 0) \geq 0$**, which pins down the measure of managers** ϕ_t
- **Measure of operating plants:** $\lambda_t(A, s) = \int_0^A x_t(a, s)\mu_t(da, s)$**, with the distribution evolving as:** $\mu_{t+1}(A', s+1) = \int_A \pi_{s+1}\left(\frac{A'}{A}\right)\lambda_t(dA, s)$
- **Factor market clearing:**
  - **capital:** $\sum_s \int_A k_t(A)\lambda_t(dA, s) = k_t$
  - **labor:** $\sum_s \int_A l_t(A)\lambda_t(dA, s) = 1$
  - **manager:** $\phi_t + \sum_s \int_A \lambda_t(dA, s) = 1$
- **Goods market clearing:** $c_t + k_{t+1} = y_t + (1-\delta)k_t$ **, where aggregate output is given by** $y_t = z_t \sum_s \int_A A^{1-\nu} F(k_t(A), l_t(A))^\nu \lambda_t(dA, s)$
- **Plant size:** $n_t(A) = \left(\frac{A}{\overline{A}_t}\right)$, $\overline{A}_t = \sum_s \int_A A\lambda_t(dA, s)$ **= aggregate specific productivity**
- **Equilibrium allocation:** $k_t(A) = n_t(A)k_t$ **and** $l_t(A) = n_t(A)l_t$
- **Equilibrium output:** $y_t(A) = n_t(A)y_t = n_t(A) z_t\overline{A}_t^{1-\nu}F(k_t, l_t)^\nu$
- **Equilibrium variable profit:** $d_t(A) = (1-\nu)y_t(A) = (1-\nu)n_t(A)y_t$

2. Generalization: Monopolistic Competition

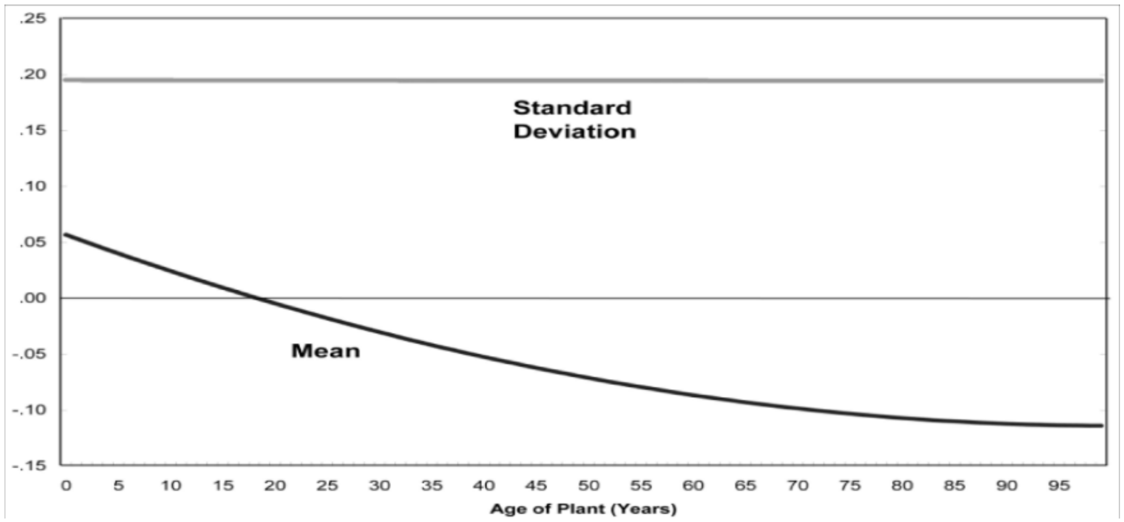
- **The competitive final good output:** $y_t = \left[\sum_s \int_A y_t(A)^\theta \lambda_t(dA,\ s)\right]^{1/\theta}$**, implying the demand schedule for intermediate goods:** $y_t(A) = p_t(A)^{-1/(1-\theta)} y_t$
- **Supply of intermediate goods:** $y_t(A) = z_t^{1/\theta} A_t^{(1-\gamma\theta)/\theta} F(k_t(A),\ l_t(A))^\gamma$**, with the powers adjusted to include the markup accrued from local monopoly power**
- **All other setups remain the same**

3. Calibration Analysis

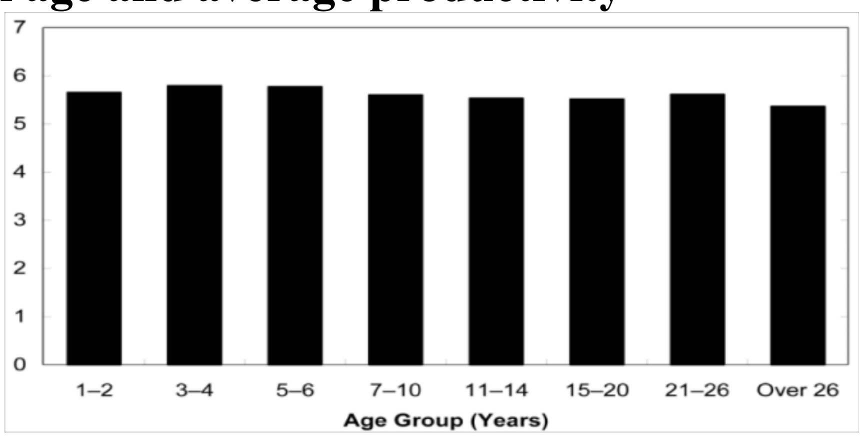
- **Use standard macroeconomics and firm-distribution parameters and set the markup parameter to θ = 0.9 and the span of control parameter to γ = 0.95**
- **The rates of job turnovers can then be computed (based on the definition by Davis-Haltiwanger-Schuh 1996):**

	Data	Model
Overall job creation rate	8.3	10.2
Overall job destruction rate	8.4	10.2

- **Mean and standard deviation of shocks to ln(n):**



- **Firm age and average productivity**



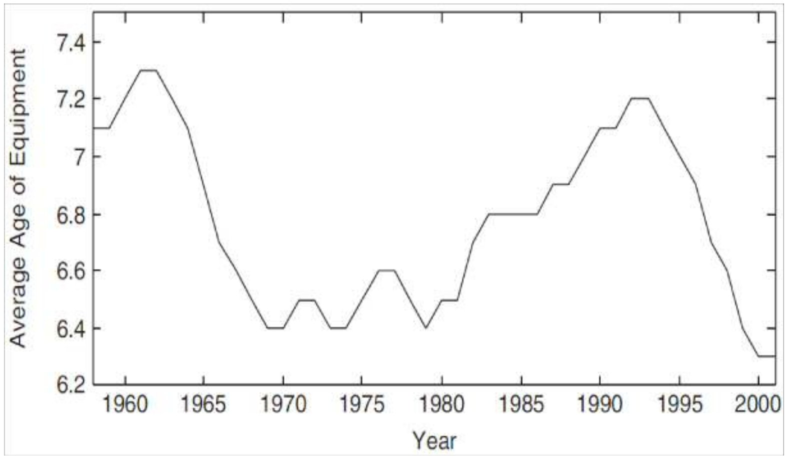
- **Measurement of organizational capital and growth accounting**
  - **physical capital income share: $\theta\gamma\alpha = 19.9\%$**

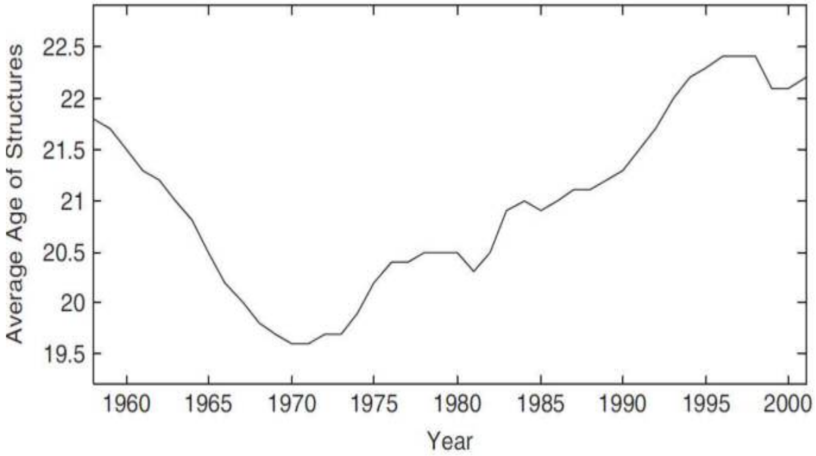
- labor income share: $\theta\gamma(1-\alpha) = 65.1\%$
- managerial and organization rent share: $1-\theta\gamma = 15\%$
  - by using the expression for w_m, managerial rent share is: 11.7%
  - organization rent share is: 3.3%
- Varying $\nu = \theta\gamma$ by 5 percentage points, we obtain:

	Data on U.S. Manufacturing*	Model $\nu = .80$	Model $\nu = .85$	Model $\nu = .90$
	Shares of Output (%)			
Labor	72.2	75.7	76.8	77.9
Workers	. . .	60.1	65.1	70.1
Managers	. . .	15.6	11.7	7.8
Physical capital	19.9	19.9	19.9	19.9
Intangible capital	8.0	. . .	. . .	. . .
Organization capital	. . .	4.4	3.3	2.2

G. Organizational Capital and Productivity Slowdown: Samaniego (2006)

- Link organizational capital to productivity growth to explain establishment lifecycles and productivity slowdown of the 1970s and the 1980s based on changing patters of technological adoption of fast-depreciating IT capital, whose share in equipment investment rose from 7% to 56%
- Average age of capital (equipment vs. structures)





  - average age of equipment capital *rose* over 1970-92
  - this, together with the fact of fast depreciation of the IT capital, implies that the adoption of new capital *decelerated* over the period
  - over the same period, Hobijn-Jovanovic (2001) find that young firms outperformed old ones

1. The Model

- **Production of a plant of age τ and experience *a*:** $y_t = \gamma_n^t \gamma_s^{t-\tau} \Omega(a) k_t^{\alpha_k} n_t^{\alpha_n}$**, where γ_n and γ_s measure general and plant-specific technologies, Ω(*a*) measures accumulated plant-level expertise, and $\alpha_k + \alpha_n < 1$**
- **Organizational capital: π =** $\gamma_s^{t-\tau}\Omega(a)$ **= plant-specific component of productivity**
- **Updating cost from age τ to age ν:** $\kappa(v) = \kappa\gamma_s^{t-v}/\gamma_s^t = \kappa\gamma_s^{-v}$
- **Measure of plant follows a process $\mu_{t+1} = \Gamma(\mu_t)$, relying on entry/exit/updating**
- **Plant values:**
  - **growing factor on BGP:** $\gamma_y = \gamma_n^{1/(1-\alpha_k)}\gamma_s^{1/(1-\alpha_k)}$
  - **plant value deflated by γ_y:** $P(\tau, a, X_t) = \max_{k_t, n_t} \{\gamma_s^{-\tau}\Omega(a)k_t^{\alpha_k}n_t^{\alpha_n} - k_t r(X_t) - n_t w(X_t)\}$
  - **continuation value:** $C(\tau, a, X_t) = \max\{W(\tau+1, a+1, \Gamma(X_t)), U(a, X_t)\}$
  - **updating value:** $U(a, X_t) = \max_{0 \leqslant v} \{W(v, a+1, \Gamma(X_t)) - \kappa\gamma_s^{-v} p(\Gamma(X_t))\}$
  - **optimal updating rule:** $\Upsilon(\tau, a, X_t) = \begin{cases} 1 & \text{if } U(a, X_t) \geqslant W(\tau+1, a+1, \Gamma(X_t)) \\ 0 & \text{otherwise.} \end{cases}$
  - **value function:** $W(\tau, a, X_t) = P(\tau, a, X_t) + \frac{\gamma_y}{1+i}\lambda(a)C(\tau, a, X_t)$
  - **properties:**
    - **updating follows (S,s)-rule, with full updating once decided to do so**
    - **adoption lags decrease with age**

- **Firm profit Π will be redistributed to households (measure one)**
- **Managerial supply: a household invests e to generate φ(e) managerial units paid at p**
- **One manager per plant: managerial units φ and new plant establishment investment (q) are homogeneous**
- **Household optimization given state X = (μ, K):**

max $\sum_{t=0}^{\infty} \beta^t \{\ln c_t + \xi(\Xi - n_t)\}$

s.t. $c_t + I_t + p(X_t)q_t \leqslant \Pi(X_t) + r(X_t)K_t + w(X_t)(n_t - e_t) + p(X_t)\phi(e_t)$

$K_{t+1} = I_t + (1-\delta)K_t$

2. Equilibrium and Productivity Slowdown

- **Stationary recursive competitive equilibrium with $\Gamma(\mu^*) = \mu^*$**
- **Productivity slowdown due to an *organizational shock*:**
  - **after $t^* = 1973$, learning accumulated before is no longer compatible with new technologies born since t^***
  - **plants established before t^* must suffer by starting from the lowest rung of the learning ladder with:** $U(a, X_t) = \max_{0 \leqslant v} \{W(v, 0, \Gamma(X_t)) - \kappa\gamma_s^{-v} p(\Gamma(X_t))\}$

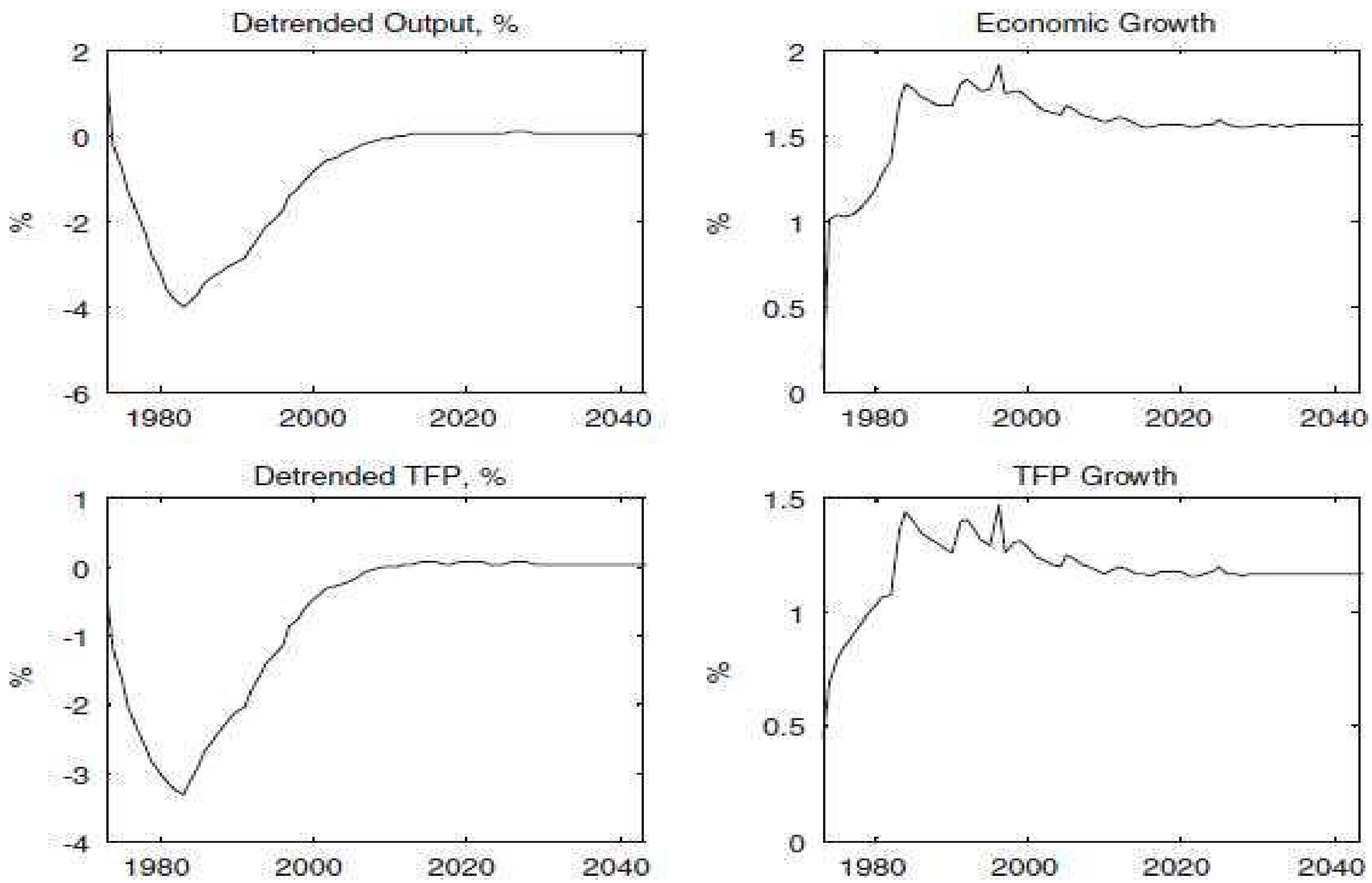
- **Responses to the organizational shock:**
  - **given fixed prices,**
    - **slower updating: adoption lags increase for all plant types**
    - **faster turnovers (entry/exit): the ratio of values of the incumbent to the new entrant drops**
    - **age-biased updating: young plants update before older ones**

3. Calibration Analysis

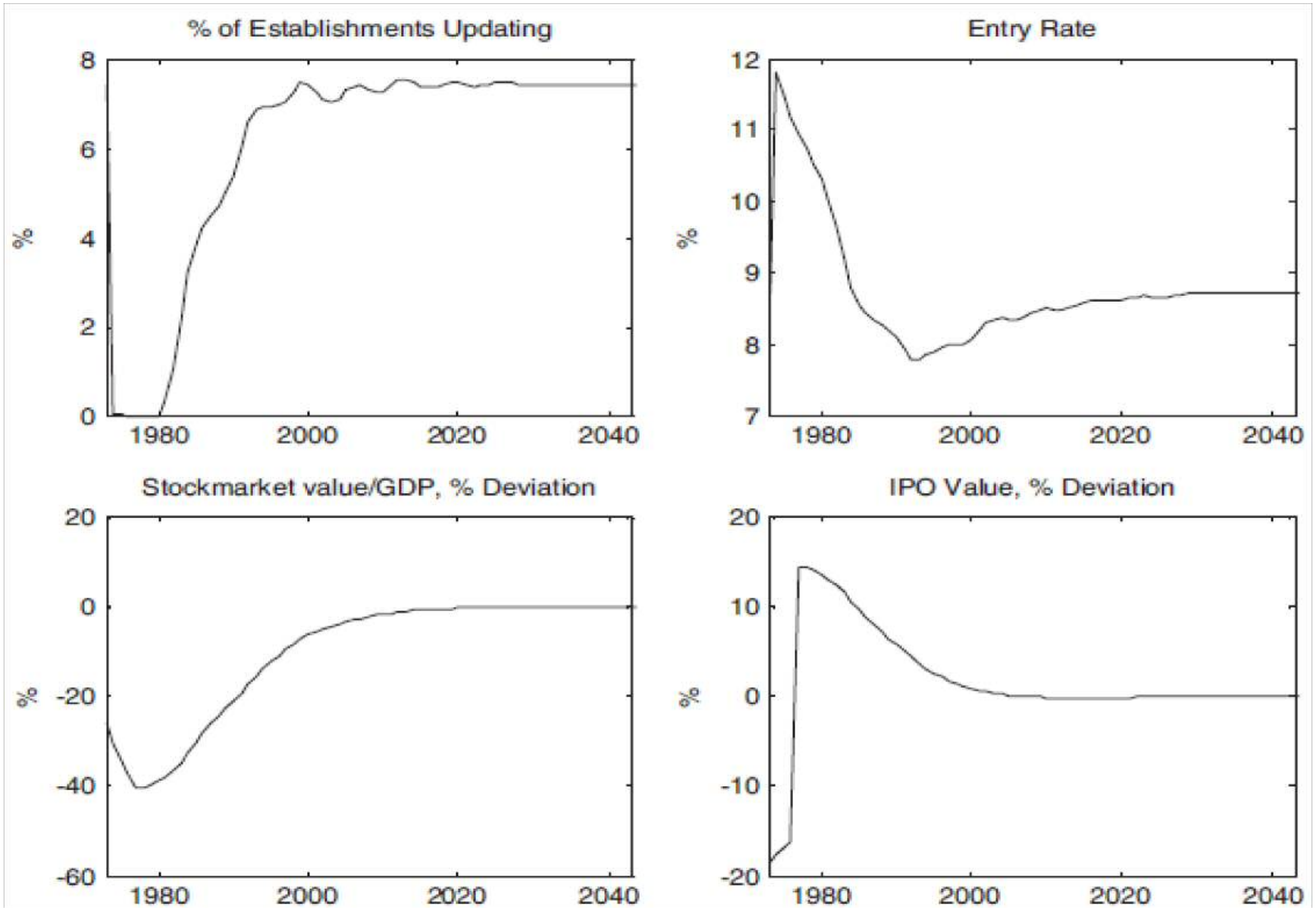
- **Lifecycle dynamics:**

Statistic	US data	Model
Plant growth (%)	35	30.5
Plant growth, young (%)	45	47.8
Relative size of the young (%)	77	67
New establishments (%)	9.2	8.6
5-year exit rate (%)	36	37
5-year exit rate, young plants (%)	39	40
Lumpy investments (%)	25	25.5
Lumpy investors (%)	8	7.5
Updating lag, years	5–8	7.5
Time to average learning	5–10	8
Average age of capital	12	11

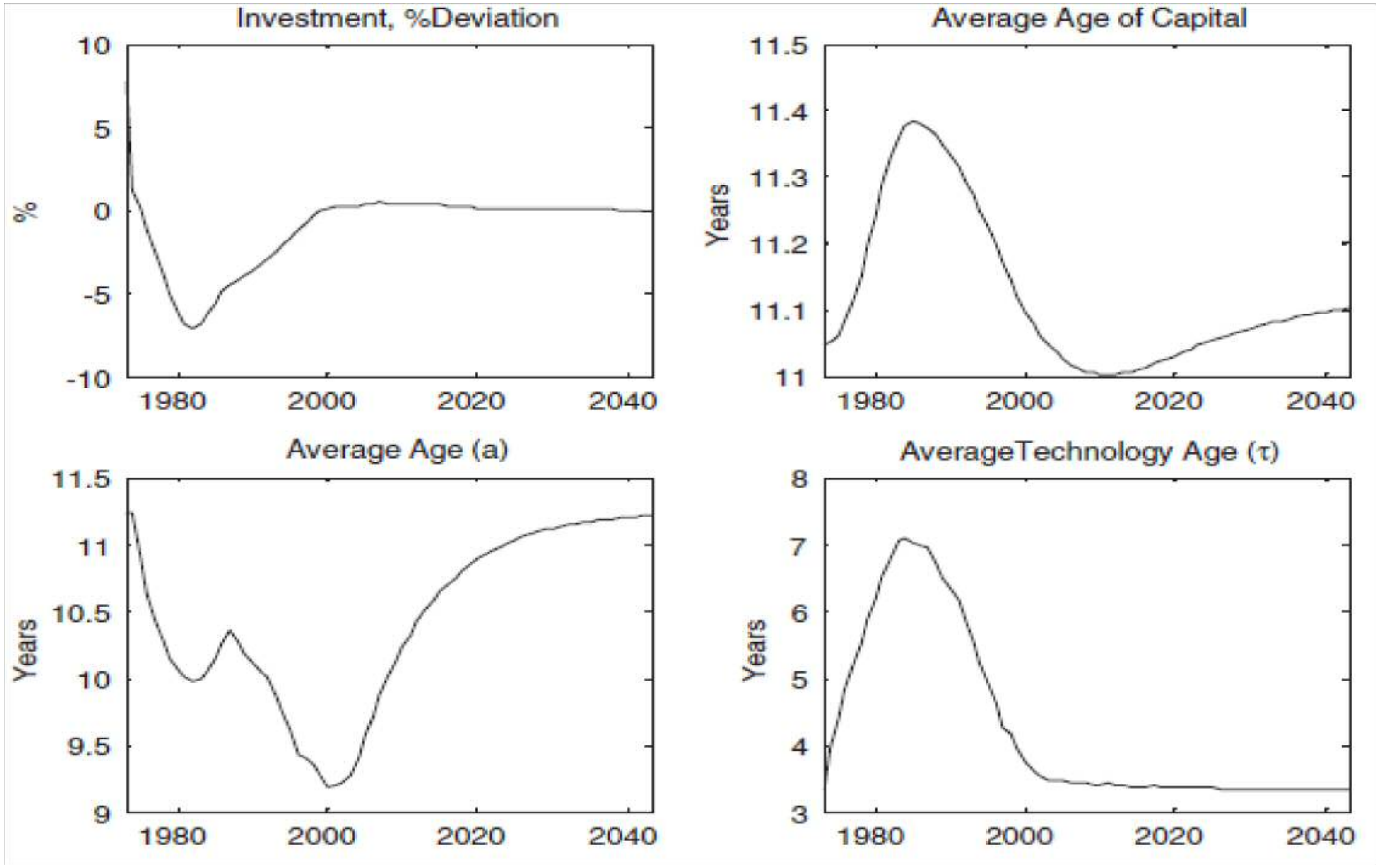
- **Impulse responses to the organizational shock**
  - **output and productivity**



- **plant dynamics and values**



- **plant investment and age**



H. Technology Assimilation and Development: Wang-Wong-Yip (2018)

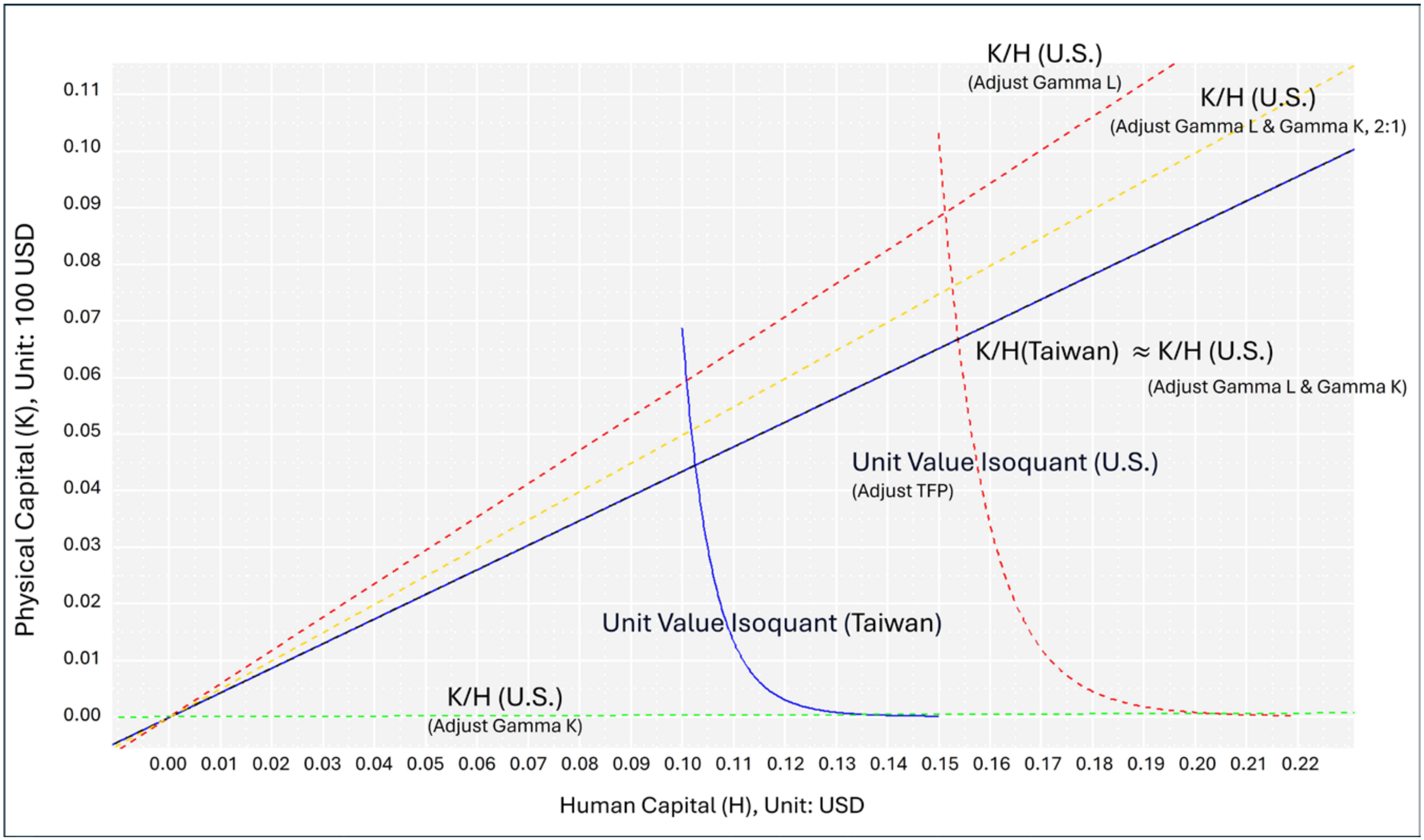
- **Country-specific assimilation ability and the gap of the factor input ratio relative to the frontier country may interact, serving to explain the cross-country relative income disparities**
- **By establishing an assimilation framework using normalized CES with country m assimilating the frontier technology of s, one may decompose output growth after assimilation as:**

$$\hat{y} = \underbrace{\hat{z}_s}_{\text{source TFP growth}} + \underbrace{\sum_{m=1,\dots,N} \alpha_m \hat{n}_m}_{\text{factor growth}} + \underbrace{\sum_{m=1,\dots,N} (\pi_m - \alpha_m)(\hat{n}_m - \hat{n}_{s,m})}_{\text{change in mismatch}}$$

  - α_m **= technology share**
  - π_m **= factor input share**
  - $\hat{n}_m - \hat{n}_{s,m}$ **= rate of change in the gap of the factor input ratio**
  - **so growth is decomposed into 3 components:**
    - **source TFP growth**
    - **country-specific factor accumulation**
    - **new mismatch component driven by ability to assimilate (zero if Cobb-Douglas) interacting with changes in the gap of the factor input ratio**

A diagrammatic illustration of assimilation in semiconductor foundry industry:

US subsidiaries vs. TSMC headquarter (Lee-P. Wang-S. Wang 2024)



- **Growth accounting:**

	#	Relative growth(pp) Income	Capital	Human capital	Contribution to growth(%) Capital	Human capital	Mismatch	TFP gap
Fraction of U.S. income in 1960								
$\leq 25\%$	39	0.66	0.36	0.39	28.84	6.38	49.93	14.85
$(25\%, 50\%]$	16	0.97	1.13	0.23	25.46	5.52	10.21	58.81
$(50\%, 75\%]$	10	0.21	0.77	0.01	36.19	3.07	4.56	56.18
$> 75\%$	3	0.01	0.36	-0.11	-28.86	20.89	-1.95	109.93
Fraction of U.S. income in 2010								
$\leq 25\%$	56	-0.15	-0.31	0.47	32.55	-14.29	60.63	21.11
$(25\%, 50\%]$	21	1.16	1.65	0.49	35.49	26.00	23.79	14.73
$(50\%, 75\%]$	16	1.33	1.73	0.24	37.74	17.18	19.70	25.38
$> 75\%$	13	1.32	1.40	0.14	31.48	8.77	6.31	53.44
Fraction of U.S. growth rate								
$\leq 50\%$	23	-1.74	-1.45	0.51	21.58	-24.76	51.24	51.94
$(50\%, 100\%]$	17	-0.28	-0.20	0.17	39.81	-54.78	55.36	59.61
$(100\%, 200\%]$	47	0.96	1.13	0.34	38.89	28.91	36.13	-3.93
$> 200\%$	19	2.84	2.46	0.61	30.53	14.57	24.96	29.94
Overall	106	0.51	0.60	0.40	33.78	1.27	40.49	24.45

- **Growth accounting-trapped economies:**

	Relative growth(pp)			Growth contribution(%)			
	Income	Capital	Human capital	Mismatch	Capital	Human capital	Country-spec assi
Burundi	-1.58	-1.37	0.30	69.13	28.95	-12.49	69.15
Benin	-1.55	-2.35	1.04	116.55	50.59	-44.85	146.03
Central African	-3.33	-3.23	0.58	67.30	33.26	-11.62	78.15
Côte d'Ivoire	-0.23	-2.15	0.22	438.35	311.38	-64.39	687.14
Congo D.R.	-2.30	-1.30	0.14	40.67	18.82	-4.08	32.78
Ghana	-1.35	-2.50	0.74	91.53	61.60	-36.39	159.60
Gambia	-1.34	0.33	0.70	15.97	-8.11	-34.62	17.41
Kenya	-0.54	-1.78	0.48	212.50	108.59	-58.73	275.92
Cambodia	-0.80	-0.68	0.54	93.17	28.31	-44.80	101.37
Madagascar	-1.52	-2.42	0.01	98.41	53.17	-0.30	106.65
Mauritania	-1.08	-0.08	0.51	17.80	2.56	-31.42	35.51
Malawi	-1.14	-0.47	0.02	26.84	13.67	-1.02	28.36
Niger	-2.51	-3.58	-0.27	55.46	47.57	7.22	87.92
Senegal	-2.01	-3.36	0.11	48.09	55.71	-3.50	108.37
Sierra Leone	-1.32	-1.75	0.63	107.48	44.30	-31.68	120.21
Togo	-2.78	-3.04	0.70	78.23	36.38	-16.85	89.61
Tanzania	-0.43	-0.45	-0.15	36.42	34.23	22.52	45.94
Zimbabwe	-3.07	-3.40	1.16	83.57	36.92	-25.28	99.02
Average	-1.60	-1.87	0.41	94.25	53.22	-21.79	127.17

- **Growth accounting-development miracles:**

	Relative growth(pp)			Growth contribution(%)			
	Income	Capital	Human capital	Mismatch	Capital	Human capital	Country-spec assi
Hong Kong	1.83	2.13	0.50	2.59	38.67	18.15	22.80
Korea	3.18	4.17	0.53	46.73	43.79	11.22	57.67
Singapore	2.95	2.01	1.05	-0.80	22.82	23.94	-0.02
Taiwan	3.05	3.51	0.68	26.58	37.39	14.87	28.95
Botswana	2.48	2.27	1.82	4.04	30.56	48.99	12.14
China	1.79	4.08	0.71	101.41	76.10	26.63	111.33
India	0.97	0.37	0.36	0.19	12.62	24.97	0.27
Malaysia	1.64	1.71	0.80	17.78	34.67	32.63	31.28
Thailand	1.83	1.94	0.66	30.81	35.39	23.94	46.84
Vietnam	1.83	2.76	0.83	65.24	50.38	30.36	69.55
Average	2.15	2.50	0.80	29.46	38.33	25.55	38.08

- **Identifying the middle-income trap:**

Country	Chow's test	Eichengreen et. al. (2014)	Wang-Wong-Yip
Denmark	1969	1968-70, 1973	1977
Spain	**1975**	1966, 1969, **1972-7**	**1974**
Finland	**1974**	**1974-5**, 2002-3	**1976**
France	**1974**	**1973-4**	**1971**
Greece	**1972**	**1969-78**, 2003	**1972**
Hong Kong	1993	**1981-2**, 1990-4	**1984**
Portugal	1974,1990	1973-4, 1977, 1990-2	1966
Taiwan	**1995**	**1992-7**	**1999**, 2011, 2012

  - **middle-income trap arises when**
    - **an relatively fast-growing country initially narrows income gap by mitigating the disadvantageous factor's disadvantage**
    - **at a later development stage, this country faces factor advantage reversal, over accumulating the originally disadvantageous factor that becomes an advantageous factor**
    - **growth slows down as a consequence**

I. Technology – On Modeling Automation: Acemoglu-Restrepo(2018)

- **Roy's (1951) task-based production model with technology advancement**
- **Production:** $Y = \widetilde{B}\left(\int_{N-1}^{N} y(i)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}$
  - **the quality of the tasks over [N-1,N] being updated over time**
  - **∃ I ∈ [N-1,N] s.t. i ∈ [N-1,I] automated, i ∈ [I,N] non-automated**
  - **non-automated tasks are produced with intermediate *q* and labor *l***

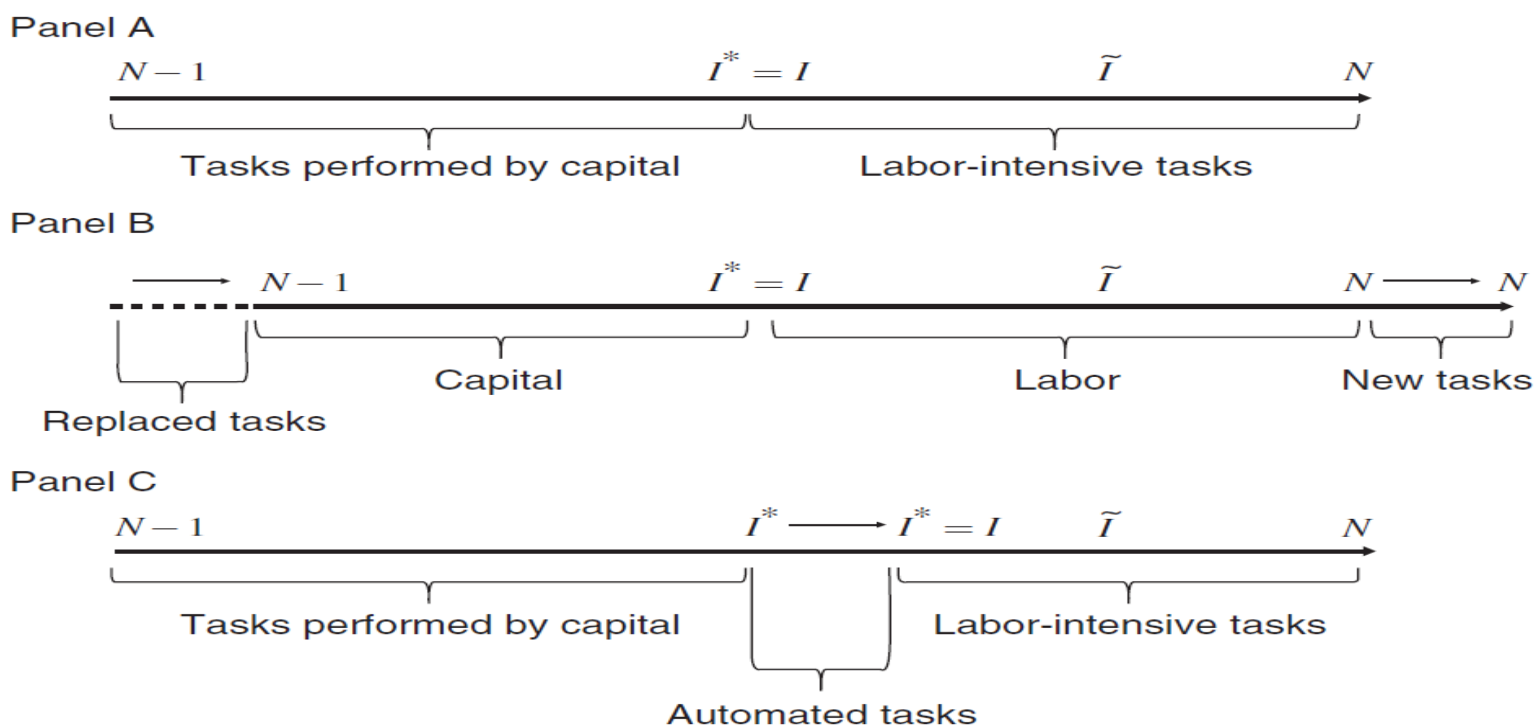
$$y(i) = \overline{B}(\zeta)\left[\eta^{\frac{1}{\zeta}} q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)^{\frac{1}{\zeta}} (\gamma(i)\, l(i))^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}$$

  - **automated tasks are produced with *q*, *l* and capital *k***

$$y(i) = \overline{B}(\zeta)\left[\eta^{\frac{1}{\zeta}} q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)^{\frac{1}{\zeta}} (k(i) + \gamma(i)\, l(i))^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}$$

  - **γ(i) is increasing in i (the higher i, the more labor intensive)**
- **Utility:** $u(C,L) = \frac{\left(Ce^{-\nu(L)}\right)^{1-\theta} - 1}{1-\theta}$**, with disutility of labor measured by *v***

- **Technology advancement via arrival of new tasks with better quality and more automated tasks**



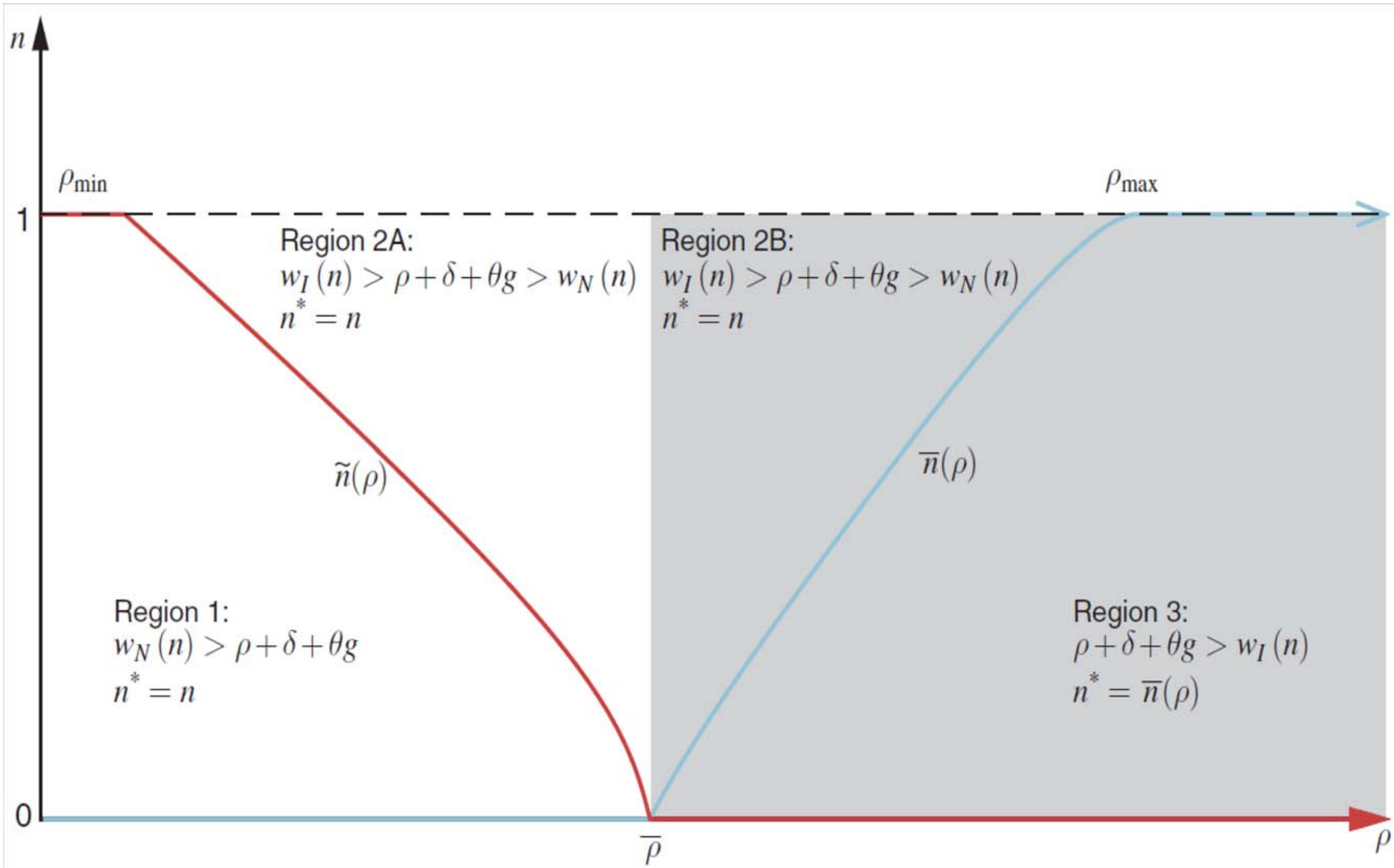
  - **threshold task** $\frac{W}{R} = \gamma(\tilde{I})$**: those below are, if unconstrained, produced with k (cost-minimized)**
  - **equilibrium** $I^* = \min\{I, \tilde{I}\}$**: if** $\tilde{I} > I$**, then firms are constrained by I, unable to produce up to the threshold (under-automation due to constraint)**

- **Static equilibrium with aggregate output captured by:**

$$Y = \frac{B}{1-\eta}\left[(I^* - N + 1)^{\frac{1}{\hat{\sigma}}} K^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} + \left(\int_{I^*}^{N} \gamma(i)^{\hat{\sigma}-1} di\right)^{\frac{1}{\hat{\sigma}}} L^{\frac{\hat{\sigma}-1}{\hat{\sigma}}}\right]^{\frac{\hat{\sigma}}{\hat{\sigma}-1}}$$

  - **without being constrained by automation technology, costs are minimized and hence I has no effect on W/R**
  - **being constrained, higher I relaxes the constrained and hence lower W/R (more relative returns on k)**
  - **regardless of the constraint, higher N (task upgrading) => higher w/R (k more likely constrained by automation technology)**
- **Introducing dynamics:**
  - **labor-biased technical progress** $\gamma(i) = e^{Ai}$
  - **standard lifetime utility with time preference rate ρ**
  - **standard capital evolution with depreciation rate δ**
  - **{I(t), N(t)} exogenous, as dose n(t) = N(t) - I(t)**
  - **BGP with g(t) = growth rate of exp(AI*(t))**
  - **Euler equation:** $\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}(F_K[k(t), L(t); n^*(t)] - \delta - \rho) - g(t)$
  - **Labor-leisure choice:** $\nu'(L(t))\, e^{\frac{\theta-1}{\theta}\nu(L(t))} = \frac{F_L[k(t), L(t); n^*(t)]}{c(t)}$

- **Equilibrium:**



- n = 1: no automation
- Interior BGP:
  - Region 1: n < $\widetilde{n}(\rho)$ => γ(N(t)) < W(t)/R(t) => new tasks would lower aggregate output, so not adopted
  - Region 2A: n > $\widetilde{n}(\rho)$ => γ(N(t)) > W(t)/R(t) => new tasks raise aggregate output, so adopted and produced with labor
  - Region 2B: n > $\overline{n}(\rho)$ => W(t) > R(t) => automated new tasks raise aggregate output, so adopted and produced with capital
  - Region 3: n < $\overline{n}(\rho)$ => W(t) < R(t) => automation would lower aggregate output, so no change in response to small changes in automation technology – eventual automation when R(t) becomes cheaper than W(t)
- n = 0: full automation

Automation need not lead to labor displacement:

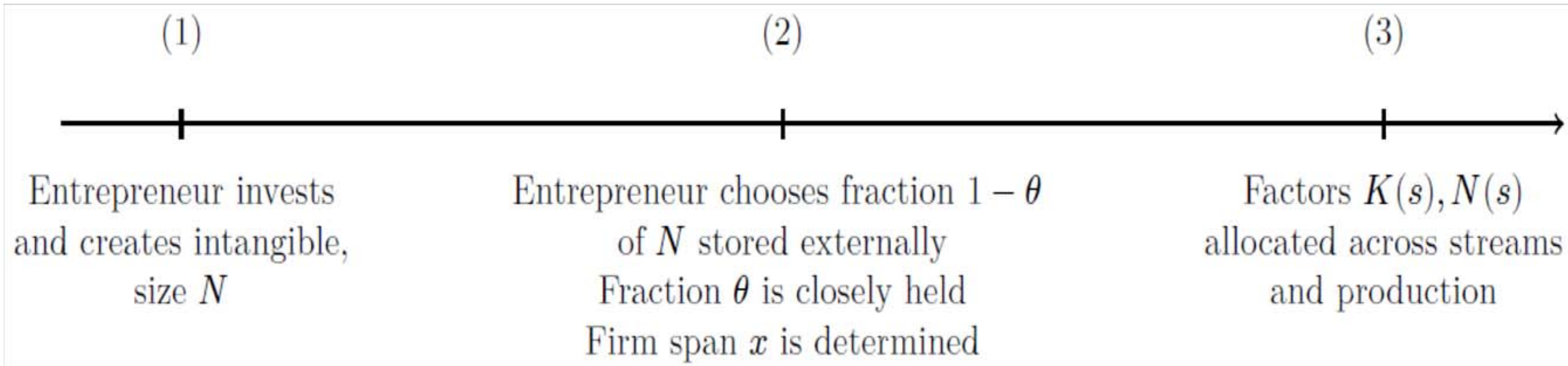
- new tasks
- automation constraints
- labor-biased technical progress

J. Technology – The Role of Intangible Capital: Crouzet-Eberly-Eisfeldt-Papanikolaou (2022)

- **Many forms of intangible capital are productive – patents, software and databases, video and audio materials, franchise agreements, consumer lists, organization capital, and brands, to name but a few**
- **Existing literature does not treat intangibles sufficiently different from tangibles**
- **Key features of intangibles:**
    - **they contain information and information storage**
        - **storage is needed in order to put intangible capital for productive uses**
        - **storage may be in forms of software, documentation or human beings (organization capital)**
    - **they are non-rivalry within the firm and limits to excludability**
        - **non-rivalry means the intangibles are scalable: the stock of intangible capital and the span/scope of firms are complements**
        - **imperfect excludability means limits to the incentive for entrepreneurs, managers, and key personnel**

- **Consider a firm managed by an entrepreneur who makes operating and investment decisions to maximize the value of the firm**

- **Timing: backward solving**



- **Stage 3: production of a variety of streams s ∈ [0,x] given x**

$$
\begin{aligned}
V(N,x) \quad = & \max_{\{N(s),K(s)\}_{s\in[0,x]},K} \quad \int_0^x F(K(s),N(s))ds - RK \\
\text{s.t.} \quad & \int_0^x K(s)\,ds \leq K \\
& \left(\int_0^x N(s)^{\frac{1}{1-\rho}}\,ds\right)^{1-\rho} \leq N
\end{aligned}
$$

  - $Y(s) = F(K(s),N(s)) = N(s)^{1-\zeta}K(s)^{\zeta}$, **ζ = intangible share**
  - **ρ∈[0,1]: ρ=0 => rivalry as tangibles, MRTS =** $\nu(N(s),N(s');\rho) = \left(\frac{N(s')}{N(s)}\right)^{\frac{\rho}{1-\rho}}$

- factor demands under symmetry:

  $N(s) = x^{-(1-\rho)} N$ and $K(N,x) = \left(\frac{A}{1-\zeta}\right)^{\frac{1}{\zeta}} N x^{\rho}$

  where $A \equiv (1-\zeta)\left(\frac{\zeta}{R}\right)^{\frac{\zeta}{1-\zeta}}$
- Firm value: $V(N,x) = A N x^{\rho}$, increasing in N and x

- **Stage 2: storage and span decisions**
  - key: trade-off between retention of N and expansion of the span of firm
  - N_e = share of intangibles to retain, $\theta = N_e/N$
  - value accruing to entrepreneur = $V_e(N,x,\theta) = \theta V(N,x)$
  - limit appropriability: $x = f(\theta,\delta) = \begin{cases} -\frac{1}{\delta}\log\left(\frac{\theta}{\delta}\right) & \textit{if} \quad \theta \in [0,\delta) \\ 0 & \textit{if} \quad \theta \in [\delta, 1] \end{cases}$
  - optimization: $\hat{V}_e(N) = \max_{\theta\in[0,1],\ x\geq 0} \theta V(N,x) \quad \text{s.t.} \quad x = f(\theta,\delta)$
  - solution: $\hat{x} = \frac{\rho}{\delta}, \quad \hat{\theta} = \delta e^{-\rho}$ and $\hat{V}_e(N) = A N \delta e^{-\rho}\left(\frac{\rho}{\delta}\right)^{\rho}$:
    - high degree of non-rivalry ρ or low storage cost δ => less retention and greater span

- **firm value:** $\hat{V}(N) = A\,N\,\hat{x}^{\rho} = A\,N\,\left(\frac{\rho}{\delta}\right)^{\rho}$

- **Stage 1: intangible investment (in effort ι with cost c(ι))**

$\max_{\iota} \int \hat{V}_e(N) f(N;\iota) dN - c(\iota)$ **(ex ante rent)**

  - **investment decision:** $A\left[\delta e^{-\rho}\left(\frac{\rho}{\delta}\right)^{\rho}\right]\frac{\partial}{\partial e}E[N;\hat{\iota}] = \frac{\partial}{\partial \iota}c(\hat{\iota})$
    - **decreasing in storage cost δ but ambiguous in degree of non-rivalry ρ**

- **Aggregate output:** $Y = \left(\frac{\rho}{\delta}\right)^{\rho(1-\zeta)} N^{1-\zeta} K^{\zeta}$

- **TFP in logs:** $tfp \equiv \log Y - \zeta \log K = \rho(1-\zeta)\,(\log\rho - \log\delta) + (1-\zeta)\log N$
  - **high degree of non-rivalry ρ or low storage cost δ or higher stock of intangibles => higher TFP**
  - **an endogenous TFP story**

K. Misallocation of Talent in Innovation: Celik (2023)

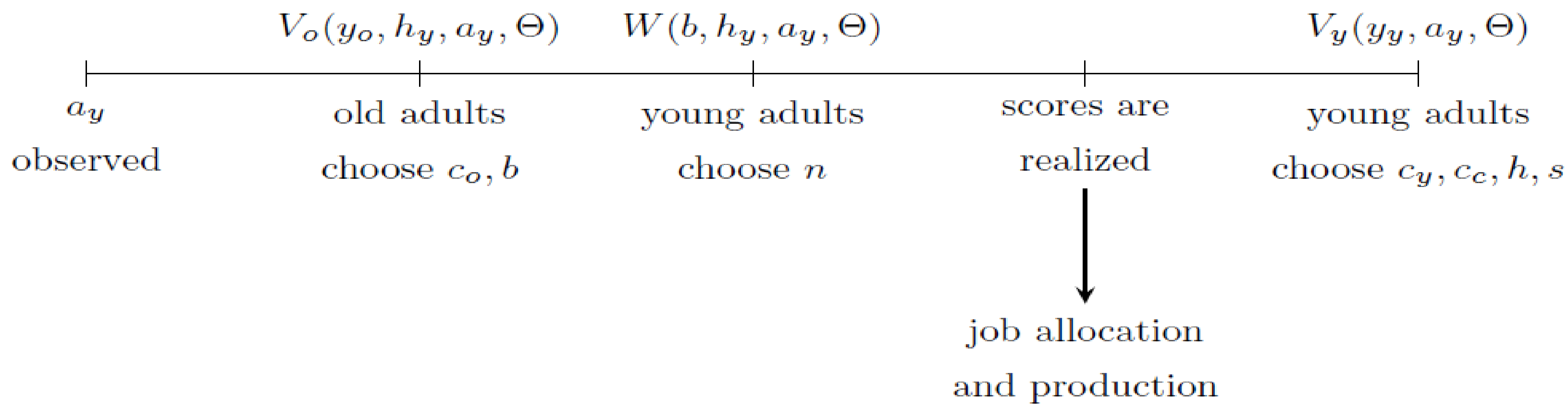
- **Was the richness of Hermann Einstein critical for the birth of a great inventor, Albert Einstein?**
- **Stylized facts (supported by empirical work):**
  - **Capital accessibility: individuals from richer backgrounds are more likely to become inventors (23.9%), but those from more educated families are not (0.1%)**
  - **Innovation ability: conditional on becoming an inventor, individuals from more educated backgrounds become more prolific inventors (17.5%), but those from richer families do not (0.1%)**
- **3-period overlapping generations: childhood (c), young adulthood (y), old (o)**
- **Intergenerational transmission: altruistic parents choose children's consumption in their childhood, invest in their education, and leave non-negative bequests**
- **Dynasty lifetime utility function of household m of generation t:**

$$U_{m,t}(\vec{c}_{m,t}) = \mathbb{E}_t\left[\frac{c_{c,m,t}^{1-\omega}}{1-\omega} + \beta\frac{c_{y,m,t}^{1-\omega}}{1-\omega} + \beta^2\frac{c_{o,m,t}^{1-\omega}}{1-\omega} + \alpha\beta U_{m,t+1}(\vec{c}_{m,t+1})\right],$$

where $\vec{c}_{m,t} = \left\{c_{c,m,T}, c_{y,m,T}, c_{o,m,T}\right\}_{T=t}^{\infty}$ **(dynasty)**

- **Firm production: final output is produced with capital k and unskilled labor l_u**
  $o(z, k, l_u) = z^{\zeta} k^{\kappa} l_u^{\lambda}$
  where $\zeta + \kappa + \lambda = 1$ **and** $z' = z + \gamma \bar{z}$ **, depending on the average productivity of the economy – those successfully innovate have γ > 0, those failed have γ = 0**
- **Firm innovation:** $i(l_s) = \chi l_s^{\xi}$ **, depending on skilled labor l_s, with χ>0, ξ∈(0,1)**
- **Household innate ability (a), child education (h) and labor productivity (l):**
  $$l_{m,t}(h_{m,t}, a_{m,t}) = \left(\psi h_{m,t}^{\frac{\epsilon-1}{\epsilon}} + (1-\psi) a_{m,t}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$
  - **cost of education:** $c_h(h, \Theta) = \kappa_h h^{\xi_h} \bar{z}^{\zeta/(\zeta+\lambda)}$**,** $\kappa_h > 0$**,** $\xi_h > 1$ **, scaled up with aggregate output due to the last term**
  - **evolution of innate ability (AR1):** $\log a' = (1-\rho)\mu_a + \rho \log a + \epsilon_a, \ \epsilon_a \sim N(0, \sigma_a^2)$ **with ρ as the inherit weight**
- **Inventor training (key of the model): skilled jobs require education**
  - **training is necessary for creative innovation**
  - **education leads to higher individual productivity:** $l' = \Lambda l$**, Λ > 1**
  - **score:** $\tilde{s}(l(h,a), n) = (1-\nu) l(h,a) + \nu n + \epsilon_j, \ \epsilon_j \sim N(0, \sigma_j^2)$ **, n is credential buildup at cost** $c_n(n) = \kappa_n n^{\xi_n} \bar{z}^{\zeta/(\zeta+\lambda)}$**, ν>0,** κ_n**>0,** ξ_n**>1 (think of "cram school")**
  - **top η fraction selected for inventor training => ∃** $\bar{s}$ **s.t.** $\tilde{s} \geq \bar{s}$ **got trained**

- **Time line:**



- **Firm optimization:**
  - **flow operative profit:** $\Pi(z,\Theta) = \max_{k, l_u \geq 0} \{z^{\zeta} k^{\kappa} l_u^{\lambda} - (r+\delta)k - w_u l_u\}$
  - **value: operative profit+continuation value depending on innovation success**

$$V(z,\Theta) = \max_{l_s \geq 0} \left\{ \Pi(z,\Theta) + \frac{\chi l_s^{\xi}}{1+r} V(z+\gamma\bar{z}, \Theta') + \frac{(1-\chi l_s^{\xi})}{1+r} V(z,\Theta') - w_s l_s \right\}$$

- **Household optimization:**
  - **The old:** $V_o(y_o, h_y, a_y, \Theta) = \max_{c_o, b \geq 0} \{u(c_o) + \alpha W(b, h_y, a_y, \Theta)\}$ s.t. $c_o + b \leq y_0$

- **The young before job allocation:**

$$\begin{aligned} W(b, h_y, a_y, \Theta) &= \max_{n \geq 0} \{E[V_y(y_y, a_y, \Theta)|\cdot]\} \text{ s.t.} \\ y_y &= \left(w_{j_y} + \frac{w'_{j_y}}{1+r'}\right) l_y(h_y, a_y) + b - c_n(n) \\ j_y &\sim F(j; l_y(h_y, a_y), n, \Theta) \end{aligned}$$

- **The young after job allocation:**

$$\begin{aligned} V_y(y_y, a_y, \Theta) &= \max_{c_y, c_c, h'_y, s \geq 0} \{u(c_y) + \alpha u(c_c) + \\ &\quad \beta \mathbb{E}[V_o(y'_o, h'_y, a'_y, \Theta')|\cdot]\} \text{ s.t.} \\ y_y &\geq c_y + c_c + c_h(h'_y) + s \\ y'_o &= (1+r')s \\ a'_y &\sim g(a_y) \\ \Theta' &= T(\Theta) \end{aligned}$$

- **Equilibrium:**
  - **labor market clearing:**

$$L_{u,t} \equiv \int_0^1 \hat{l}_{u,t}(z,\Theta)dZ(z) = 2(1-\eta)\int l(h,a)d\Phi_{u,t}(h,a)$$

$$L_{s,t} \equiv \int_0^1 \hat{l}_{s,t}(z,\Theta)dZ(z) = 2\eta\int l(h,a)d\Phi_{s,t}(h,a)$$

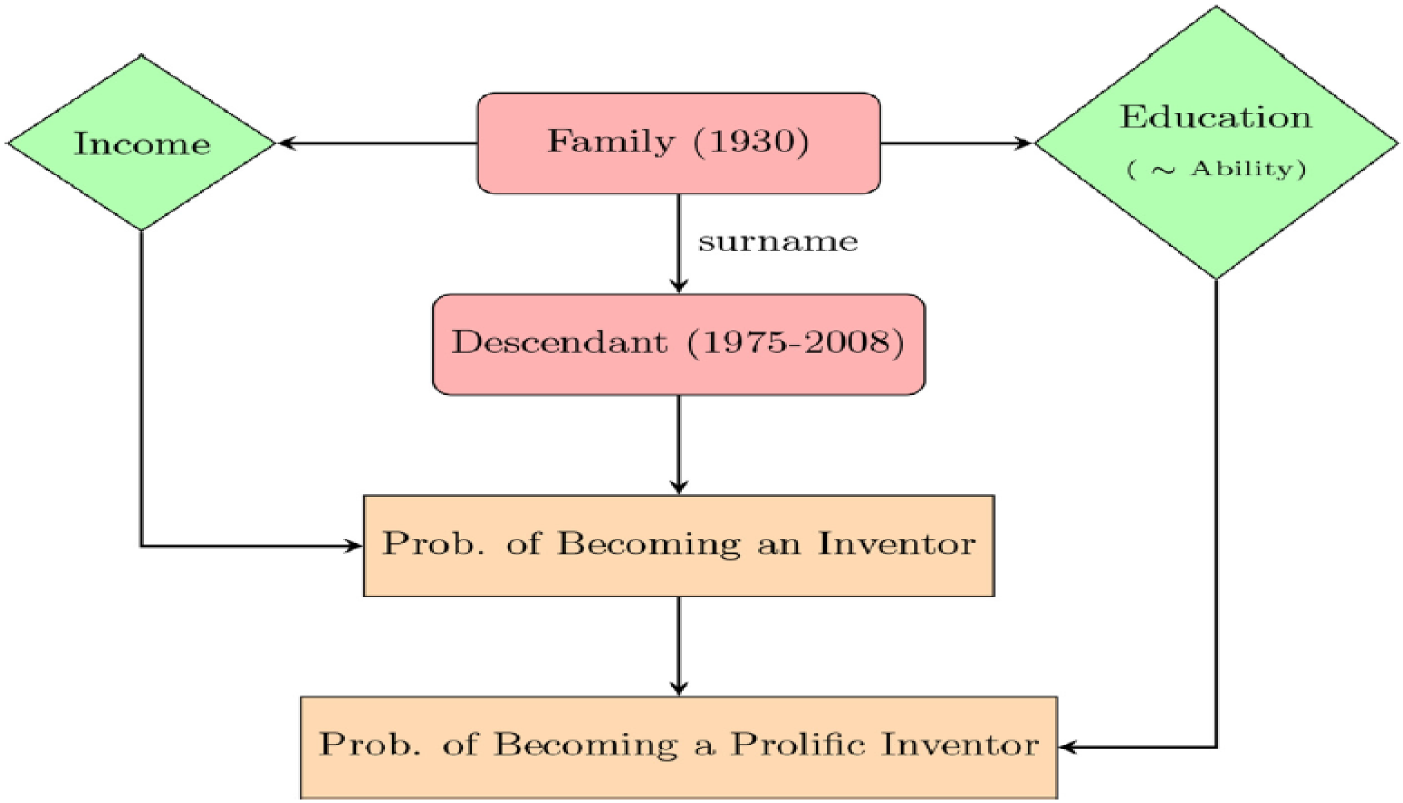
  - **inventor market clearing:** $\eta = \int_{\bar{s}_t}^{\infty} \tilde{s}d\tilde{S}_t(\tilde{s})$
  - **loanable fund market clearing (saving = investment):**
    - **aggregate saving:** $A_{t+1} \equiv \int \tilde{a}_{m,t-1}d\tilde{A}(\tilde{a})$**, with**

      $\tilde{a}_{m,t} \equiv s_{m,t} - l(h_{m,t}, a_{m,t})w_{j_{m,t},t+2}/(1+r_{t+2})$
    - **aggregate capital:** $K_t \equiv \int_0^1 \hat{k}_t(z,\Theta)dZ(z)$
    - **capital evolution/goods market clearing:**

      $O_t = C_t + K_{t+1} - (1-\delta)K_t + N_t + H_t$

- **BGP with common growth g: when** $O_t, K_t, N_t, H_t,$ and C_t **all grow at rate g**

- **Empirical strategy by using surnames in IPUMS data:**



$$\text{inventor probability (surname)} = \frac{\text{number of inventors (surname)}}{\text{number of individuals (surname)}}, \quad \text{relative representation (surname)} = \frac{\text{inventor probability (surname)}}{\text{unconditional inventor probability}}$$

- **Empirical finding support the two stylized facts**

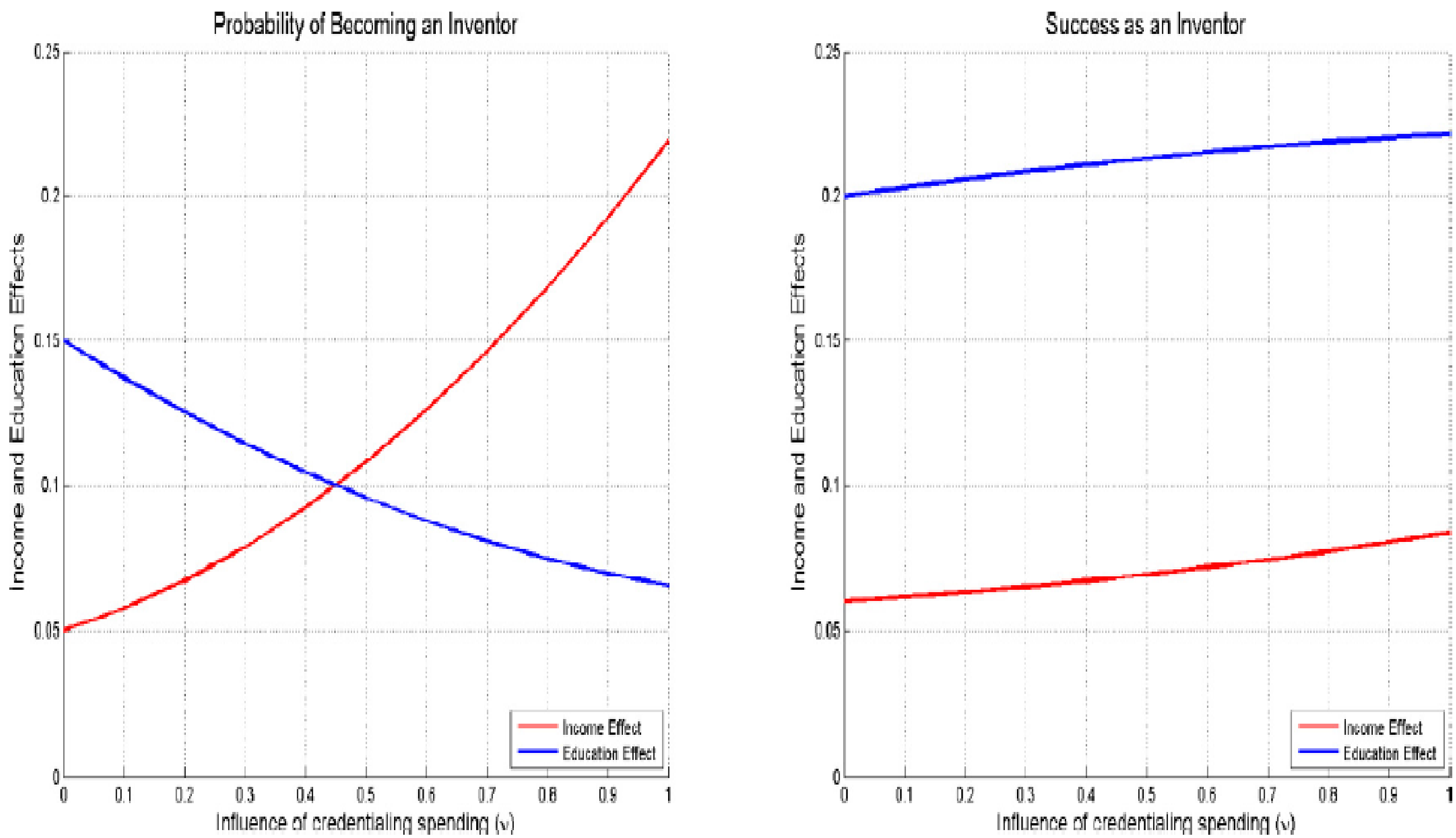
- **Calibration:**
  - **parameter values**

Parameter	*Description*	*Identification*
	External Calibration	
$\omega = 2.00$	CRRA parameter	Kaplow (2005)
$\alpha = 0.50$	Parental altruism	Aiyagari et al. (2002)
$\kappa = 0.25$	Capital's share in production	Corrado et al. (2009)
$\lambda = 0.60$	Labor's share in production	Corrado et al. (2009)
$\delta = 0.82$	Depreciation rate	U.S. NIPA
$\xi = 0.50$	Concavity of innovation production	Hall and Ziedonis (2001)
$\sigma_a = 0.70$	St. dev. of innate ability shock	Knowles (1999)
$\eta = 11.6\%$	Fraction of skilled jobs	U.S. Census Bureau (2013)
	Internal Calibration	
$\beta = 0.28$	Discount factor	Real interest rate
$\Gamma = 0.92$	Innovation productivity increase	GDP growth rate
$\rho = 0.70$	Persistence of innate ability	IG corr. of earnings
$\kappa_h = 0.04$	Cost of pre-college education investment	Education spending/GDP
$\kappa_n = 0.05$	Cost of credentialing investment	Inequality targets
$\xi_h = 1.30$	Convexity of pre-college education inv.	Inequality targets
$\xi_n = 2.50$	Convexity of credentialing inv.	Inequality targets
$\psi = 0.40$	Education share of ind. productivity	Regression targets
$\epsilon = 1.90$	Ind. productivity elasticity	Regression targets
$\nu = 0.89$	Influence of credentialing spending	Regression targets
$\sigma_j = 0.80$	St. dev. of job shock	Regression targets

- **targets**

Target	*U.S. Data*	*Model*
Aggregate targets		
Yearly real interest rate	4.00%	4.00%
Yearly GDP growth rate	2.00%	2.00%
Education spending/GDP	7.30%	8.55%
Intergenerational correlation targets		
IG corr. of earnings	0.70	0.70
IG corr. of wealth	0.37	0.33
Inequality targets		
Wage income Gini index	0.48	0.52
Log 90/10 ratio	1.08	1.17
Log 90/50 ratio	0.46	0.52
Log 50/10 ratio	0.62	0.65
Regression targets		
Becoming an inventor, income effect	0.24	0.19
Becoming an inventor, education effect	0.00	0.07
Productivity as an inventor, income effect	0.00	0.08
Productivity as an inventor, education effect	0.18	0.22

- **Quantitative results: probability of becoming an inventor and success as an inventor – income effect (rich parents) versus education effect**



- **Counterfactual: shutting down the credential build up channel**

Variable	*Baseline*	$\nu = 0$	Change
Becoming an inventor, income effect	0.19	0.05	-73.7%
Becoming an inventor, education effect	0.07	0.15	114%
Productivity as an inventor, income effect	0.08	0.06	-25.0%
Productivity as an inventor, education effect	0.22	0.20	-9.09%
Yearly GDP growth rate	2.00%	2.21%	10.4%
Education spending/GDP	8.55%	10.2%	19.1%
Aggregate skilled labor, L_s	0.48	0.62	28.4%
Aggregate unskilled labor, L_u	1.91	2.00	4.69%
Mean innate ability of skilled workers, a	2.08	2.57	23.4%
Mean pre-college education of skilled workers, h	2.27	2.96	30.1%
Mean parental wealth of skilled workers, y_o	0.87	0.84	-4.32%
Mean bequests received of skilled workers, b	0.49	0.25	-49.5%
Wage income Gini index	0.52	0.56	6.61%
Log 90/10 ratio	1.17	1.20	3.10%
Log 90/50 ratio	0.52	0.57	9.30%
Log 50/10 ratio	0.65	0.64	-1.88%

  - **the credential build up channel is crucial for explaining the role played by parental background in becoming an inventor (income effect), pre-college education of the skilled and bequest received by the skilled**