# Fundamentals

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# A. Growth Theory

# Part-1: Neoclassical Growth Theory

# A. What Motivates Neoclassical Growth Theory?

## 1. 3 Kaldorian stylized facts:

- On-going increasing capital-labor (K/L) and output-labor (Y/L) ratios
- Increasing (real) wage rate (w) with stationary (real) interest rate (r)
- Stationary wage income and capital income shares (wL/Y, rK/Y)

## 2. The Harrodian knife-edge problem:

- **Capital accumulation:**
  - net investment (i.e., net changes in the capital stock) in continuous time: $\dot{K} = I - \delta K$ = gross investment – capital depreciation
  - to convert into per capita measure (k = K/L), we apply log calculus: $\dot{K}/K = \dot{k}/k + \dot{L}/L = \dot{k}/k + n$ (n = population growth rate)
  - we thus have a fundamental relationship: $\dot{k} = \frac{I}{L} - (n + \delta)k$ (KA)
    - depreciation shrinks the size of the pie
    - population growth shrinks the per capita share of the pie
    - both reduces the speed of accumulation of capital per worker

- **Goods market equilibrium:**
  - **per capital investment: I/L**
  - **per capita saving: S/L, where saving is assumed proportional to total income/output, i.e., S = sY with s measuring the exogenous saving rate (0 < s < 1)**
  - **goods market equilibrium:** $\frac{I}{L} = \frac{S}{L} = s\frac{Y}{L} = sy$ **(GE)**
- **Constant capital-output ratio: K/Y = v > 0, or, k = K/L = vY/L = vy**
- **Steady state: defined as when *per capita* capital ceases to grow ($\dot{k} = 0$):**

$$\frac{I}{L} = (n + \delta)k = (n + \delta)vy \quad \text{(SS)}$$

- **Combining goods market equilibrium (GE) and steady state (SS), we obtain: (n+δ)vy = sy, or, s/v = n + δ, which equates 4 constants, leading to a knife-edge outcome**
- **To solve the knife-edge problem requires:**
  - **variable v => neoclassical production f(k) with v(k) ↑ in k**
  - **variable s => Keynes-Wicksell model with variable s(k) ↑ in k (wrong prediction: higher δ raises k in equilibrium)**
  - **variable n => Nagatani model with variable n(k) ↑ in k (but why?)**

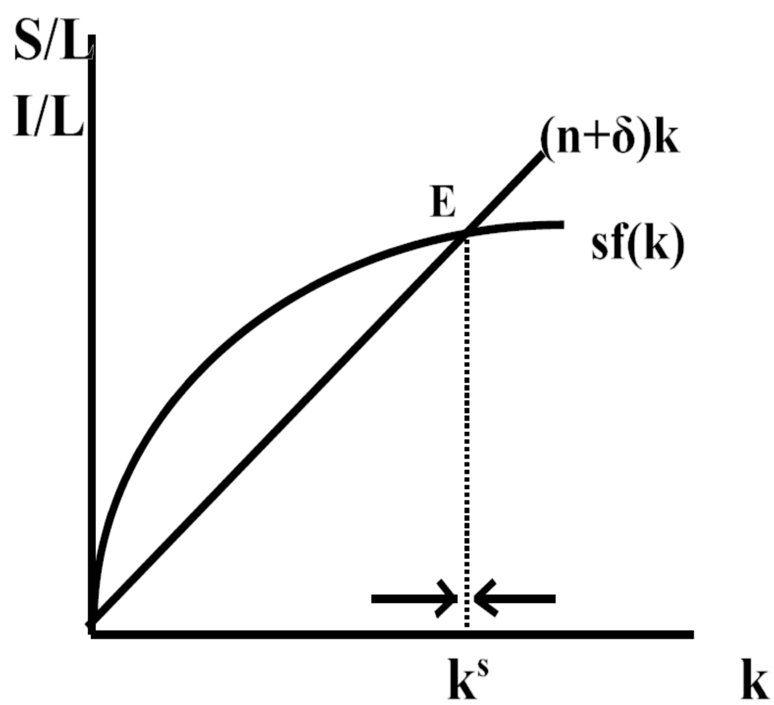
## B. The Basic Solow-Swan Model

- **Basic reference: Jones (secs 2.1, 3.1)**
- **Neoclassical production: Y = F(K,L), satisfying:**
  - **strictly increasing: MPK > 0, MPL > 0**
  - **strictly concave: diminishing MPK and diminishing MPL**
  - **constant returns to scale: for any constant a>0, F(aK,aL) = aF(K,L)**
  - **input necessity: F(0,L) = F(K,0) = 0, implying the iso-quants do not intersect with the two axes**
  - **Examples:**
    - **Cobb-Douglas: F(K,L) = A $K^{\alpha}$ $L^{1-\alpha}$ => constant factor shares (Kaldorian 3rd stylized fact, which is our benchmark)**
    - **Leontief: F = min{aK, bL}, a > 0, b > 0 => Y/K = a, Y/L = b)**
    - **Constant elasticity of substitution (CES):**

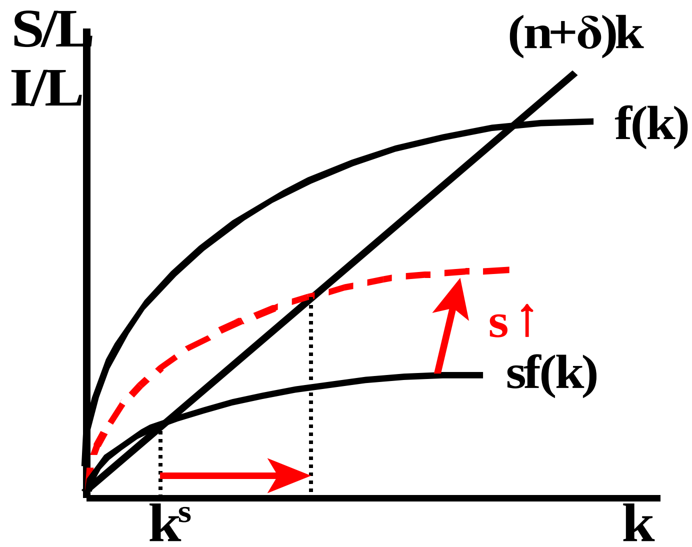
$$F(K,L) = A\left[\alpha K^{(\xi-1)/\xi} + (1-\alpha)L^{(\xi-1)/\xi}\right]^{\xi/(\xi-1)}, \; 0 \le \xi \le 1$$

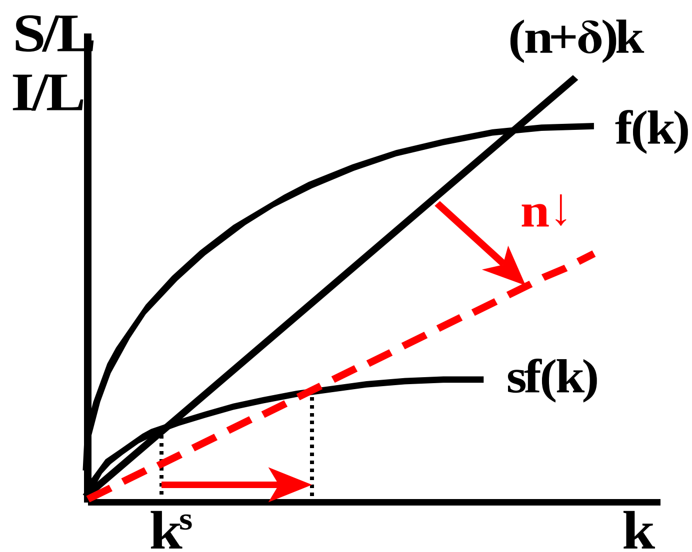
      - **$\alpha$ and 1- $\alpha$ are long-run factor shares**
      - **$\xi$ = 0: Leontief**
      - **$\xi$ = 1: Cobb-Douglas**
      - **$\xi$ > 1 is ruled out to ensure steady-state convergence**

- **Per capita output:** $y = \frac{Y}{L} = F(\frac{K}{L},1) = f(k)$, **where variable capital output ratio, k/f(k), is the key to resolve the knife-edge problem**
- **Capital accumulation (KA):** $\dot{k} = \frac{I}{L} - (n+\delta)k$, **same as before**
- **Goods market equilibrium:** $\frac{I}{L} = sy = sf(k)$
- **Steady state (SS,** $\dot{k} = 0$ **=> $k^s$):**
  $\frac{I}{L} = (n+\delta)k$
- **Convergence: off steady-state k will always converge to $k^s$**
  - **when $k > k^s$: excessive investment**
  - **when $k < k^s$: excessive saving**
  - **empirical evidence: no convergence between sub-Saharan and OECD countries**



- **Comparative statics:**
  - **An increase in the exogenous saving rate raises per capita saving and funds available for investment, thus raising capital accumulation in the steady state**
  - **A reduction in population growth lowers the depletion of per capita capital investment, leading to greater accumulation in the steady state**
  - **Thus, $k^s$ ↑ in s and ↓ in n (or δ)**
  - **Examples:**
    - **s: Japan/Taiwan**
    - **n: Kenya/Zaire**
- **Problems: it is a non-optimizing model driven by the mechanics of production, ignoring the intertemporal consumption-saving trade-off**



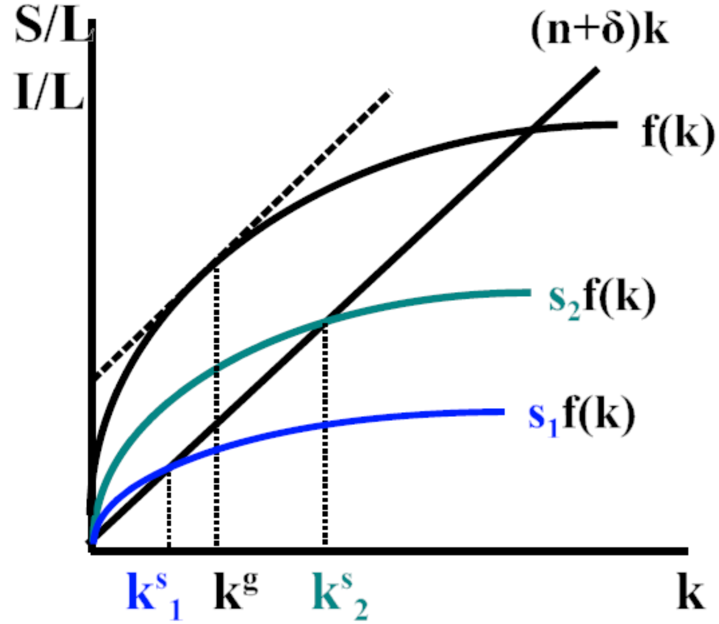
## C. Allais-Phelps Golden Rule: Endogenizing the Saving Decision

- **Steady-state per capita consumption:**

$$\frac{C}{L} = \frac{Y}{L} - \frac{S}{L} = \frac{Y}{L} - \frac{I}{L} = f(k) - (n+\delta)k$$

- **Optimization:**

$$\max \quad \frac{C}{L} = f(k) - (n+\delta)k$$




- **Optimizing condition (*golden rule*):**

  **$f_k = n + \delta$ => solution $k^g$**

  - **it differs from the non-optimizing Solovian solution by allowing the saving rate s to be endogenous to achieve highest SS per capita consumption**
  - **the condition equates the MPK with the cost of capital (depreciation + population growth)**
  - **Kaldorian 1st stylized fact: higher A => higher K/L (k), Y/L (f)**

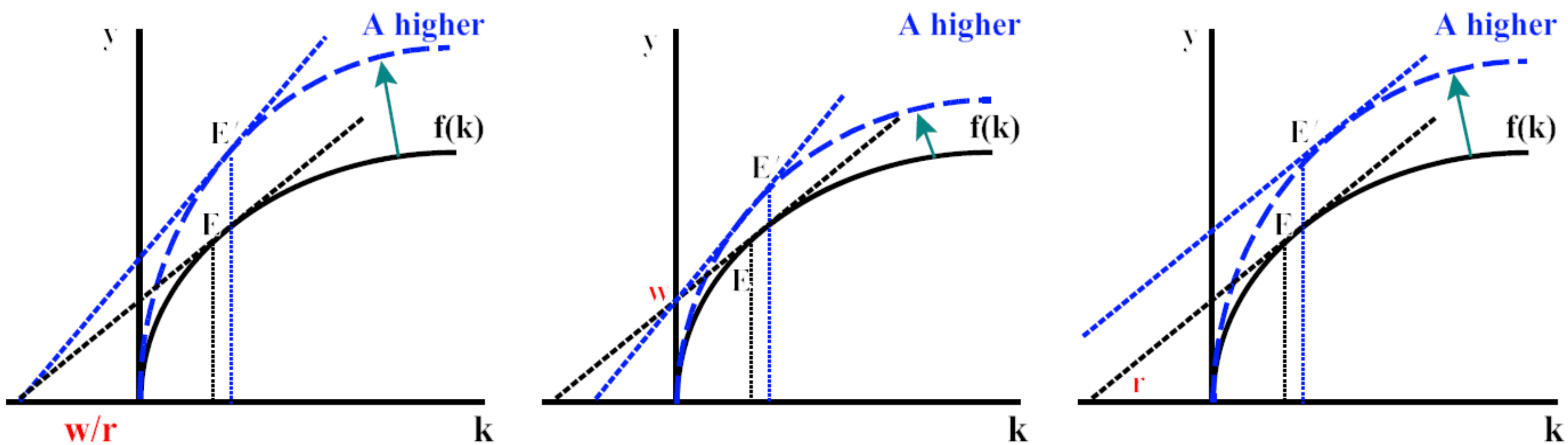
- **Appendix: Consider a Cobb-Douglas production function $Y = AK^{\alpha}L^{1-\alpha}$**
  - **per capita output: $y = f(k) = Ak^{\alpha}$**
  - **MPK: $f_k = \alpha Ak^{\alpha} = \alpha f(k)/k$**
  - **Solow-Swan:**
    - **$k^s$ solves: $sf(k) = (n+\delta)k$**
    - **so, $k^s = [sA/(n+\delta)]^{1/(1-\alpha)}$**
  - **Golden rule:**
    - **$k^g$ solves, $\alpha f(k)/k = n+\delta$, or, $\alpha f(k) = (n+\delta)k$**
    - **so the solution becomes: $k^g = [\alpha A/(n+\delta)]^{1/(1-\alpha)}$**
  - **Thus, $k^g > (<) k^s$ if $\alpha > (<)$ s.**

- **In the U.S., $\alpha = 1/3$, $s = 0.12$, $n = 1.4\%$, $\delta = 5\%$, and $k/y = 2.75$**
  - **since $\alpha > s$, the Solow-Swan solution under-accumulates relative to the golden rule**
  - **under the Solow-Swan setup, $sf(k) = (n+\delta)k \Rightarrow k/y = s/(n+\delta) = 1.88$**
  - **under the golden rule, $\alpha f(k)/k = n+\delta \Rightarrow k/y = \alpha/(n+\delta) = 5.21$**
  - **so, neither model fits with the empirical observation – we shall return to this after building up “optimal growth theory”**

## D. Exogenous Technical Progress

- Consider a neutral technical progress A:
  - Hicks neutral: $Y = AF(K,L)$, i.e., all round
  - Solow neutral: $Y = F(AK,L)$, i.e., capital-augmenting (K-biased), potentially useful to explain wage stagnation
  - Harrod neutral: $Y = F(K,AL)$, i.e., labor-augmenting (L-biased)
- The three neutral technical progress cases feature:
  - Hicks neutral: constant w/r
  - Solow neutral: constant w
  - Harrod neutral: constant r (matching Kaldorian 2nd stylized fact)

**Hicks neutral** **Solow neutral** **Harrod Neutral**



- Let the exogenous Harrod neutral technical progress rate be $\eta = \frac{\dot{A}}{A} > 0$
- Redefine variables in *per capita efficiency unit* (AL): $k = \frac{K}{AL}$, $y = \frac{Y}{AL}$
- Per capita output in efficiency unit: $y = \frac{F(K, AL)}{AL} = F(\frac{K}{AL}, 1) = f(k)$
- Capital accumulation: $\dot{k} = \frac{I}{L} - (n + \delta + \eta)k$
- Goods market equilibrium: $\frac{I}{AL} = sf(k)$
- Solow-Swan steady-state solution ($k^s$): $sf(k) = (n + \delta + \eta)k$
- Golden rule solution ($k^g$): $f_k = n + \delta + \eta$
- Model implications:
  - The capital-effective labor ratio k is decreasing in $\eta$
  - Per capita real GDP (Y/L) growth in steady state is $\eta$, i.e., driven only by the exogenous technical progress rate
- Denison accounting: under Cobb-Douglas production function, we can decompose the rate of output growth by capital accumulation (20%), employment enhancement (< 10%) and technical progress (> 70%)

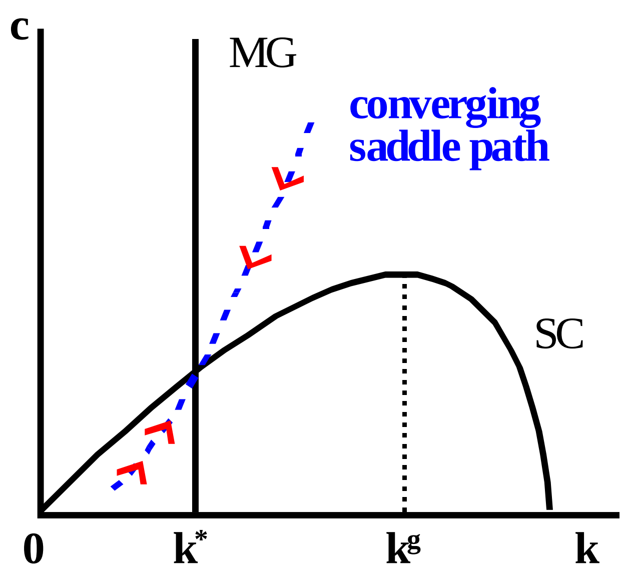
## E. The Optimal Growth Model: Ramsey-Cass-Koopman

- **Basic reference: Aghion-Howitt (sec. 1.3)**
- **Lifetime utility: $U = \int_0^\infty u(c)e^{-\rho t}dt$, with $u(c) = \dfrac{c^{1-\sigma^{-1}} - 1}{1-\sigma^{-1}}$, where σ is the elasticity of intertemporal substitution:**
  - **$\sigma = 0$: no substitution between current and future consumption**
  - **$\sigma = 1$: $u(c) = \ln(c)$**
  - **$\sigma = \infty$: $u(c) = c$, with perfect substitution between current and future consumption**
  - **in the U.S., σ is estimated in between 0.25 and 2/3**
- **Lifetime budget: consumption + gross investment = income = output (all in per capita), where depreciation includes both physical depreciation and depletion due to population growth**
  - **per capita consumption: c**
  - **per capita gross investment: $\dot{k} + (n + \delta)k$**
  - **per capita output: f(k)**
  - **so the lifetime budget yields the following capital accumulation equation: $\dot{k} = f(k) - (n+\delta)k - c$ , $k(0) = k_0 > 0$ (KA)**

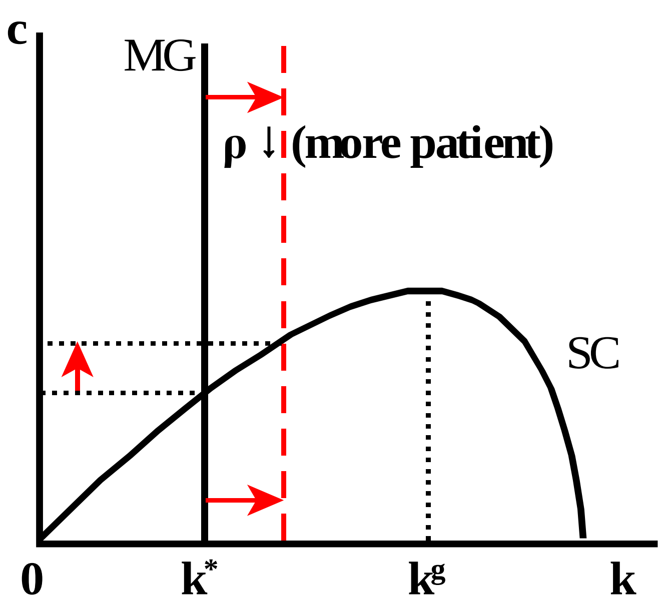
- **Optimization: max U s.t. (KA)**
- **Optimizing condition – the Keynes-Ramsey equation (KR),**

$$\theta = \frac{\dot{c}}{c} = \sigma[f_k - (\rho + n + \delta)]$$

  - **MPK measures the marginal benefit of capital**
  - **$\rho + n + \delta$ measure the marginal cost of capital (inclusive of time cost due to delayed consumption)**
  - **when the marginal benefit exceeds the marginal cost, capital is accumulated, as is consumption, implying positive growth**
- **Steady state ($\dot{c} = 0, \dot{k} = 0$):**
  - **$f_k(k) = \rho + n + \delta$ (modified golden rule, MG)**
  - **$c = f(k) - (n + \delta)k$ (steady-state consumption, SC): the SC locus**
    - **starts from 0, given f(0) = 0**
    - **peaks at MPK = (n+δ), i.e., the golden rule solution**
    - **reaches zero eventually (diminishing returns)**



- **The optimizing steady-state solution is $k^* < k^g$, implying that the golden rule is not golden – it indeed *over*-accumulates capital!**
- **Dynamic equilibrium path:**
  - o **there is a converging saddle path, leading to the steady state $(c^*,k^*)$**
  - o **along the converging path, consumption and capital are co-moving (both decreasing or both increasing)**
- **Comparative-static analysis:**
  - o **$k^* \downarrow$ in n, δ, ρ**
  - o **new result: more patience (1% in the case of China) raises SS capital accumulation and per capita consumption**




- **Returning to the empirics:**
  - o **using the U.S. observations delineated above ($\alpha = 1/3$, $s = 0.12$, $n = 1.4\%$, and $\delta = 5\%$), we can compute from (MG) $k/y = \alpha/(\rho + n + \delta) = 2.92$**
  - o **though not perfect, it is much closer to the observed value ($k/y = 2.75$), compared to Solow-Swan and Alais-Phelps**

# F. Empirical Applications

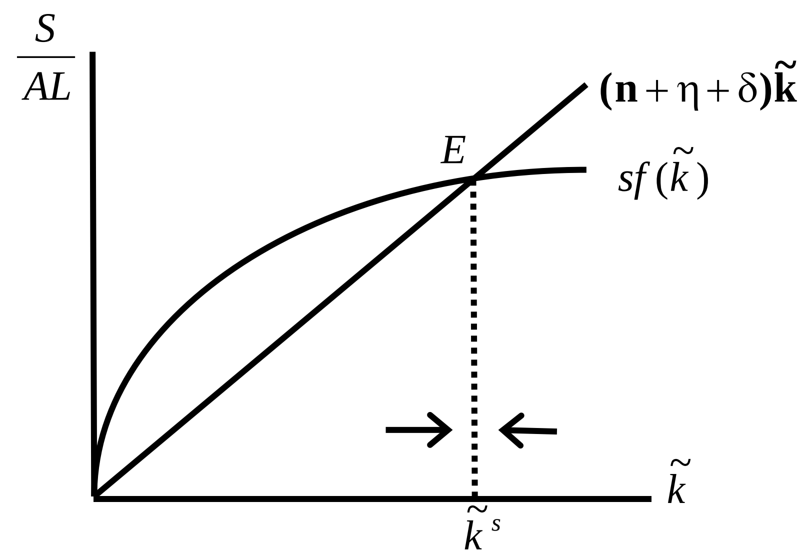
## 1. The Solow-Swan model with Human Capital

- **Basic reference: Jones (secs 3.1), Mankiw-Romer-Weil (Quarterly Journal of Economic, 1992)**
- **Cobb-Douglas production with physical capital and skilled labor**
  - **Aggregate output: $Y = K^{\alpha} (AH)^{1-\alpha}$**
  - **$A$: labor-augmenting technology growing at rate $\eta = \frac{\dot{A}}{A}$**
  - **$H$: effective skilled labor, measured by $hL$, where $h = e^{\psi u}$ measures human capital and $u$ is exogenous time investment in education**
    - **This is following the Mincer approach, which matches with the observed trend in real wages**
    - **In aggregate data, $u$ is measured by the average year of schooling of the economy as a whole**
  - **Per capita output: $y = \frac{Y}{L} = k^{\alpha}(Ah)^{1-\alpha}$**
- **Capital accumulation: $\dot{K} = sY - \delta K$, same as the Solow-Swan model**

- **Appendix: transforming variables in "efficiency units" by dividing throughout the level of the technology:** $\tilde{y} = \frac{Y}{A}$, $\tilde{k} = \frac{K}{A}$
  - **The production function becomes:** $\tilde{y} = \tilde{k}^{\alpha}$
  - **The capital accumulation equation is now:** $\dot{\tilde{\mathbf{k}}} = \mathbf{s}\tilde{\mathbf{y}} - (\mathbf{n} + \eta + \delta)\tilde{\mathbf{k}}$

- **Steady state (**$\dot{\tilde{k}} = 0$**):**
  - $\mathbf{s_K}\tilde{\mathbf{y}} = \mathbf{s}\tilde{\mathbf{k}}^{\alpha} = (\mathbf{n} + \eta + \delta)\tilde{\mathbf{k}}$
  - $\tilde{\mathbf{y}}^{*} = \left(\frac{\mathbf{s}}{\mathbf{n} + \eta + \delta}\right)^{\alpha/(1-\alpha)}$,

    **similar to the original Solow-Swan model with exogenous technical progress**




- **Remark: should the aggregate skill level u increase in the process of economic development, η must be augmented by skilled growth** $\frac{\dot{\mathbf{u}}}{\mathbf{u}}$

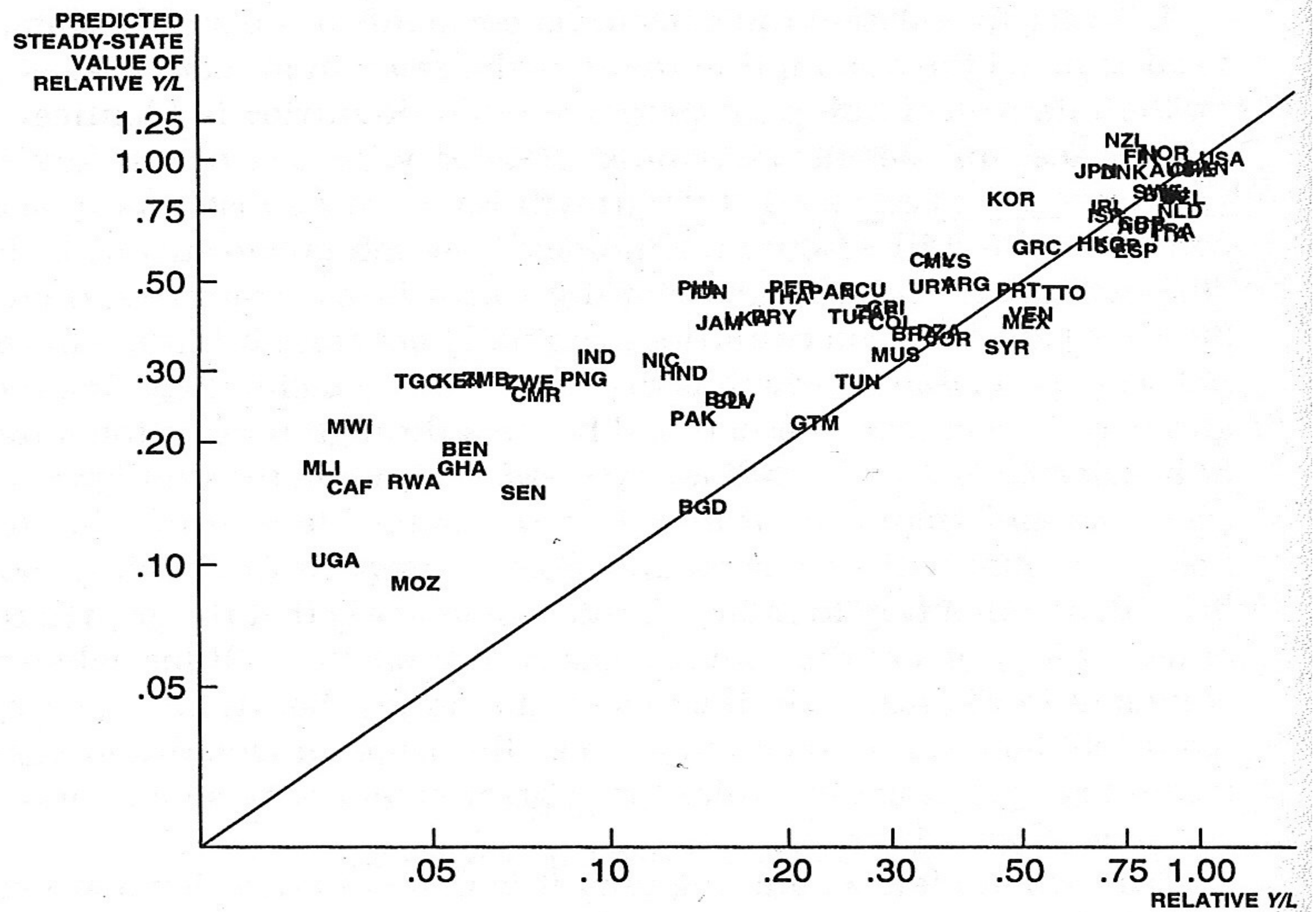
## 2. Evaluate the Human-Capital Augmented Solow-Swan Model

- **Basic reference: Jones (secs 3.2)**

- **The Solow-Swan model with human capital says that differences in the income level are resulting from**
  - o **saving rate (s): differ in different countries**
  - o **population growth rate (n): differ in different countries**
  - o **technology growth rate (η)**
  - o **depreciation rate (δ)**

- **Evaluate the performance of the model in terms of relative income to the U.S., that is:** $\hat{y}^* = \left(\frac{y^*}{y^*_{US}}\right) = \left[\frac{(s/s_{US})}{(n+\eta+\delta)/(n+\eta+\delta)_{US}}\right]$ **allowing the initial state (say, measured by the initial level of A) to differ in different countries:**
  - o **Higher relative saving => higher relative income**
  - o **Higher relative population growth => lower relative income**

FIGURE 3.1 THE "FIT" OF THE NEOCLASSICAL GROWTH MODEL, 1990

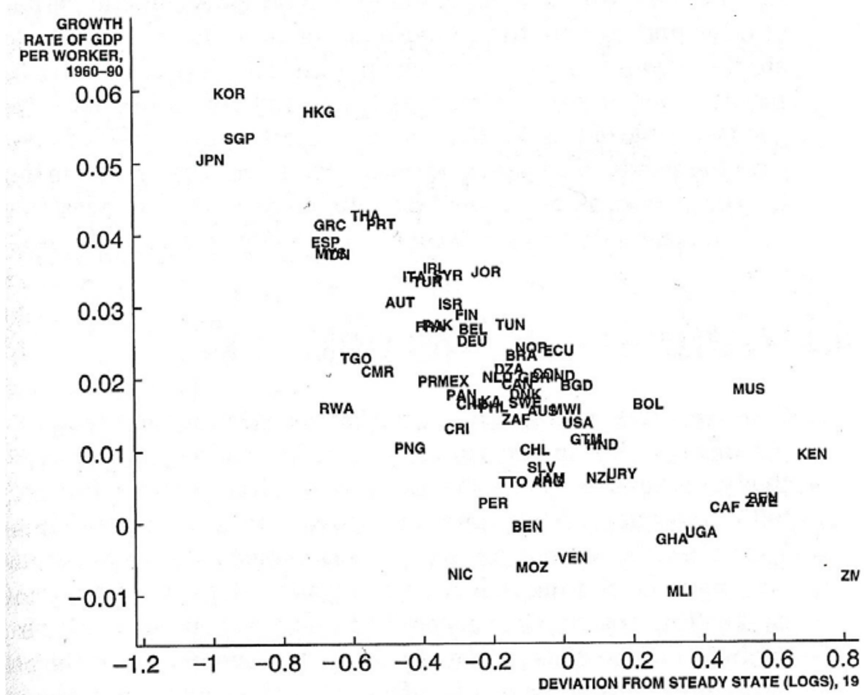



- The mis-fit is associated with Solow residuals and hence cross-country TFP gaps

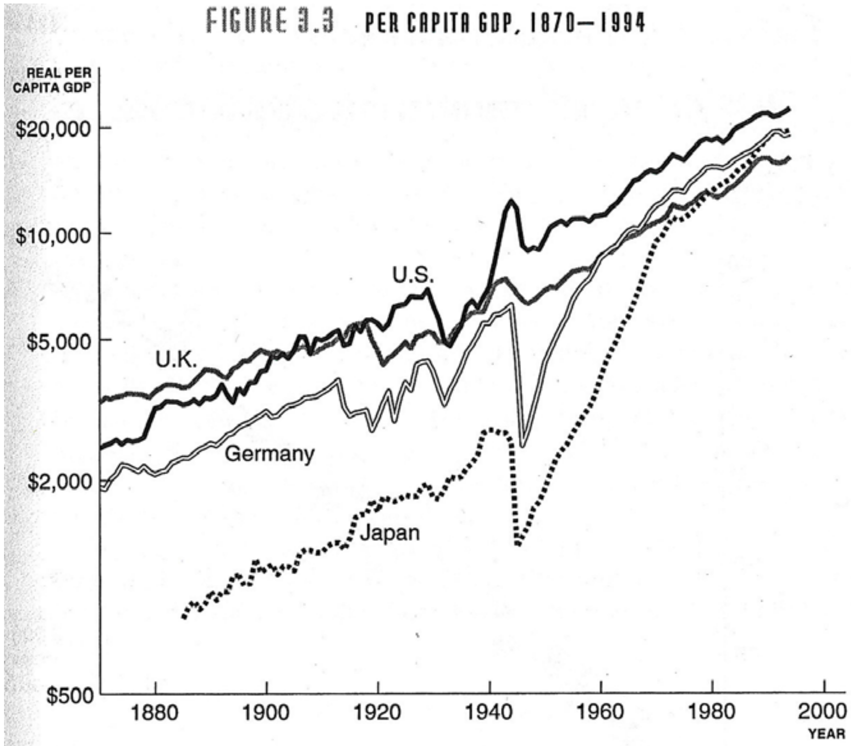
# G. Convergence and Differences in Growth Rates

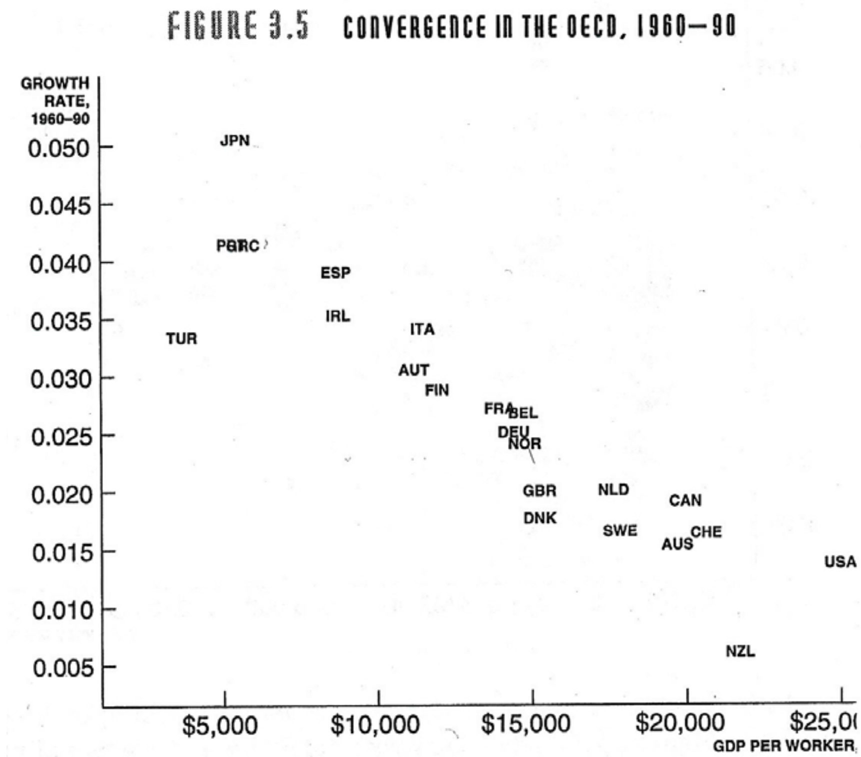
- **Convergence:**
  - o **The further an economy is below its steady state, the faster the economy should grow, and vice versa (β-convergence)**
  - o **Among countries that have the same steady state, poor countries grow faster on average than rich countries (σ-convergence)**

FIGURE 3.8 "CONDITIONAL" CONVERGENCE FOR THE WORLD, 1960–90

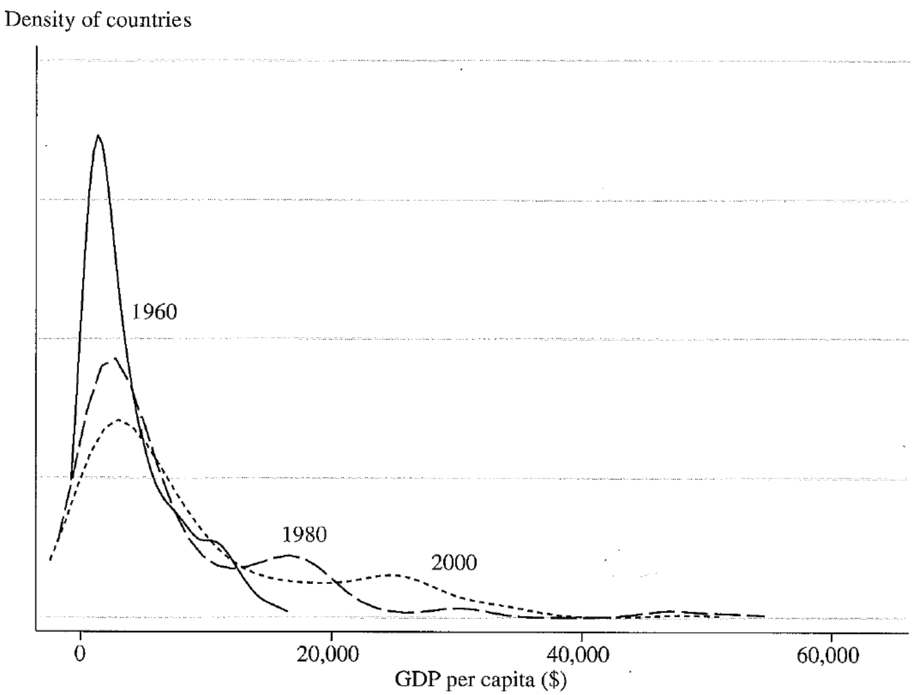


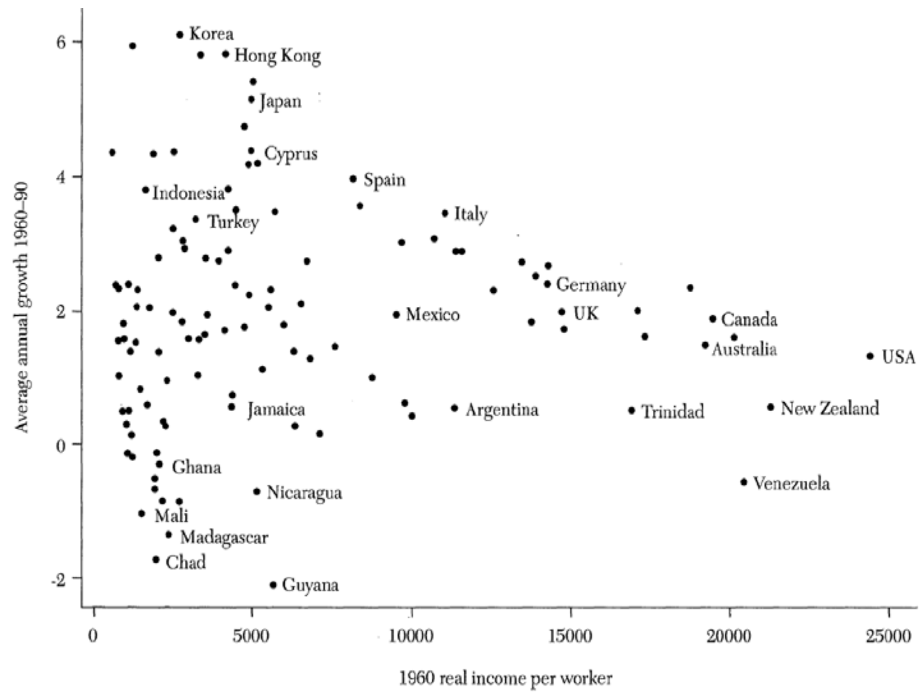
- **Convergence among OECD countries**



- **No convergence across the world, particularly between the group of countries in low-growth development trap and others**




  - **Non-convergence of many sub-Saharan countries**
  - **Non-convergence of some South Asian countries**

## H. From Exogenous to Endogenous Growth Theory

**There are quite a few shortcomings of neoclassical exogenous growth theory, including at least:**

- **Long-run steady state without growth except exogenous technical progress**
- **Unrealistic result for exogenous technical progress accounting for more than 70% of output growth**
- **Predicted convergence lack of strong evidence when considering the entire world economy rather than the developed world (e.g., OECD)**
- **Failure to explain widened growth disparities, particularly, TFP gaps fail to explain widened real GDP per capita gap**
- **Failure to explain long-run imbalance between capital/raw labor**
- **Policy irrelevance unless it can affect the rate of technological progress**

**These shortcomings lead to the birth of endogenous growth theory, which has been formally developed since mid-1980s following the pivotal works by Romer (1986, 1990) and Lucas (1988) and which has soon become the dominant framework for studying economic growth and development.**

# Part-2: Endogenous Growth Theory

## A. Development of Endogenous Growth Theory

- **Early work prior to 1980:**
  **von Neumann (1937) – linear production & balanced growth**
  **Solow (1956) – increasing returns & sustained growth**
  **Pitchford (1960) – decreasing returns with capital-labor as substitutes & sustained growth**
  **Arrow (1962) – learning by doing & growth**
  **Uzawa (1965) – human capital, education & growth**
  **Nelson and Phelps (1966) – human capital & productivity growth, A(H)**
  **Shell (1966) – inventive activity & growth**
  **Arrow (1969) – technology spillover & growth**
  **Wan (1970) – learning, innovation & growth, A(learning by doing)**
- **Pivotal studies of new growth theory:**
  **Romer (1986) – general or knowledge capital**
  **Lucas (1988) – human capital**
  **Stokey (1988) – learning by doing**
  **Rebelo (1991) – the AK model**
  **Romer (1990), Grossman-Helpman (1991), Aghion-Howitt (1992) – R&D and horizontal/vertical innovation**

- **New stylized facts:**

**(S1)** **Club-convergence: bi-mode (Quah 1996)**

**(S2)** **Divergent factor accumulation paths:**

| | | | |
|---|---|---|---|
| **US** | **high K** | **high H** | |
| **Macao** | **low K** | **high H** | **(Taiwan - H-intensive)** |
| **Congo** | **high K** | **low H** | **(Korea - K-intensive)** |
| **Zaire** | **low K** | **low H** | |

**(S3)** **Increasing rates of growth for the leading economy:**

| | | |
|---|---|---|
| **1700-1785** | **Netherlands** | $\theta = -0.07\%$ |
| **1785-1820** | **UK** | $\theta = 0.5\%$ |
| **1820-1890** | **UK** | $\theta = 1.4\%$ |
| **1890-1970** | **US** | $\theta = 2.3\%$ |

**(S4)** **Migration of both skilled/unskilled to rich countries: why would the unskilled migrate if paid at their marginal product?**

**(S5)** **Over-taking/lagging behind development experiences:**
**why did Korea/Taiwan over-take the Philippines?**
**why did Argentina drop from the high rank in 1900?**

**(S6)** **Cross-country productivity disparities and widened world income differences**

- **Birth of endogenous growth theory**

| | One-Sector | Two or Multi-Sector |
|---|---|---|
| **Constant Returns**<br>**or**<br>**Long-run constant return** | **(S1)** ***Romer (1986)***<br>***Rebelo*** **(1991)**<br>***Barro*** **(1990)**<br>**Jones-Manuelli (1990)** | **(S2) Bond-Wang-Yip (1996)**<br>**(S4)** ***Lucas*** **(1988)** |
| **Increasing Returns**<br>**or**<br>**Imperfect competition**<br>**or**<br>**Distortions** | **(S3) Romer (1986)**<br>**Xie (1991)** | **(S5) Xie (1994)**<br>**Benhabib-Perli (1994)**<br>**Bond-Wang-Yip (1996)**<br><br>**(S6)** ***Romer (1990)***<br>**Grossman-Helpman (1991)**<br>***Aghion-Howitt*** **(1992)**<br>**Acemoglu-Aghion-Zilibotti (2006)**<br>***Wang (2010)*** |

**Note:** **Those highlighted (in *italic*) will be discussed in this note – thus, our focus will only be on stylized facts (S1), (S3), (S4) and (S6) for now.**

## B. One-Sector Endogenous Growth Theory

**To allow for sustained growth, the one-sector production must feature constant (our focus) or increasing returns, implying that the marginal product of capital (MPK) is strictly positive even when the capital stock grows to infinity. We will present the basic setup, followed by three useful economic models with different focuses that can be applied to different growth and development issues. For simplicity, we normalize the population to 1, so the population growth rate is zero (n = 0).**

### 1. The Basic Setup

- **Lifetime utility: U = $\int_0^\infty u(c)e^{-\rho t}dt$, with $u(c) = \dfrac{c^{1-\sigma^{-1}} - 1}{1-\sigma^{-1}}$**
- **Periodic budget constraint & capital evolution:**

  $$\dot{k} = f(k) - \delta k - c$$

  **where the initial capital stock per capita is $k(0) = k_0 > 0$**
- **Main feature: The setup is similar to the Ramsey optimal growth model except that the production function f(k) now features strictly positive MPK (e.g., $f(k) = AK$ or $f(k) = AK+BK^\alpha$)**

- In competitive markets, capital efficiency is reached when MPK equals the market rental rate r:

  $f_k = r$ (KE locus)

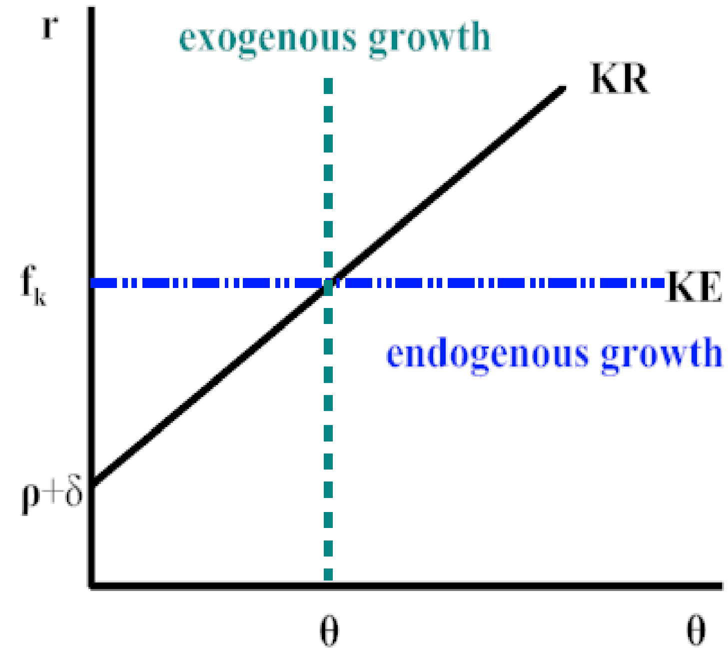
  which is drawn under constant returns (MPK = constant)

- The Keynes-Ramsey equation:

$$\theta = \frac{\dot{c}}{c} = \sigma\left[f_k - (\rho + \delta)\right]$$

  which, with the use of (KE), can be rewritten as,

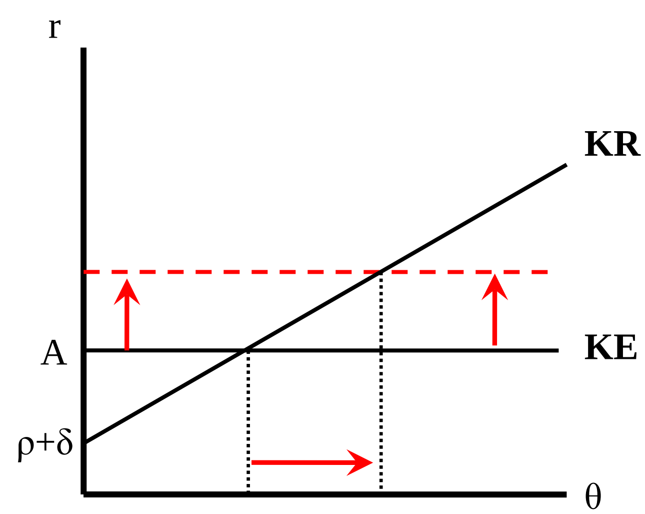
$$r = (\rho + \delta) + \sigma^{-1}\theta \quad \text{(KR)}$$




- Endogenous vs. exogenous growth:
  - the KR locus is the same in both exogenous and endogenous growth models
  - θ is exogenous (= η) in the exogenous growth model
  - the endogenous *balanced growth* rate (θ) is determined in the endogenous growth model by the KE and KR loci jointly, where under constant returns, c, k and y all grow at this common rate θ

## 2. The AK Model: Rebelo (1988)

- **Production function: y = f(k) = Ak**
- **Endogenous growth rate:** $\theta = \sigma[A - (\rho + \delta)] > 0$
  - **higher productivity A => MPK↑ => encourage capital accumulation & raise growth**
  - **higher intertemporal substitution σ or lower time discounting ρ => encourage saving => raise capital and output growth**




- **Main implications:**
  - **Countries with different A will grow at different growth rates, implying non-convergence (S1)**
- **Problem: why China, with low A, has grown so rapidly**
  - **Improvement in human capital**
  - **International trade & FDI**
  - **Institutions toward pro-market**

- **Any policy affecting the *level* of A has a *growth* effect**
  - **This represents a “standing on the giant’s shoulder” effect, where persistent & continual good policy matters:**
    - **political cycles in democratic countries may be potentially harmful**
    - **long lived ruling party is bad as well, most of time much worse due to lack of check and balance**
  - **Such growth-promoting policy includes**
    - **well-established political economy**
    - **benevolent or non-corruptive government**
    - **strong IPR protections**
    - **good infrastructure**
    - **well-designed incentives toward investment in physical and human capital**

## 3. Uncompensated Knowledge Spillover: Romer (1986)

- **Production function: $y = f(k, \bar{K}) = Ak^{1-\beta}\bar{K}^{\beta}$**
  - **a firm's output depends on its own capital and the society's aggregate capital $\bar{K}$**
  - **higher aggregate capital raises individual output as a result of *uncompensated knowledge spillover***
  - **to each individual, $\bar{K}$ is taken as given**
  - **in equilibrium, $\bar{K}$ = k (recall that population = 1)**
- **Remarks:**
  - **$\bar{K}$ includes physical, human, knowledge and firm organizational capital**
  - **Externality (or the spillover) causes free-rider problem and results in underinvestment**
- **Capital efficiency:** $r = MPK - \delta = A(1-\beta)\left(\frac{\bar{K}}{k}\right)^{\beta} - \delta = A(1-\beta) - \delta > 0$
- **Endogenous growth rate: $\theta = \sigma[A(1-\beta) - (\rho + \delta)] > 0$**
  - **$\theta$ is increasing in A and $\sigma$ but decreasing in $\rho$, same as AK-model**

- o **new insight:**
  - **the stronger the uncompensated knowledge spillover (β) is, the lower the endogenous growth rate will be**
  - **intuition: the free-rider problem leads to under-investment**
  - **with no free-rider problem (β = 0), it reduces to the AK-model**
- **Policies to correct the underinvestment problem**
  - o **Free education: education subsidy**
  - o **Patent protection policy: provide public subsidy to inventors but do not prohibit the use of the invented knowledge**
  - o **Public provision of investment incentives: investment-tax credit, successful cases found in East Asian countries**

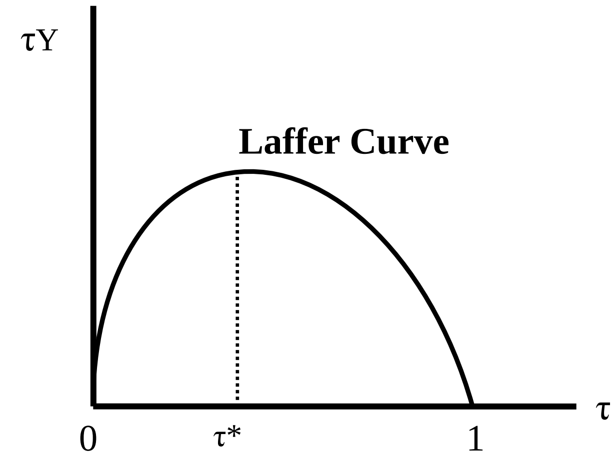
## 4. Public Capital: Barro (1990)

- **Setup: $y = f(k) = Ak^{1-\beta} G^{\beta}$**
- **Public capital $G = \tau y$ in equilibrium; each individual takes G as given**
- **3 types of public provision of capital**
  - o **Infrastructure: highways, satellite**
  - o **Public provision of education or knowledge**
  - o **Public provision of physical capital: science park, utility, etc.**

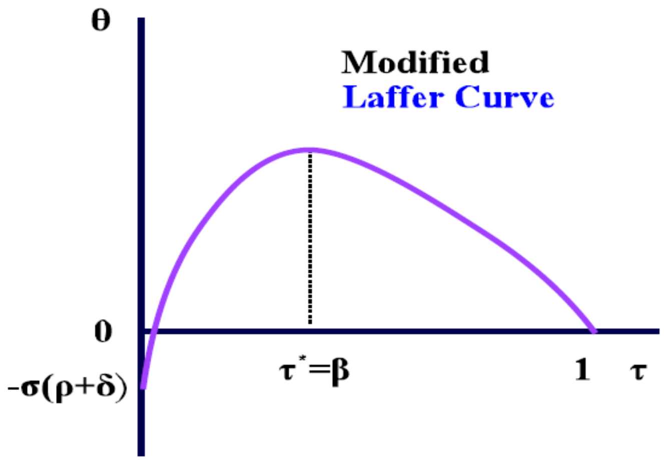
- **Endogenous balanced growth rate now depends on $\tau$:**

$$\theta = \sigma\left[(1-\beta)A^{1/(1-\beta)}\tau^{\beta/(1-\beta)}(1-\tau) - (\rho+\delta)\right]$$

**where $\tau = 0$ or $1 \Rightarrow \theta = -\sigma(\rho+\delta) < 0$**




- **Growth-maximizing flat tax rate: $\tau^* = \beta$**
  - **For $\tau^* < \beta$, higher tax rate corrects free rider problem and raises growth**
  - **For $\tau^* > \beta$, higher tax rate reduces investment incentive and growth**
  - **This yields the modified Laffer curve**
  - **To maximize economic growth, the tax rate should be set the productive public good's share ($\beta$)**




- **Growth-maximizing government size: $G/y = \tau^* = \beta$**
- **Empirical observations: ambiguous cross-country relationships between government size and growth**
  - **Northern EU countries: much larger government size than US**
  - **Problem: Singapore has $\tau$ but high G (non-tax revenues)**

## C. Multi-Sector Endogenous Growth Theory

**While the one-sector model offers clean setups and useful explanations for (S1) and (S2), it is too stylized to capture other real world observations. We will thus move to more realistic multi-sector frameworks, with endogenous human capital accumulation or endogenous technical progress.**

### 1. Human Capital Based Endogenous Growth Model

- **Uzawa (1965): the rate at which human capital is accumulated depends positively on both the stock of human capital and the education input**
- **Nelson and Phelps (1966): if the prevailing technology A is below the frontier technology $\bar{A}$, then the productivity growth rate is increasing in both this technology gap and the stock of human capital H**
- **Lucas (1988) utilizes the Uzawa argument to develop a two-sector endogenous growth model with perpetual physical and human capital accumulation. He begins with a model with constant returns, followed by one with increasing returns in the presence of positive uncompensated human capital spillovers**

## a. The Basic Constant-Returns Model: Lucas (1988)

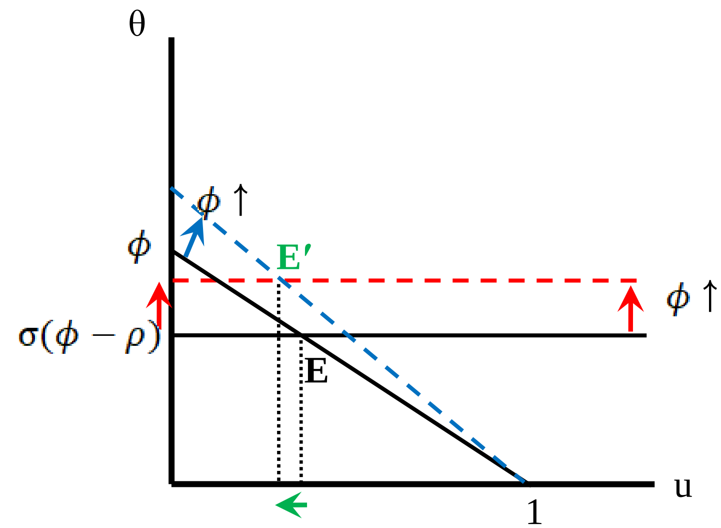
- **By incorporating human capital, the new concept of the labor input is now "effective labor," defined as the multiple of work hours (u) and human capital (H): L=uH**
- **Lifetime utility:** $U = \int_0^\infty \frac{c^{1-\sigma^{-1}} - 1}{1-\sigma^{-1}} e^{-\rho t} dt$ **(same as before)**
- **Goods production:** $F = AK^{1-\beta}(uH)^{\beta}$ **(constant-returns in the long run)**
- **Physical capital accumulation (budget constraint):**

  $\dot{K} = F(K, uH) - \delta K - c$ **, with** $K(0) = K_0 > 0$
- **Human capital accumulation (linear & hence constant-returns):**

  $\dot{H} = \phi(1-u)H$ **, with** $H(0) = H_0 > 0$

  **where** $\phi$ **measures the maximal human capital growth rate with 1-u=1, governing education efficacy**
- **Constant-returns technologies with constant intertemporal elasticity of substitution =>** $\frac{\dot{K}}{K} = \frac{\dot{H}}{H} = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = \theta$**, i.e., common growth**

- **Human capital accumulation (HA) thus leads to:** $\theta = \dfrac{\dot{\mathbf{H}}}{\mathbf{H}} = \phi(1-\mathbf{u})$
- **Keynes-Ramsey (KR):** $\theta = \dfrac{\dot{\mathbf{c}}}{\mathbf{c}} = \sigma[\mathbf{MPK} - (\rho + \delta)]$
- **Intertemporal no-arbitrage (IN) in K and H:** $\mathbf{MPK} - \delta = \mathbf{MPH} = \phi$
- **(KR) and (IN) => (KR) governing endogenous growth:** $\boldsymbol{\theta = \sigma(\phi - \rho)}$
- **Main findings:**
  - **Other than preference parameter (σ, ρ), growth is driven *only* by education efficacy ϕ**
  - **Higher ϕ => the human capital accumulation locus and the endogenous growth locus shift up**
    - **higher economic growth**
    - **higher education time due to higher MPH:** $1-\mathbf{u} = \sigma(1-\rho/\phi)$
    - **example: rising year of schooling with reducing work hours**
    - **policy: mandatory education (6-12 years); on-the-job training; talent pool matching**



## b. Increasing-Returns with Human Capital Spillover: Lucas (1988)

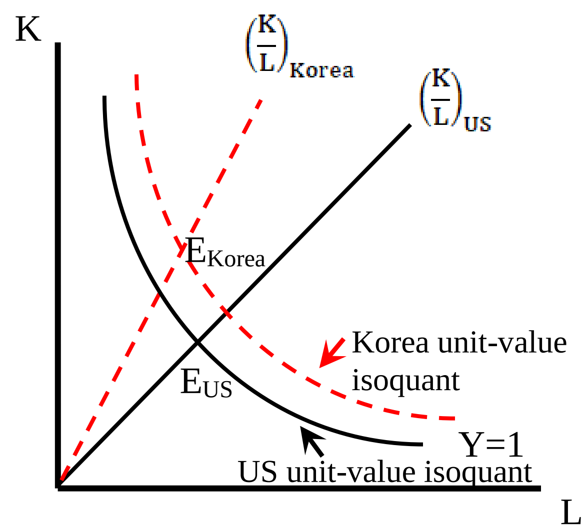
- **Generalized goods production function: $F(K, H, \overline{H}) = AK^{1-\beta}(uH)^{\beta}\overline{H}^{\gamma}$**
  - **$\overline{H}$ captures uncompensated human capital spillover (similar to the argument by Romer 1986) and $\gamma$ measures the degree of spillover**
  - **$\overline{H}$ is taken as given by individuals and $\overline{H}$ = H in equilibrium**
- **Main findings:**
  - **Other than preference parameters, economic growth depends not only on the maximal human capital accumulation rate $\phi$ but on the degree of human capital spillover $\gamma$ (positive growth effect) – higher $\gamma$ lowers u and hence raises the wage rate**
  - **Similar to Romer, there is a free rider problem as a result of uncompensated spillover, implying under-investment in education by individuals (inefficiency can be mitigated by education subsidy)**
  - **So, H grows slower than K (or Y, C), explaining stylized fact (S2)**
  - **Due to uncompensated positive human capital spillover, even the unskilled would migrate to rich countries where the aggregate human capital stock is high and spillover is stronger to offset higher cost of living, thereby explaining stylized fact (S4).**

## 2. R&D Based Endogenous Growth Model

**We begin by developing a basic framework modifying the Nelson-Phelps (1966) technology advancement setup and the Acemoglu-Aghion-Zilibotti (2006) distance-to-frontier setup, a simplified version of Wang (2010). We then shift to the horizontal innovation (product variety) model of Romer (1990) and the vertical innovation (product quality) model of Aghion-Howitt (1992). For now, we ignore cost-reducing process innovation.**

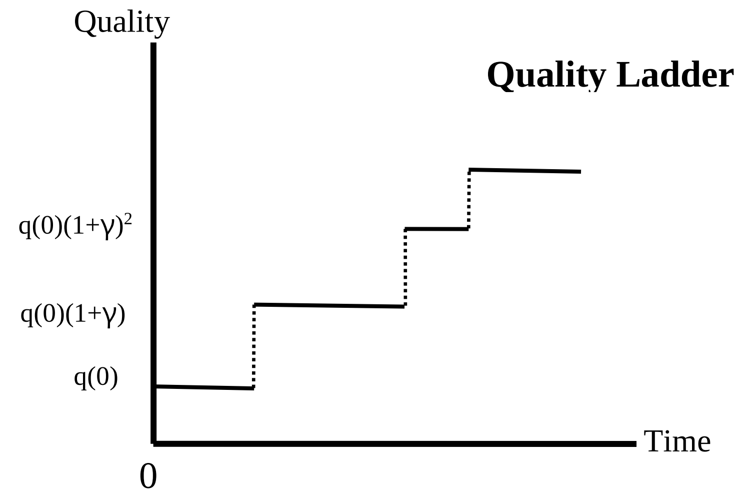
**Remarks:**

- **Three forms of technological progress: invention, imitation, adoption**
  - **Datsun/Toyota**
  - **Hyundai**
- **Two ways of improving production performance:**
  - **Improve efficiency along the same K/L ray**
  - **Change K-L composition toward an advanced economy**



## a. Technology Gap and R&D: Wang (2010)

- **Innovation versus implementation:**
  - o **the leading-edge frontier technology: $\bar{A}$**
    - **growing on the *quality ladder* at rate γ(n), depending on R&D effort n**
      **Note: the "shape" of the quality ladder depends on**
      - **a. The size of the leap γ**
      - **b. The speed of arrival**
    - **$\bar{A}$ is taken as given by individuals**




  - o **fraction of sectors on the frontier (innovating sectors): η**
  - o **fraction of sectors below the frontier (implementing sectors): (1-η)**
  - o **technology gap: $\bar{A} – A$, with its effect on technical progress depending on implementation/imitation effort m**

- **Technology advancement (sector 2):**

$$\dot{A}/A = \eta\gamma(n) + (1-\eta)\psi(m)(\bar{A} - A)/A$$

**= own innovation + technology adoption/licensing/imitation**

  - **$\eta\gamma(n)$ = the growth rate in the frontier sector**
  - **$\gamma = \gamma_0 n$ captures the frontier technology expansion rate**
  - **$\psi = \psi_0 m^b$, 0<b<1 captures more rewarding technology adoption/imitation with a larger technology gap ratio ($\bar{A}$-A)/A= a**
- **Effective labor: L=A(1-n-m)**
- **Goods production (sector 1): $F(K,L) = K^{1-\beta}(A(1-n-m))^{\beta}$**
- **Capital accumulation (budget constraint):**

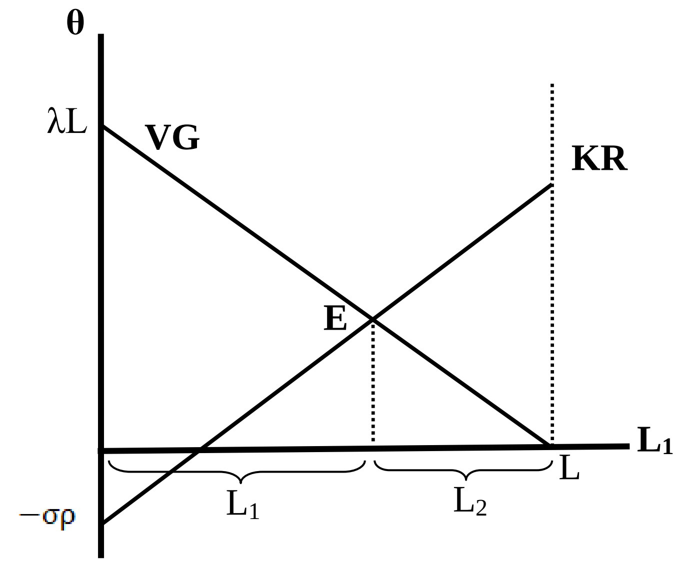
$$\dot{K} = K^{1-\beta}(A(1-n-m))^{\beta} - \delta K - c, \text{ with } K(0) = K_0 > 0$$

- **Cross-sector allocation of time implies: MPL must be equalized =>**
  **$MPn_1 = MPn_2$ and $MPm_1 = MPm_2$**
- **Main findings: both innovation and imitation are valuable**
  - **Higher frontier growth $\gamma_0$ widens the technology gap ratio, promotes economic growth and leads to larger firm dispersion**
  - **A larger fraction of frontier sectors $\eta$ increases technology gap ratio and increase growth $\theta$**

## b. R&D and Horizontal Innovation: Romer (1990)

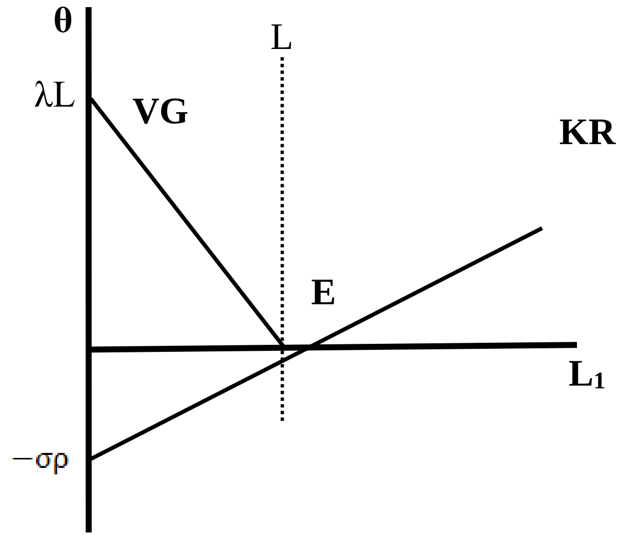
- **Labor allocation: $L_1$ for production and $L_2$ for R&D, with $L_1 + L_2 = L$**
- **Final good production (numeraire):**
  - **perfectly competitive**
  - **produced with labor and a basket of M intermediate goods $x_i$**
    - **longer production line M => higher specialization/productivity**
    - **the sophistication of the production line can growth, depending on R&D labor:** $\dot{M}/M = \lambda L_2$ **(variety growth, VG)**
  - **production function:** $\mathbf{Y = L_1^{1-\alpha} \int_0^M x_i^{\alpha} di}$
  - **labor demand:** $MPL_1 = (1-\alpha)L_1^{-\alpha}Mx^{\alpha} = w$ **(*ex post* symmetry $x_i$=x)**
- **Intermediate goods production:**
  - **monopolistically competitive**
  - **monopoly rent measured by markup:** $\eta = (1-\alpha)/\alpha$ **(= 0 if α = 1)**
  - **maximized profit (earned forever with new entry):** $\Pi = \eta x$
- **R&D decision:**
  - **profit:** $\Pi_R = (\Pi / r_D)\dot{M} - wL_2 = (\Pi / r_D)\lambda L_2 M - wL_2$
  - **labor demand:** $MPL_2 = (\Pi / r_D)\lambda M = w$

- **The two labor demand conditions together with maximized intermediate firm profit yield the R&D return per incremental variety:**
  $\mathbf{r_M = \alpha\lambda L_1}$ **(RD)**
  **where R&D return depends on R&D productivity ($\lambda$) and MPx ($\alpha L_1$)**
- **Intermediate varieties growth**
  **(VG):** $\theta = \dfrac{\dot{M}}{M} = \lambda L_2 = \lambda(L - L_1)$
    - **downward sloping in $L_1$**
    - **higher $\lambda$ improves R&D efficacy => higher growth**

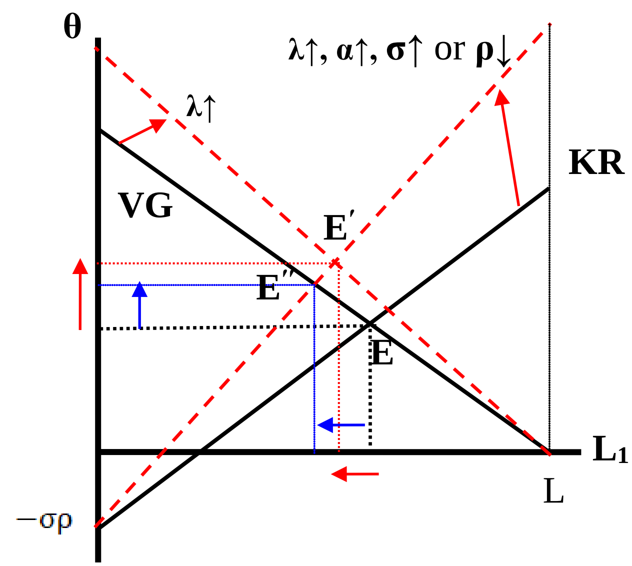



- **Keynes-Ramsey (KR):**
  $\theta = \sigma(r_M - \rho) = \sigma(\alpha\lambda L_1 - \rho)$
    - **upward sloping in $L_1$**
    - **higher $\lambda$ improves R&D efficiency, raises intermediate firm's profitability/rent and enhances output growth**
    - **higher $\alpha$ raises MPx and R&D returns => higher growth, but lower production labor)**
- **Equilibrium: point E where RD and KR intersect with each other**

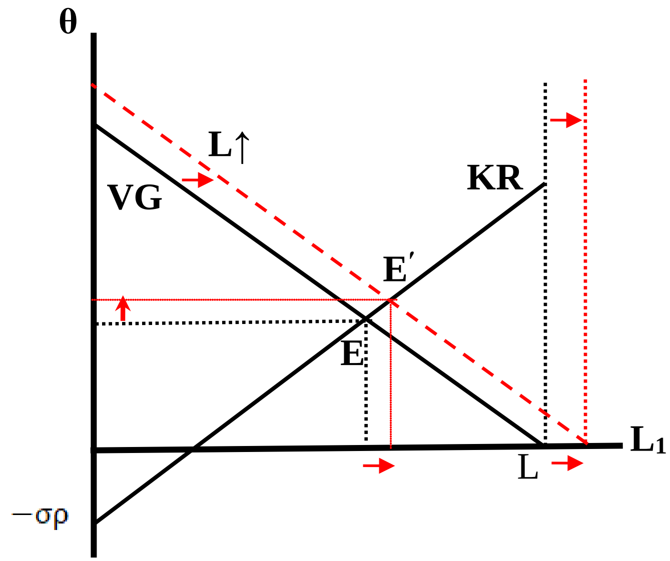
- **Condition for nondegenerate BGP: $\alpha\lambda L > \rho$**
  - **this condition requires that the economy be productive enough such that both sectors are operative**
  - **this condition is met when:**
    - **R&D productivity and labor scale are high**
    - **time discounting and markup are low**
    - **this signifies a low productivity trap**



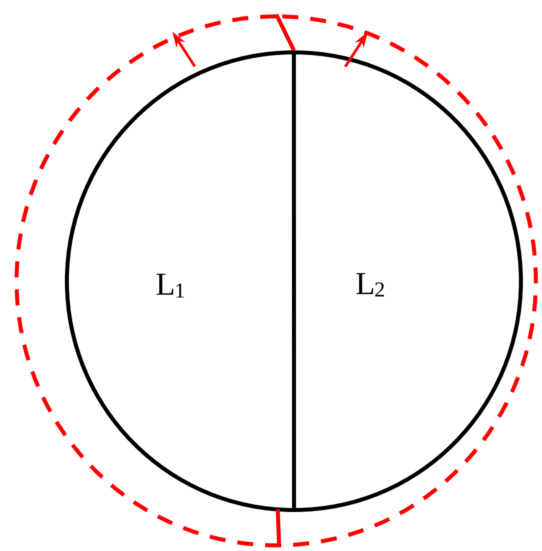

- **Main findings:**
  - **higher R&D productivity λ encourages R&D, reallocates labor away from production and raises economic growth (E′)**



- o **higher α (lower markup) raises profitability in R&D and increases growth (E″)**
- o **more intertemporal substitution (σ↑) or more patient (ρ↓) reallocates labor toward R&D and raises growth (E″)**
- o **larger employment size (L↑) raises production labor, R&D labor as well as growth**




- **Examples:**
  - o **TSMC: superior technology in customized and low nanometer chips => higher λ => higher growth/R&D**
  - o **Traditional industries: do not require much of specialization, which is equivalently to lower λ => lower incentives for R&D and lower growth**



- Highly capital intensive OEM: shortened product cycle => lower markup => *higher* growth?

- **Problems:**
  - Lower markup leads to higher R&D is counterfactual – an issue with this production-line setting
  - The scale effect – larger countries grow faster – is unrealistic, as pointed out by Jones (1995)

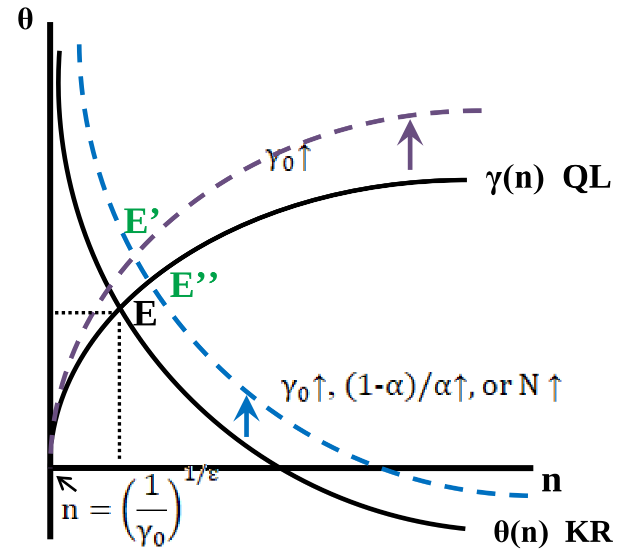
## c. R&D and Vertical Innovation: Aghion-Howitt (1992)

- **Vertical innovation: the quality of goods rises with more R&D – the size of the "quality ladder" (QL) is $\gamma(n)$ and hence technology evolves according to $\dot{A} = \gamma(n)A$, where $\gamma(n) = \gamma_0 n^{\varepsilon}$, with $0 < \varepsilon < 1$**
- **Labor allocation:**
  - Total non-research labor =N (exogenous)
    - N can be devoted to manufacturing (L) and R&D (n)
  - researcher =R (exogenous) (general researcher, outside of the firm)

- **Final good production (numeraire):**
  - **perfectly competitive**
  - **produced with only intermediate good**
  - **production function: $\mathbf{Y = Ax^{\alpha}}$, $\mathbf{0 < \alpha < 1}$ (allow for rent by intermediate producer)**
- **Intermediate goods**
  - **monopolizing over final good producers**
  - **production function (linear): x = L = N - n**
  - **labor demand: $\mathbf{MPL} = \alpha^2 \mathbf{x}^{\alpha-1} = \mathbf{w} = \mathbf{W/A}$ (effective wage)**
  - **maximized profit: $\Pi = \eta \mathbf{Awx}$, with $\eta = (1-\alpha)/\alpha$ measuring the monopoly markup**
- **Innovator's decision:**
  - ***ex ante* perfect competition**
  - **once innovating successfully and entering the intermediate sector, the innovator becomes an *ex post* monopolist**
  - **profit: $\Pi_{\mathbf{R}} = \gamma(\mathbf{n})(\Pi/\mathbf{r}) - \mathbf{Wn} - \mathbf{W_R R}$**
  - **R&D labor demand: $MPL_n = (\Pi/r)\varepsilon\gamma_0 n^{\varepsilon-1} = wA$**

    **Remark: ex ante perfect competition (price taker, here, "value taker" – innovator takes Π as given)**

- **Plugging maximized profit $\Pi = \eta Awx$ into R&D labor demand $MPL_n = Aw$ yields R&D return (RD), $r(n) = \eta\varepsilon\gamma_0 n^{\varepsilon-1}(N-n)$:**
  - **decreasing in n due to diminishing return in R&D, ε < 1 and reduced production labor (less N - n)**
  - **three channels for higher return:**
    - **technological improvement**
    - **higher markup**
    - **larger scale of production (x = N-n)**




- **Endogenous growth is driven by vertical innovation, which is given by: $\theta = \gamma(n) = \gamma_0 n^{\varepsilon}$ (QL) which is upward sloping in n and increasing in R&D efficacy $\gamma_0$**
- **Keynes-Ramsey (KR): $\theta(n) = \sigma(r-\rho) = \sigma(r(n)-\rho)$ depending on net R&D returns, which is**
  - **decreasing in n (diminishing return and production labor effects), implying downward sloping KR**

- **increasing in R&D efficacy $\gamma_0$**
- **increasing in the monopoly markup $\eta$**

- **Main findings:**
  - **Economic growth $\theta$ is driven positively by,**
    - **markup** $\eta = (1-\alpha)/\alpha$
    - **labor size N (scale effect)**
    - **R&D productivity $\gamma_0$**
  - **R&D labor depends positively on markup and labor size**
  - **The effect of $\gamma_0\uparrow$ on R&D labor:**
    - **direct incentive effect (+)**
    - **indirect "R&D labor saving effect" (-)**
    - **by "normality" arguments, when $\gamma_0\uparrow$, it is anticipated that n$\uparrow$ (because R&D is more efficient) – this requires that the direct incentive effect dominate the indirect effect**

- **Problem: there is still a counterfactual scale effect with larger countries growing faster.**

# Part-3: Growth Accounting

## A. Basic Organizing Tool – Neoclassical Production Theory:

- **Consider a neoclassical production function in constant-returns-to-scale Cobb-Douglas form with Harrod-neutral technical progress:**

  $$Y = F(K,L) = K^{\alpha} (AL)^{1-\alpha}$$

- **It can be rewritten in per worker form as:**

  $$y = Y/L = B\,k^{\alpha}$$

  **where *total factor productivity* (TFP) $B = A^{1-\alpha}$ and $k = K/L$**

- **By log differentiation, the economic growth rate can be expressed as:**

  $$\theta = \frac{\dot{y}}{y} = \frac{\dot{B}}{B} + \alpha\frac{\dot{k}}{k}$$

## B. Growth Accounting

- **Thus, economic growth is decomposed into capital deepening (measured by growth in capital per worker) and TFP growth**
- **Denison, Jorgenson and Solow estimate TFP as Solow residual – the residual of output per worker not be explained by capital deepening: Solow residual = $\ln(y) - \hat{\alpha}\ln(k)$, with $\hat{\alpha}$ = 1/3 (capital income share)**

- **Growth accounting estimates in OECD countries**

| % of Growth Driven by TFP Growth | Countries |
|---|---|
| 50-59 | Iceland, Italy, Spain, US |
| 60-69 | Austria, Belgium, Canada, France, Germany, Portugal, UK |
| 70-79 | Australia, Denmark, Finland, Ireland, Netherlands, New Zealand, Norway, Sweden, Switzerland |
| 80-90 | Greece, Japan |

**Thus, TFP growth accounted for at least half of the economic growth of OECD countries, from 55% (Spain) to 86% (Greece), averaging about 68% (which is 1.61% of the average growth rate of 2.41%)**

- **Some earlier work uses raw labor, but the later ones include human capital as part of the capital deepening component**

- **Using data from East Asian Tigers, Young (1995) shows very different TFP growth estimates from the above figures:**

| Country | Economic Growth (%) | TFP Growth (%) | % of Growth Driven by TFP Growth |
|---|---|---|---|
| Hong Kong | 5.7 | 2.3 | 40 |
| Korea | 6.8 | 1.7 | 25 |
| Singapore | 6.8 | 0.2 | 3 |
| Taiwan | 6.7 | 2.1 | 31 |

- **Using data from Taiwan, Tallman and Wang (1994) develops a framework to identify the contribution by human capital separately from physical capital.**
  - **generalized production function with both disembodied technology and human capital:**

    $$Y = F(K,L) = AK^{\alpha} (HL)^{1-\alpha}$$
  - **output per worker,**

    $$y = Y/N = A\, k^{\alpha} H^{1-\alpha}$$

    **where $k = K/L$**

- **conventional studies use crude measures of human capital, such as:**
  - **literacy rate**
  - **primary (P)/secondary (S)/higher (H) education enrollment**
  - **P/S/H education attainment**
  - **years of schooling**
- **it is more appropriate to use refined measures:**
  - **Bils and Klenow (2000) use weighted enrollment rate: E=6×P+6×S+5H**
  - **Tallman and Wang (1994) use weighted attainment rates: E=1×P+1.4×S+2×H, or, 1×P+2×S+4×H**
- **setting H = $E^{\nu}$ and log-differentiating,**

$$\frac{\dot{y}}{y} = \frac{\dot{A}}{A} + \alpha\left(\frac{\dot{K}}{K} - n\right) + (1-\alpha)\nu\frac{\dot{E}}{E}$$

- **estimation shows that human capital accounted for 45% of output growth in Taiwan**
- **using similar approach, Lee, Liu and Wang (1994) and Thanapura and Wang (2002) find the comparable figures in Korean and Thailand are 20% and 28% (15-50% in most countries)**

## C. Problems with Growth Accounting

- **Difficult to separate productivity growth from capital deepening:**
  - **technology is likely embodied in new capital goods:**
    - **Gordon (1990) and Cummins and Violante (2002) find the relative price of capital goods falling dramatically over several decades**
    - **this cannot be explained without technological improvements**
  - **inventive knowledge or new productive idea is likely embodied in human capital**
    - **the real cost of education has risen sharply, but people over-educate to gain wage premium**
    - **such a wage premium is paid only because human capital generates productive returns**
- **National accounts systematically overestimate the accumulation of capital:**
  - **government corruption (Prichett 2000)**
  - **firm misallocation due to capital and institutional barriers (Hsieh and Klenow 2007)**

- **Estimation of TFP based on the production function is biased:**
  - **should Young (1995) be right, Singapore must have fallen rate of returns to capital: Hsieh (2002) finds a roughly constant rate of return – so it must be productivity growth to prevent capital from facing diminishing marginal products**
  - **Hsieh (2002) thus proposes to use the *dual* method by estimating TFP based on unit cost equal to unit price p:**

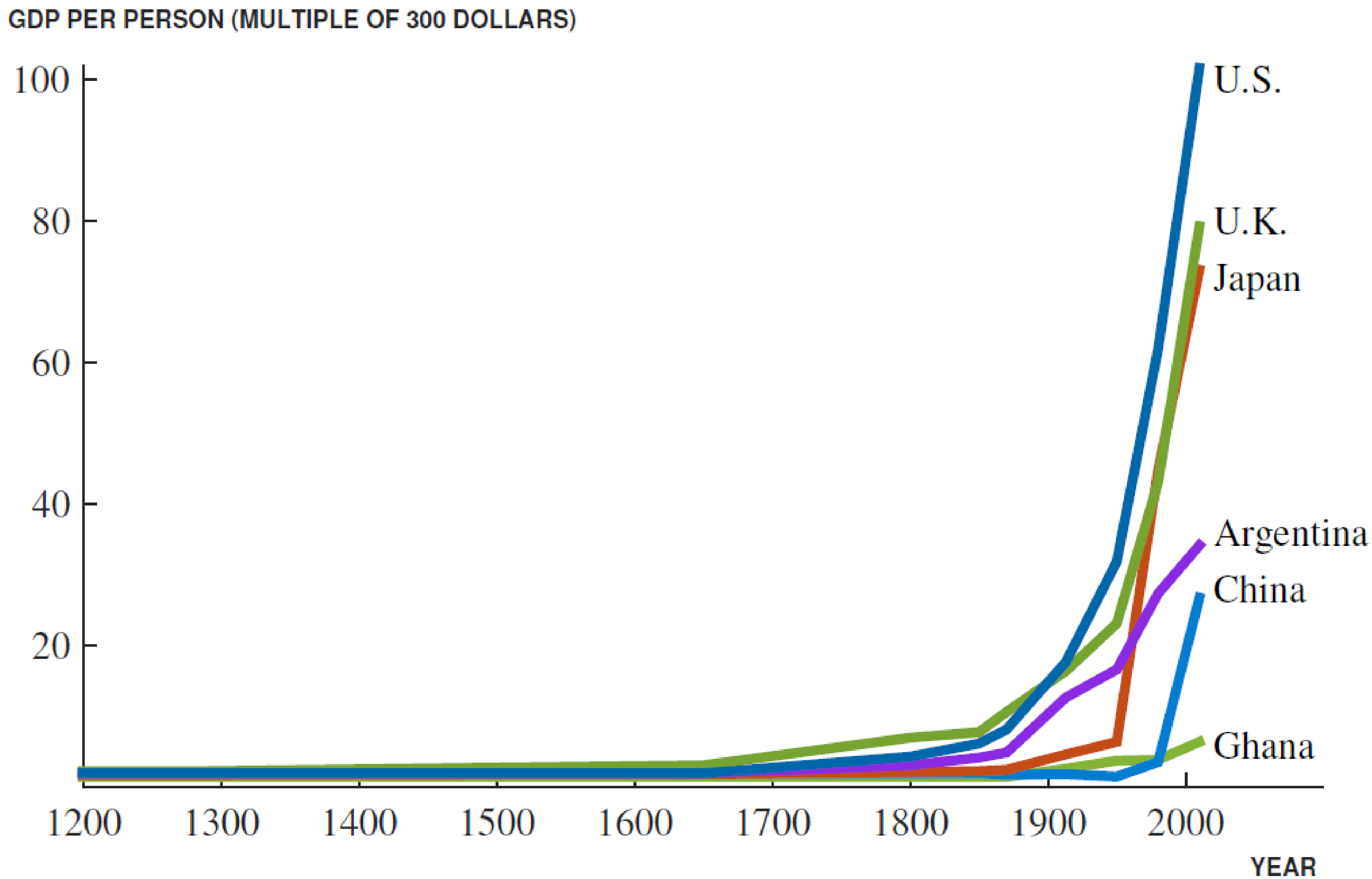
$$\text{unit cost} = \frac{1}{B}\left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha} = p$$

    **since factor prices (r, w) and goods price (p) are observable, TFP (B) can be estimated, which turns out to be 2.2% for the case of Singapore (2 percentage points higher than Young's estimate)**
- **Could Theory Help? The answer is definitely yes. Theory facilitates microfoundation for better understanding of**
  - **consumers' and firms' behavior and response to government policies**
  - **the workings of the markets and the macroeconomy**
  - **the roles of various constraints, market frictions and institutional barriers.**

## D. The Birth of Development Accounting

- **While one may apply the growth accounting framework to many countries, an integrated approach is to compare countries with a benchmark country (typically the U.S. or a reference country of particular interest) – called development accounting**
- **This is especially useful for understanding why some countries are rich, some are poor and why we have seen a great divergence over the past few decades**
- **Development accounting:**
    - **TFP gap:** $\frac{(\mathbf{Y}/\mathbf{L})_{\mathbf{i}}}{(\mathbf{Y}/\mathbf{L})_{\mathbf{US}}} = \frac{\mathbf{A_i}}{\mathbf{A_{US}}}\left[\frac{(\mathbf{K}/\mathbf{L})_{\mathbf{i}}}{(\mathbf{K}/\mathbf{L})_{\mathbf{US}}}\right]^{\alpha}\left[\frac{\mathbf{H_i}}{\mathbf{H_{US}}}\right]^{1-\alpha}$
- **It requires a TFP gap of 40-95 times which is viewed implausible particularly with international capital flows, technology adoptions, technology spillovers and technology assimilation**

# Great Divergence (Jones 2015)



# Development Accounting (Jones 2015)

| | GDP per worker, $y$ | Capital/GDP $(K/Y)^{\alpha/(1-\alpha)}$ | Human capital, $h$ | TFP | Share due to TFP |
|---|---|---|---|---|---|
| U.S. | 1.000 | 1.000 | 1.000 | 1.000 | ... |
| Hong Kong | 0.854 | 1.086 | 0.833 | 0.944 | 48.9% |
| Singapore | 0.845 | 1.105 | 0.764 | 1.001 | 45.8% |
| France | 0.790 | 1.184 | 0.840 | 0.795 | 55.6% |
| Germany | 0.740 | 1.078 | 0.918 | 0.748 | 57.0% |
| U.K. | 0.733 | 1.015 | 0.780 | 0.925 | 46.1% |
| Japan | 0.683 | 1.218 | 0.903 | 0.620 | 63.9% |
| South Korea | 0.598 | 1.146 | 0.925 | 0.564 | 65.3% |
| Argentina | 0.376 | 1.109 | 0.779 | 0.435 | 66.5% |
| Mexico | 0.338 | 0.931 | 0.760 | 0.477 | 59.7% |
| Botswana | 0.236 | 1.034 | 0.786 | 0.291 | 73.7% |
| South Africa | 0.225 | 0.877 | 0.731 | 0.351 | 64.6% |
| Brazil | 0.183 | 1.084 | 0.676 | 0.250 | 74.5% |
| Thailand | 0.154 | 1.125 | 0.667 | 0.206 | 78.5% |
| China | 0.136 | 1.137 | 0.713 | 0.168 | 82.9% |
| Indonesia | 0.096 | 1.014 | 0.575 | 0.165 | 77.9% |
| India | 0.096 | 0.827 | 0.533 | 0.217 | 67.0% |
| Kenya | 0.037 | 0.819 | 0.618 | 0.073 | 87.3% |
| Malawi | 0.021 | 1.107 | 0.507 | 0.038 | 93.6% |
| Average | 0.212 | 0.979 | 0.705 | 0.307 | 63.8% |

- **What are the possible factors to narrow the TFP gap?**
  - **Measurement problem:**
    - **Capital barriers: Buera-Shin (2013)**
    - **Education quality: Schoellman (2012), Cubas-Ravikumar-Ventura (2014)**
  - **Missing inputs:**
    - **Intangible capital: ideas (Romer 1993) and organizational capital (Lucus 1978, Prescott-Visscher 1980)**
    - **Health: health capital (Acemoglu-Johnson 2007, Weil 2007, Y. Wang 2013), health barriers (Wang-Wang 2014) and health productivity (Ravikumar-Wang-Wang 2016)**
  - **Sectoral disparity:**
    - **Agricultural disparities (Colin et al 2012, Lagakos-Waugh 2013, Adamopoulos-Restuccia 2014, Lai 2014)**
    - **Service accounting and the cost disease (Baumol 1985, Young 2014, Liao 2014)**
  - **Country-specific technology assimilation (Wang-Wong-Yip 2016)**

- **Growth theory through the lens of development economics: Banerjee and Duflo (2004) provide a comprehensive list of factors underlying the divergent development process:**
  - **Access to technology**
  - **Human capital externalities (Lucas 1988)**
  - **Coordination failure**
  - **Government failure**
    - **Excessive intervention**
    - **Lack of appropriate regulations**
  - **Credit constraints**
  - **Insurance market failures**
  - **Local complementarities (Romer 1986)**
  - **Incomplete contracts within and across generations**
  - **Behavioral issues**
  - **Micro distortions**
  - **The way forward: disaggregation supported by micro data (the new microfoundation)**

# B. Development Theory

## A. Introduction

**Main development theory: *big push* and *industrial modernization*, which enable economic take-off from poverty and structural transformation from traditional to modern sectors, toward the "mass consumption" state.**

- **Big push: Rosenstein-Rodan (1943, 1961) emphasized "coordinated investment" as the key:**
  - **When many sectors have simultaneously adopted increasing returns technologies, they would all create income as well as demand for goods in other forward and backward linked sectors.**
  - **Such income creation and demand enhancement would then enlarge the market, leading to industrialization.**
  - **Key: scale economies as well as interactions between firms across industries and between demands and supplies**
  - **To overcome a scale barrier requires a "big push" – the forces of big push could be history or expectations (self-fulfilling prophecies)**
- **Industrial modernization: Rostow (1960) and Tsiang (1964) built the foundation with full dynamic GE analysis development only since late 1990s – the key is to qualify how modern industry is activated, which will be fully analyzed here.**

## B. A Simple Theory of Big Push: Murphy-Shleifer-Vishny (1989)

- **Main contribution: to formalize the demand spillovers**
- **Preference over a basket of industrial goods $x_i$: $U = \int_0^1 \ln(x_i)\,di$, with [0,m] featuring traditional and [m,1] modern goods**
- **Production:**
  - **traditional technology (m sectors): constant returns – each unit of labor produces one unit of output (cottage production, zero profit)**
  - **modern technology (n sectors): increasing returns – upon paying a fixed amount of labor input F, the monopolist in each sector turns each additional unit of labor into a unit of output**
- **Profit of each increasing returns sector: by taking labor as numeraire (wage = 1), $\pi = \left(\frac{a-1}{a}\right)y - F = \mu y - F$**
  - **$\mu$ is the markup facing each sector as a result of local monopoly**
  - **the profit is positive as long as the sector is operative**
  - **the profit is higher the greater the markup or the lower the fixed cost is**
- **Aggregate profit (AP): $\Pi = n(\mu y - F)$**

- **Budget constraint (BC): $D = y = \Pi + L$**
- **Output: (AP) and (BC) together yield, $y = \left(\frac{1}{1-\mu n}\right)(L - Fn)$**
  - **$L - Fn$ is total labor used in production (rather than fixed cost investment in the modern sector technology)**
  - **$\frac{1}{1-\mu n}$ is the multiplier, increasing in n and exceeding one due to increasing returns**
  - **y is increasing in n as long as each sector is operative**
- **Demand spillovers – interactions between demands and supplies:**
  - **more firms in the modern sector (n higher) => larger multiplier => higher aggregate demand (D higher)**
  - **when goods are normal, higher aggregate demand will raise demand for goods (reinforcing between firms across sectors) => more shifts from traditional to modern sectors**
- **To have sufficient firms in the modern sector (initial n) requires a big push, which is history-dependent**
- **Case study: international coordination failure in global value chain can stagnate world development**

## C. Human Capital, R&D and Big Push

**Consider a two-period setup with x denoting current and x′ denoting future. There are two types of two-period lived agents: workers and entrepreneurs. Each unit of effective labor turns into one unit of output. There is no capital market to facilitate intertemporal borrowing/lending.**

- **Workers:**
  - **current period:**
    - **undertaking education (fraction of time = v)**
    - **being self-employed to yield a cottage output A(1-v)h**
  - **future period: working with an entrepreneur to receive a share β of the factory output y′**
  - **human capital accumulation:** $h' = (1+\gamma(\bar{v})v^{\alpha})h$ **, where** $\gamma(\bar{v}) = \gamma_H$ **if** $\bar{v} > v_c$ **and** $\gamma(\bar{v}) = \gamma_L$ **if** $\bar{v} < v_c$ **(**$\gamma_H > \gamma_L$**) (threshold externality in *h*)**
  - **utility:** $u = c + \left(\frac{1}{1+\rho}\right)c'$
  - **budget constraints: c = A(1-v)h (current) and c′ = βy′ (future)**
  - **optimization: maximize u subject to budget constraints**

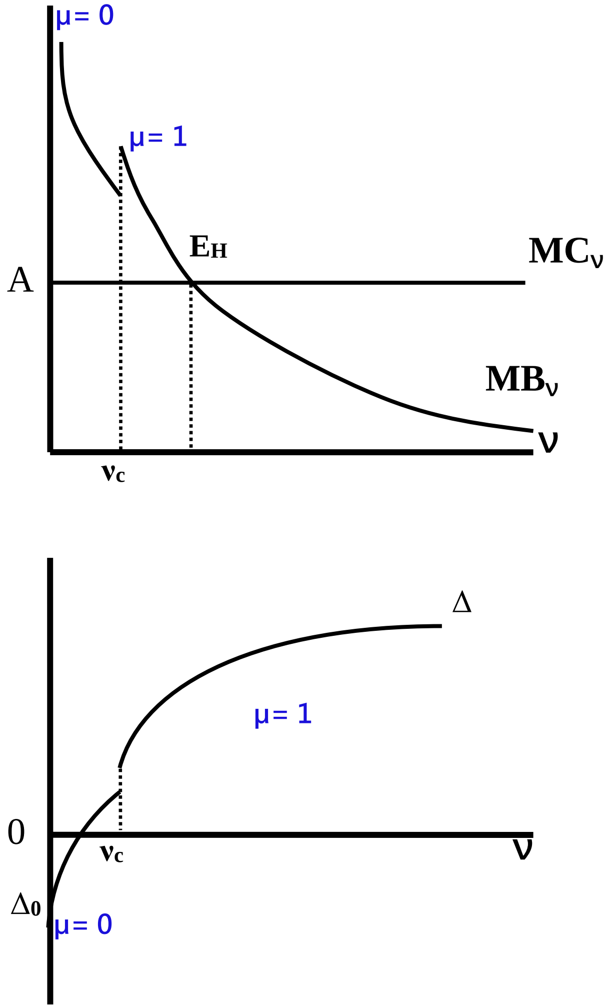
- **Entrepreneurs:**
  - **current period: determine a probability μ to invest in R&D that incurs a sunk cost of ϕA**
  - **future period:**
    - **by undertaking R&D in the current period (probability μ), factory output is enhanced by λ > 1:** $y' = \lambda A(1+\gamma(\bar{v})v^{\alpha})h$
    - **without R&D in the current period (probability 1-μ), factor output is simply:** $y' = A(1+\gamma(\bar{v})v^{\alpha})h$
  - **optimization: maximize discounted sum of flow profits given by,**

$$\Pi(\mu) = -\mu\phi A + \left(\frac{1}{1+\rho}\right)(1-\beta)[\mu\lambda + (1-\mu)]A(1+\gamma(\bar{v})v^{\alpha})h$$

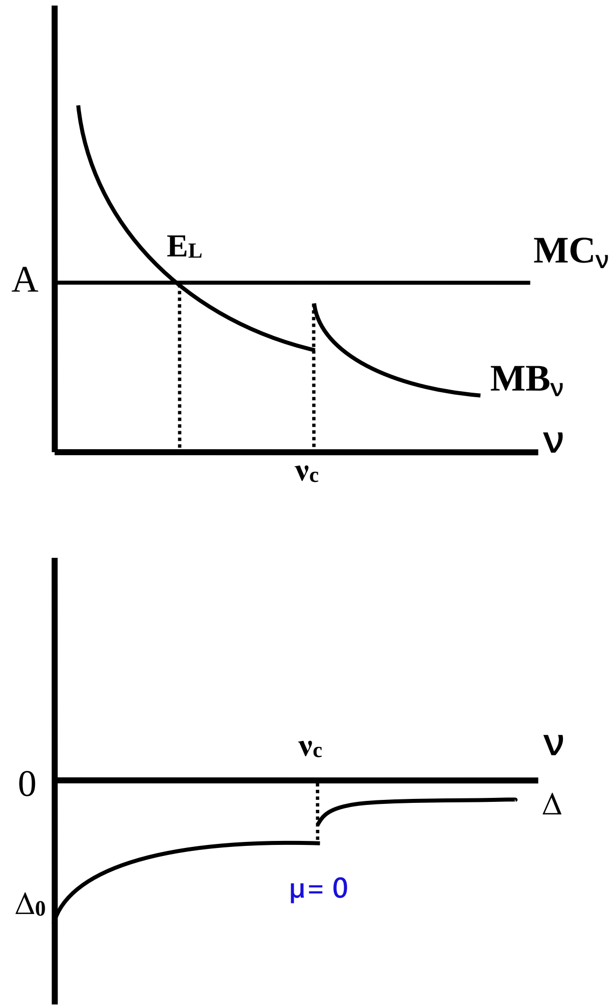
- **Stationary equilibrium: by normalizing h = 1, all other variables become time invariant (i.e., reaching a steady state)**
- **Workers' Optimization:**
  - **$MC_v$ = A (foregone marginal benefit from current production)**
  - **$MB_v$ =** $\left(\frac{1}{1+\rho}\right)\beta[1+(\lambda-1)\mu]A\alpha\gamma(\bar{v})v^{\alpha-1}$ **, falling in v & jump up in** $\bar{v}$
  - **Workers' education is increasing in entrepreneur's R&D effort μ**

- **Entrepreneurs:**
  - **Value:**
    - $\Pi(0) = \left(\frac{1}{1+\rho}\right)(1-\beta)\mathbf{A}(1+\gamma(\mathbf{v})\mathbf{v}^{\alpha})$
    - $\Pi(1) = \left(\frac{1}{1+\rho}\right)(1-\beta)\lambda\mathbf{A}(1+\gamma(\mathbf{v})\mathbf{v}^{\alpha}) - \phi\mathbf{A}$
    - **difference: $\Delta(v) = \Pi(1) - \Pi(0)$, which is increasing in workers' education effort v**
    - **denote $\Delta(0) = \Delta_0$**
  - **Discrete choice with $\mu = 1$ or $0$ if $\Delta(v) >$ or $< 0$**
  - **Entrepreneur's R&D is increasing in worker's education v**
- **Coordination between workers and firms play a key role**
- **Equilibrium outcomes:**
  - **Strategic complementarity: worker's education effort and entrepreneur's R&D effort reinforce each other**
  - **Equilibrium:**
    - **high**
    - **low**
    - **multiple equilibria (high/low co-exist)**

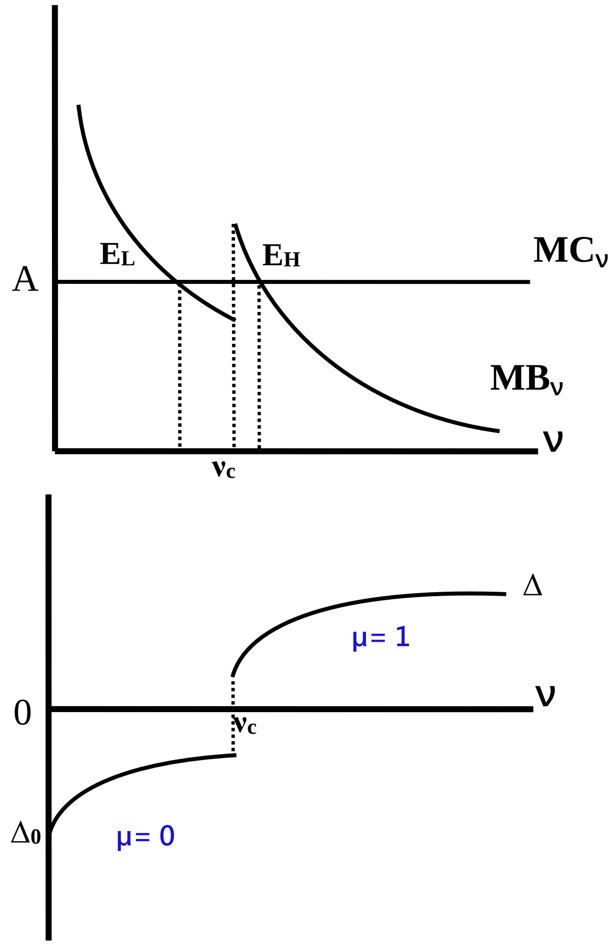
# (i) High Equilibrium




# (ii) Low Equilibrium



## (iii) Multiple Equilibria (arising when γ is large)




- **Main findings:**
  - **coordinated worker investment in education and entrepreneur investment in R&D can generate a big push over the required threshold $v_c$, giving a high-human capital and high-output equilibrium, $y' = \lambda A(1+\gamma_H v_H{}^{\alpha})$**
  - **coordination failure => equilibrium with low-human capital and low-output with $y' = A(1+\gamma_L v_L{}^{\alpha})$**
  - **expectations are crucial:**
    - **if workers expect entrepreneurs to invest in R&D (μ = 1), they exert $v_H > v_c$**
    - **if entrepreneurs expect workers to exert $v_H > v_c$, they invest μ = 1**
    - **optimistic/proactive behavior => high equilibrium**

- **Case study:**
  - **(East Asian miracle) Laing, Palivos and Wang (1995): with job matching interactions and on the job learning, it may only require a small shift to move out of a low-growth trap to reach a high-growth equilibrium**
  - **(1997 Asian crises) Becsi, Wang and Wynne (1999): with market participation externality, a small shift can turn a big push into a big crash**
- **Problems with Big Push Theory:**
  - **History or expectations (Krugman 1991, Matsuyama 1991):**
    - **initial conditions relative to the threshold matter (histories)**
    - **self-fulfilling prophecies matter (expectations)**
  - **The underlying mechanism of big push is unspecified:**
    - **how to implement a policy to lift over the threshold when histories are important?**
    - **how to affect people's self-fulfilling prophecies when expectations play a key role?**

## D. Activation of a Modern Industry: Wang and Xie (2004)

**The key issue to be addressed is to explore the important channels through which a modern industry is activated. The key features considered are:**

- **production in the modern industry requires high-skilled labor in addition to new technologies**
- **the modern sector needs industrial coordination to overcome the scale barrier concerned by big push theory**
- **the modern goods are not necessary for survival**

### 1. The Organizing Framework

- **Two sectors (i=1,2):**
  - **sector 1: traditional industry**
    - **requiring unskilled labor and capital**
    - **producing an agricultural product necessary for survival**
  - **sector 2: modern industry**
    - **requiring skilled labor ($L_2$) and capital, facing a scale barrier**
    - **producing a modern industrial good unnecessary for survival**

- **Production technologies:**
  - **traditional industry:** $\mathbf{Y}_1 = \mathbf{A}_1\mathbf{K}_1^{\alpha_1}\mathbf{L}_1^{1-\alpha_1}$
  - **modern industry:** $\mathbf{Y_2 = A_2K_2^{\alpha_2}L_2^{1-\alpha_2}\overline{K}_2^{1-\alpha_2}}$
  - **key features:**
    - **$L_1$ can be either unskilled or skilled, whereas $L_2$ must be skilled**
    - **$A_2 > A_1$: modern sector is more technologically advanced**
    - **$\alpha_2 > \alpha_1$: modern sector is more capital intensive (manufacturing vs. agriculture)**
    - **there is an uncompensated positive knowledge spillovers in the production of the modern goods represented by $\overline{\mathbf{K}}_2$ as in Romer (1986), where $\overline{\mathbf{K}}_2 = \mathbf{K}_2$ in equilibrium**
    - **as a result of the presence of uncompensated positive knowledge spillovers, there is a scale barrier in the sense that the modern sector's production is not as effective when its size is too small – that is, the coordination issues in big push theory can be captured**
    - **the modern sector is not operative if either $K_2 = 0$ (insufficient funding to be allocated to modern production) or $L_2 = 0$ (unattractive wage for skilled labor to work in modern sector)**

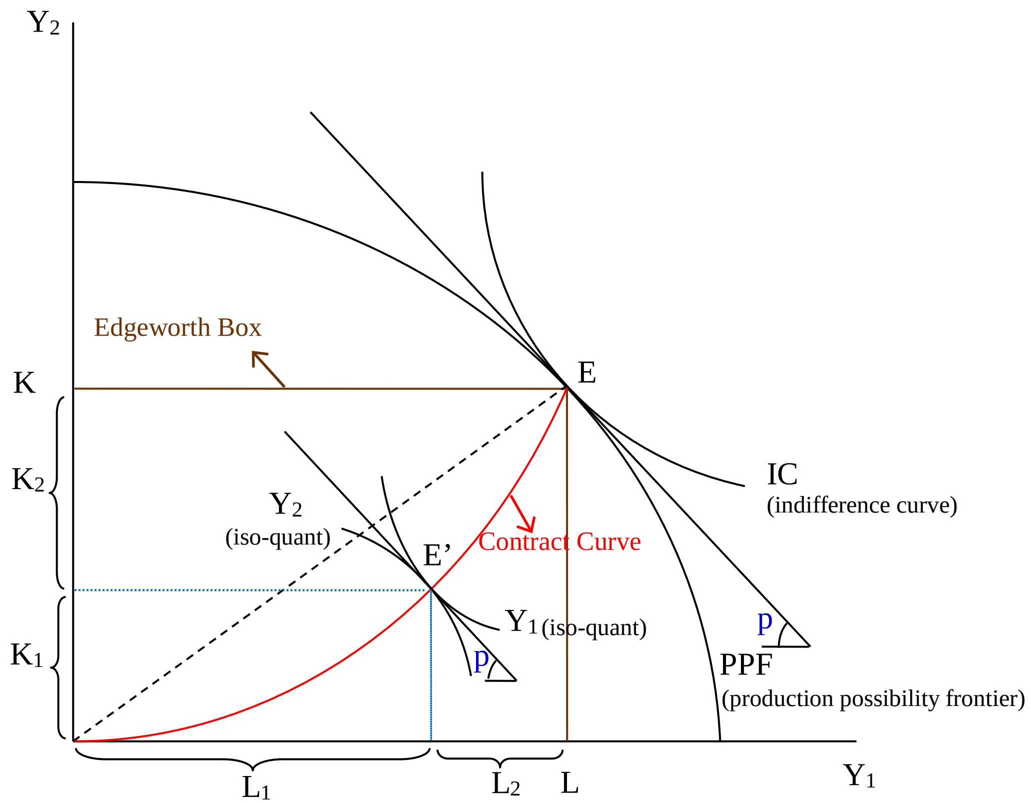
- **Factor allocation (FA) constraints:**
    - **capital: $K_1 + qK_2 \leq F$,**
        - **total available funds F depends on domestic savings as well as foreign aid/FDI**
        - **$q > 1$ captures capital allocation barriers: more costly to invest in modern industry – entry cost, licensing fee, misallocation**
        - **Example: q is lower with**
            - **lower uncertainty or technological/institutional barriers to modern sector investment**
            - **higher modern-industry investment subsidies or investment tax credit/rebate**
    - **skilled labor:**
        - **supply: $N_2$ (fixed)**
        - **constraint: $L_2 \leq N_2$, i.e., modern production requires skills**
    - **unskilled labor:**
        - **supply: $N_1$ (fixed)**
        - **constraint: $L_1 + L_2 \leq N_1 + N_2$ (skilled can work as unskilled)**
        - **$L_2 < N_2$ => integrated labor market with all paid at $W_1$**
        - **$L_2 = N_2$ => segmented labor market with $W_1 < W_2$ (positive skill premium)**

- **Aggregate output: with Y1 as numeraire, real GNP=$Y_1$+p$Y_2$**
- **Preferences: U = ln($C_1$) + ln($C_2$ + θ)**
  - **θ > 0 indicates that $C_2$ is *not* necessary for survival**
  - **Goods market equilibrium (GE) conditions: $C_1 = Y_1$ and $C_2 = Y_2$**
- **Optimization: to attain highest utility given the 3 (FA) constraints, the two production technologies and the 2 (GE) conditions.**

## 2. Competitive Equilibrium:

- **Equalization of the relative price and the marginal rate of substitution (MRS): $\mathbf{p = MRS = \dfrac{A_1 K_1^{\alpha_1} L_1^{1-\alpha_1}}{A_2 K_2 L_2^{1-\alpha_2} + \theta}}$, which depends negatively on the preference bias (the luxury nature of the modern good)**
- **Should the modern industry be operative,**
  - **the values of MPK must be equalized between the two sectors: $\mathbf{MPK_1 = r = (p/q)MRK_2}$**
  - **the values of MPL of the unskilled must be less than that of the skilled in segmented labor market: $\mathbf{MPL_1 = W_1 < W_2 = p \cdot MRL_2}$**

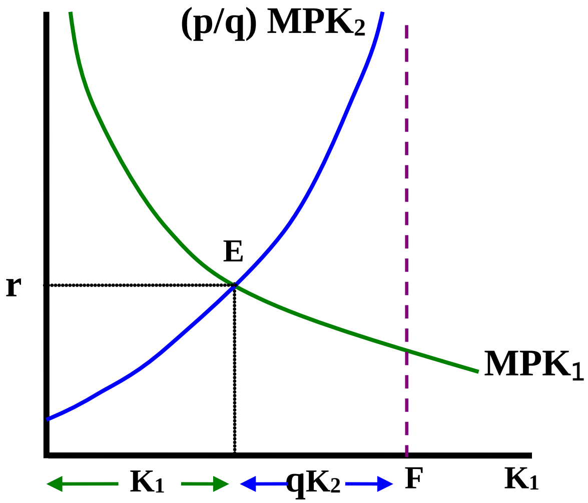
- **Remark: Expanded Edgeworth Box with q = 1**

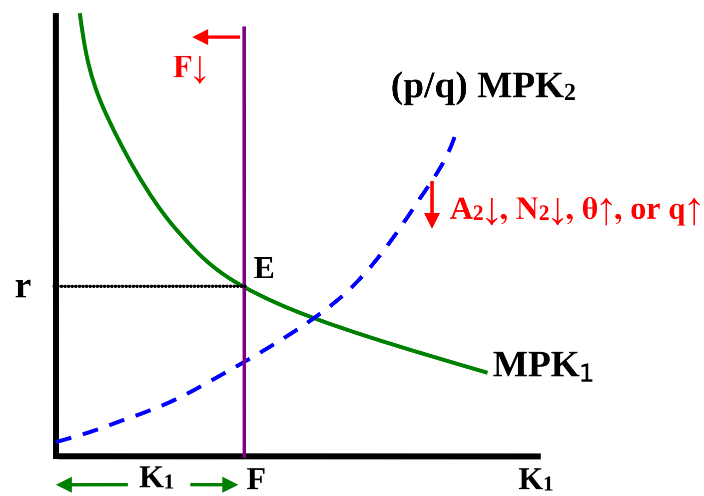


# 3. Conditions for Activating a Modern Industry

- **Industry 2 can become operative only if it is sufficiently productive to attract funding:**

**Case I** **Case II**



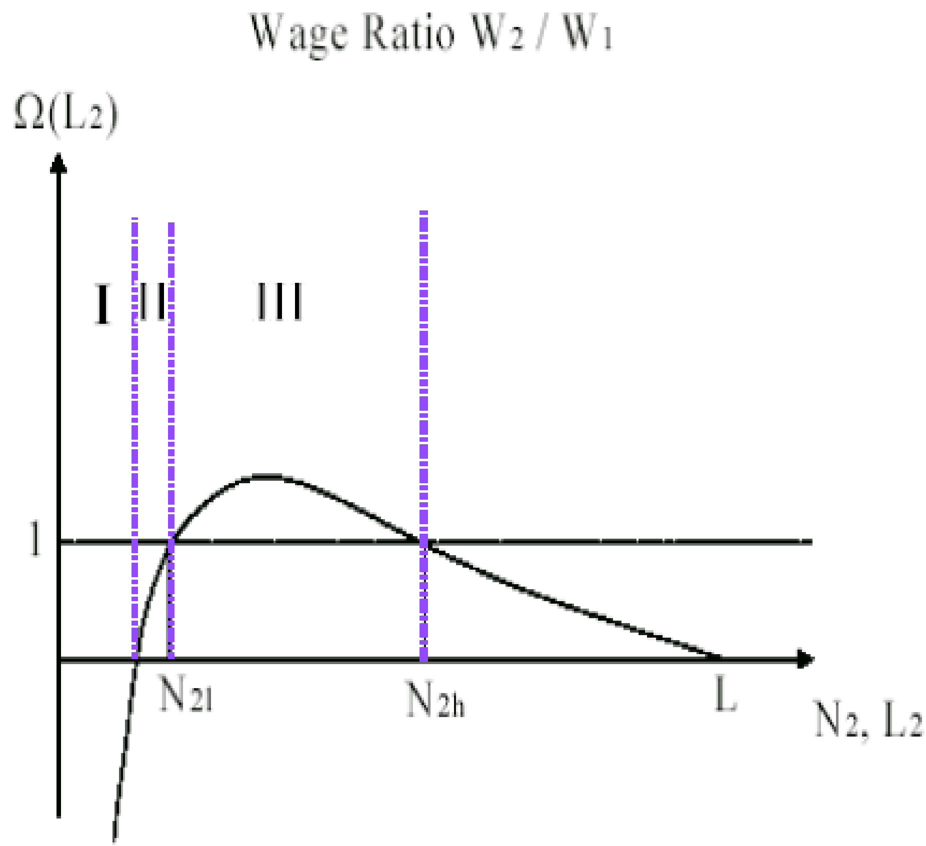




- o **Case I (post-takeoff), the modern sector is operative and the allocation of capital is allocated based on the equilibrium point E**
- o **Case II (pre-takeoff trap), the modern sector is not productive enough to attract funds that are subject to investment barriers**

- **the modern sector remains inactive with**
  - **insufficient funding or skilled labor (low F or $N_2$)**
  - **imperfect adoption of advanced technology (low $A_2$)**
  - **strong preference bias (high $\theta$)**
  - **large modern capital investment barriers (high q)**

- **This above condition is necessary but not sufficient, because it considers only capital but not labor allocation, which is driven by the relative wage of modern to traditional industry ($\Omega = W_2/W_1$)**
  - **in order for the modern sector to attract skilled labor, $\Omega$ must exceed 1 – that is, the skilled can earn higher wages in the modern sector than working as unskilled in the traditional sector**
  - **this condition is more likely to hold true with**
    - **better modern technology $A_2$**
    - **weaker preference bias $\theta$**
  - **when this condition is met, the labor markets are segmented:**
    - **all the unskilled work in the traditional sector ($L_1 = N_1$ ), earning $W_1$**
    - **all the skilled work in the modern sector ($L_2 = N_2$), earning $W_2 > W_1$**

- **In summary, we have:**
  - **I: $\Omega \le 0$, insufficient funding => industry 2 is unprofitable ($MPK_2 < 0$)**
  - **II: $0 < \Omega \le 1$, relative wage too low for skilled workers to participate in industry 2**
  - **III: $\Omega > 1$, the modern industry is activated (takeoff), requiring:**
    - **sufficient resources of capital funds**
    - **sufficient resources of skilled labor**
    - **good access to new technologies**
    - **sufficiently low capital barrier**
    - **sufficiently low preference bias**
  - **Flying geese: more modern sectors developed before reaching $N_{2h}$**



## 4. Development Policy

**The theoretical analysis above leads to policy prescriptions both in the short and in the long run.**

- **Short-term policy prescriptions:**
  - **receiving external assistance to raise F (European Investment Bank loans/U.S. Aid)**
  - **obtaining technological transfer from developed countries to advance $A_2$ (technology adoption, licensing, assimilation)**
  - **attracting immigrants of high skill to increase $N_2$ (Singapore human capital policy, Australian/Canadian immigration policy)**

- **Long-term policy prescriptions:**
  - **greater saving incentives (postal saving, interest/dividend income tax exempt up to a certain amount)**
  - **more R&D investments (patent, R&D subsidy, tax break)**
  - **better education (mandatory education, subsidy to tech schools or higher education, subsidy to on-the-job training, subsidy to on-the-job learning)**