Money, Finance and Growth

Ping Wang©
Department of Economics
Washington University in St. Louis

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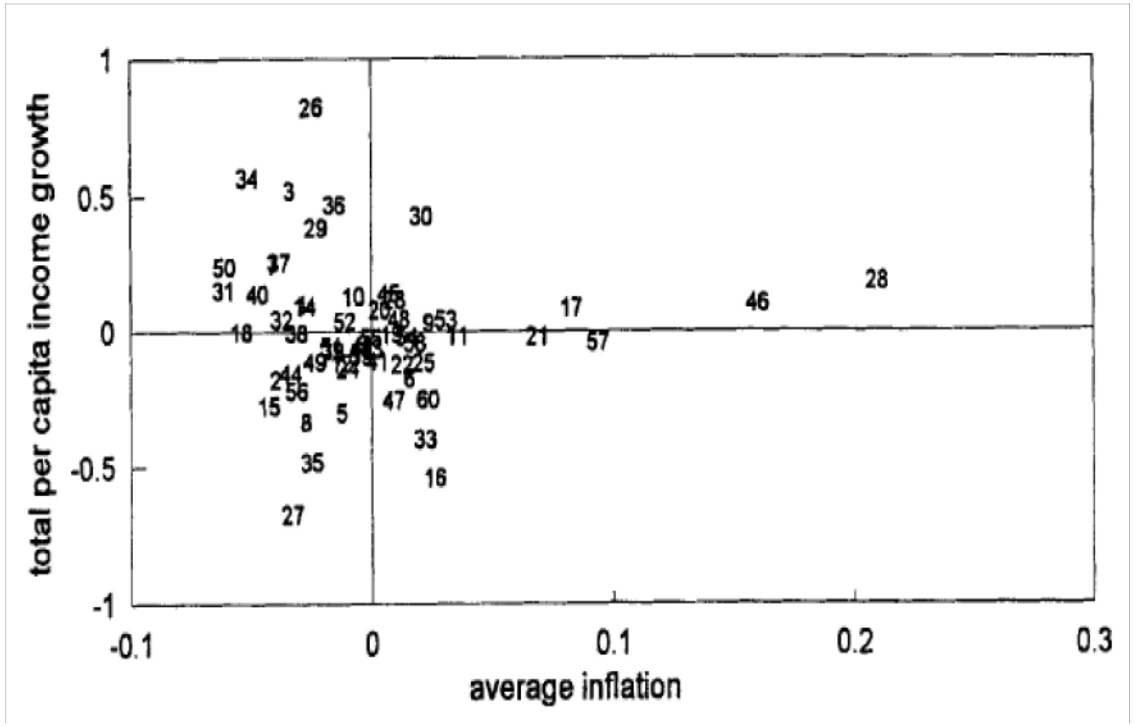
A. Introduction

I. Empirical Regularities

1. Money and growth (Friedman-Schwartz; Walsh, ch. 1):

- a. positive correlation in levels
- b. largely negative correlation in growth rates
- c. presence of a liquidity effect

Summer-Heston data (1960-85, excluding poor quality data, 60 countries):



Note: (i) **High inflation countries: 28 = Indonesia, 46 = Iceland, 57 = Turkey, 17 = Mexico, 21 = Columbia;**

(ii) **High growth countries: 26 = Hong Kong, 34 = Singapore, 3 = Morocco, 30 = South Korea, 29 = Japan.**

2. Finance and growth (Becsi-Wang 1997; Levine 1997):

- **a. positive correlation in levels (Goldsmith 1969, McKinnon 1973)**
- **b. mixed relation in growth rates:**
  - **i. zero correlation for OECD (Fernandez-Galetovic 1994)**
  - **ii. weakly negative correlation for Latin America (DeGregorio-Guidotti 1995)**
  - **iii. strongly positive correlation for Asia (King-Levine 1993)**

Financial Deepening	**Per Capita Income**		
	High	**Middle**	**Low**
High	**US, France, Italy, Switzerland**	**Chile, Venezuela**	**Kenya, Jamaica Honduras**
Middle	**Norway, Germany, Denmark**	**Malaysia Trinidad & Tobago**	**Liberia, Uganda**
Low		**Ireland, Hungary, Yugoslavia**	**Sri Lanka, Philippines Zimbabwe, Indonesia**

Notes:

(i) Per capita income is measured by 1985 real GNP per capita in US$ at 1980 constant prices where high income takes values of $7,500 or above, middle income from $3,000 to $6,000, and low income up to $2,000.

(ii) Financial deepening is measured by the financial intermediation ratio (FIR) defined as in Goldsmith (the ratio of M to GNP) where high deepening takes values of 13% or above, middle deepening from 8% to 12%, and low deepening up to 7%.

(iii) Gaps are allowed to ensure more definitive classification.

II. Key Literature

- **Use of money:**
  - **money in the utility function (direct value, wealth or transactions time reduction): Samuelson (1947), Patinkin (1965), Sidrauski (1967), Brock (1974), *Wang-Yip (1992a)*, Wang-Yip (1992b)**
  - **cash in advance: Tsiang (1966), Clower (1967), Lucas (1980), Stockman (1981), Lucas-Stokey (1987), Cooley-Hansen (1989), *Wang-Yip (1992a)*, Gomme (1993), Ireland (1994), *Jones-Manuelli (1995)*, Chang-Chang-Tsai-Wang (2017)**
  - **transactions cost: Saving (1973), Drazen (1979), Grossman-Weiss (1983), Rottemberg (1984), , *Wang-Yip (1992a)*, Jha-Wang-Yip (2002)**
  - **medium of intergenerational transactions: Samuelson (1958), Wallace (1980), McCullum (1983), Wang (1993), Van der Ploeg-Alogoskoufis (1994)**
  - **liquidity service: Feenstra (1986), Chang-Chang-Lai-Wang (2008)**

- **money and search: Wicksell (1898), Jones (1976), Wang (1987), Kiyotaki-Wright (1989, 1993), Trejos-Wrigth (1995), Lagos-Wright (2002), Laing-Li-Wang (2007, 2013)**

- **Major Roles of Financial Intermediation:**
  - **liquidity management: Diamond-Dybvig (1993), *Bencinvinga-Smith (1991)***
  - **risk pooling: Townsend (1978), Greenwood-Jovanovic (1990), Bencivenga-Smith (1993)**
  - **productive loan services: *Tssidon (1992)*, *Aghion-Bolton (1997)***
  - **effective monitoring: Williamson (1986), Greenwood-Jovanovic (1990), and the literature on:**
    - **borrowing/collateral/pledgeability constraints**
    - **venture capitalism**
    - **micro finance**
  - **funds pooling: Besley (1994), Becsi-Wang-Wynne (1999), and the literature on micro finance**

B. Money in Dynamic General Equilibrium: Wang-Yip (1992)

- **Provide a unified framework to study 3 main dynamic general equilibrium models of money**

- **Main issue: is money superneutral?**

  - **Tobin (1965): via asset substitution, higher money growth reduces real balances but encourages capital accumulation and output growth (Tobin effect)**
  - **Sidrauski (1967): even if money is valued directly, money growth has no effect on steady-state output**
  - **Stockman (1981): higher money growth reduces real balances, limits capital investment, and lowers output growth (reversed Tobin effect)**

a. Money in the Utility Function

$$\max \int_0^\infty W(c(t),\ell(t),m(t))e^{-\rho t}dt$$

$$\text{s.t. } c(t) + \dot{k}(t) + \dot{m}(t) = f(k(t), \ell(t)) - nk(t) - (\pi(t) + n)m(t) + \tau(t)$$

- **FOCs and S-S BC:**

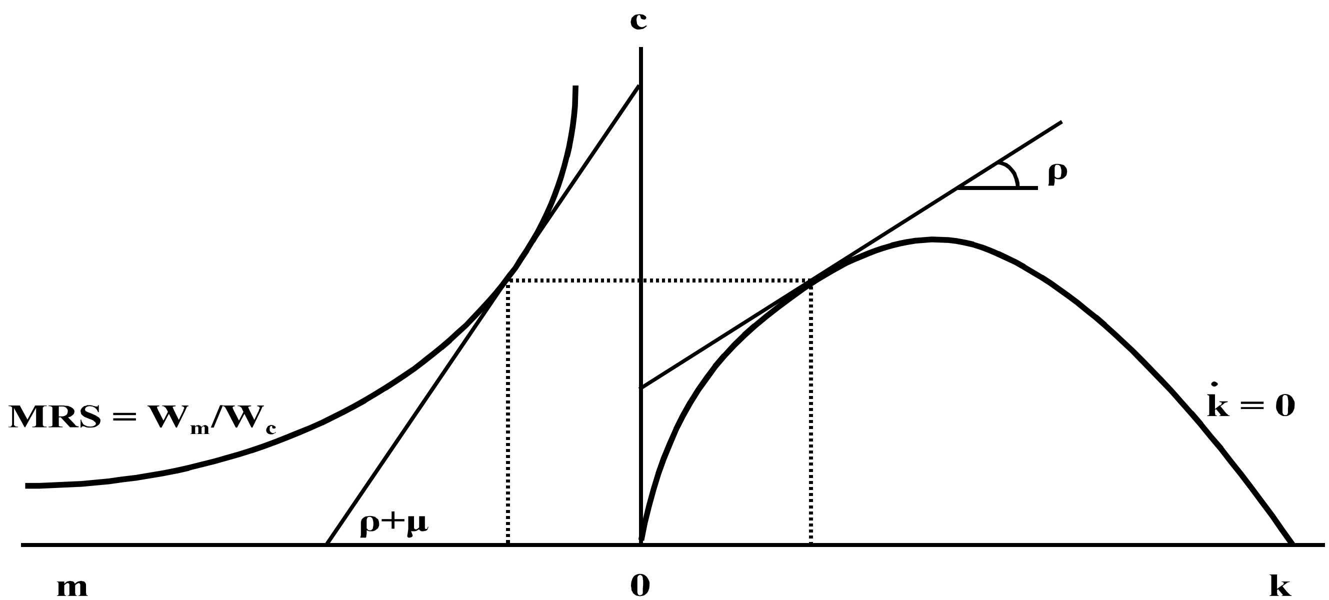
$$f_k(k, \ell) = \rho + n$$

$$W_c(c, \ell, m)f_\ell(k, \ell) = -W_\ell(c, \ell, m)$$

$$W_m(c, \ell, m) = (\rho + \mu)W_c(c, \ell, m) .$$

$$c = f(k, \ell) - nk.$$

- **Equilibrium**



- **Comparative statics**

higher μ => W_m/W_c increases
=> m lowers (if $W_{c\ell} < 0$, i.e., c and x are complem.)
=> (W1) $W_{cm} > 0$, $W_{\ell m} > 0$: c, ℓ, k all fall
(W2) $W_{cm} < 0$, $W_{\ell m} < 0$: c, ℓ, k all rise

b. Cash in Advance

$$\max \int_0^\infty U(c(t), 1 - \ell(t))e^{-\rho t}dt$$

s.t. $c(t) + \dot{k}(t) + \dot{m}(t) = f(k(t), \ell(t)) - nk(t) - (\pi(t) + n)m(t) + \tau(t)$

$$m(t) \geq c(t) + \Gamma\dot{k}(t)$$

where $\Gamma = 0$, $U_\ell = 0$ => Lucas; $\Gamma = 1$, $U_\ell = 0$ => Stockman

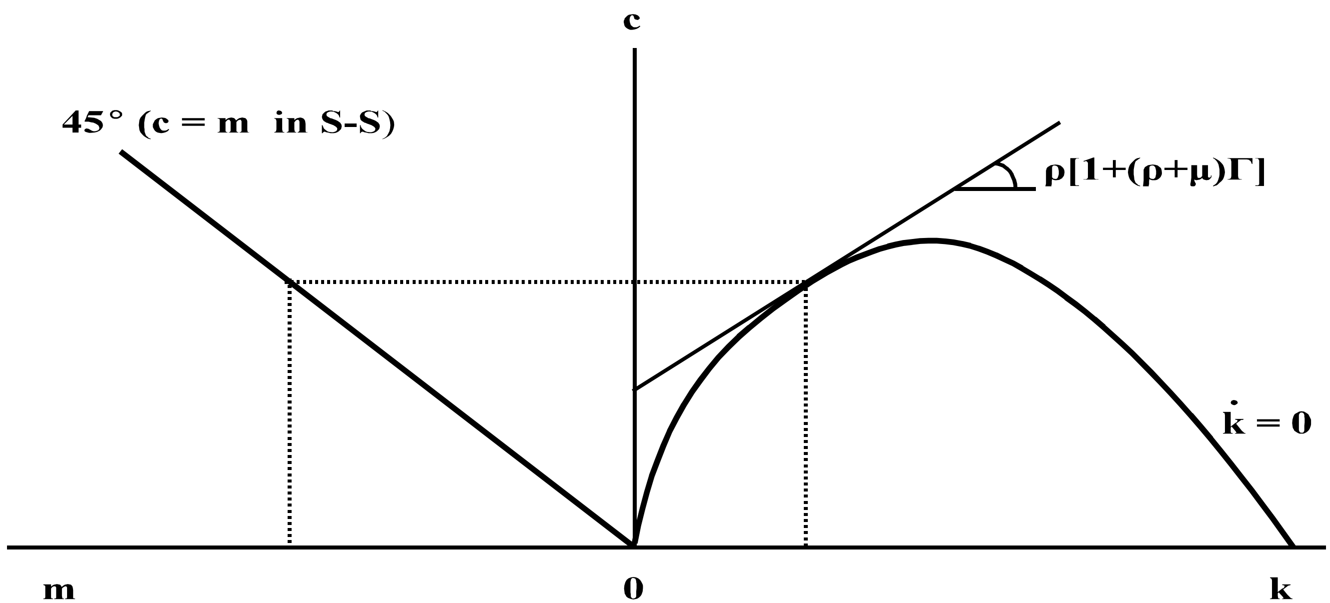
- **FOCs:**

$$f_k = \rho + n + \rho(\rho + \mu)\Gamma$$

$$f_\ell U_c = (1 + \rho + \mu)U_x \,.$$

Thus, money growth affects MPK directly

- **Equilibrium**



- **Comparative statics**

higher μ => k decreases

=> ℓ lowers ($f_{k\ell} > 0$)

=> y and c fall

=> (1-ℓ) reduces ($U_{cx} > 0$) and so ℓ rises

=> net effect on ℓ ambiguous

c. Transactions Cost Model

$$\max \int_0^{\infty} U(c(t),\, 1 - \ell(t) - T(t))e^{-\rho t}dt$$

s.t. $c(t) + \dot{k}(t) + \dot{m}(t) = f(k(t), \ell(t)) - nk(t) - (\pi(t) + n)m(t) + \tau(t)$

where $T(t) = T(c(t), m(t))$, **satisfying:**

$T_c > 0,\ T_m < 0,\ T_{cc} < 0,\ T_{mm} > 0,\ T_{mc} \leq 0$

- **FOCs:**

$$f_k = \rho + n$$

$$f_\ell U_c = (1 + T_c f_\ell)U_x$$

$$-T_m f_\ell = \rho + \mu\ ,$$

where money growth affects output via MPL

- **Comparative statics**

higher μ => k/ℓ unchanged, as does f_ℓ
=> $(-T_m)$ increases and hence m decreases
=> T increases and ℓ decreases
=> k decreases, as does c
=> but the decrease in c lowers T
=> net effect on $x = 1 - \ell - T$ ambiguous

d. Qualitative equivalence between the three models if $W_{c\ell} < 0$, $W_{cm} > 0$, $W_{\ell m} > 0$, $\Gamma = 0$, $T(c,m) = 0$ if $c \leq m$ and $= 1$ otherwise

e. Questions: As the three most used "conventional" approaches to money seem to yield qualitatively similar theoretical findings, can "newer" approaches deliver more insights along the lines of money, inflation and growth?

C. **Money and Endogenous Growth: Jones and Manuelli (1995)**

- **Modified Cooley-Hansen (1989) model to permit endogenous growth**

a. **Basic One-Sector Endogenous Growth Model of Money**

- **Consumer's optimization:**

$$\max \sum_t \beta^t u(c_{1t}, c_{2t}, 1 - n_t)$$

s.t.

$$m_t + b_{t+1} \le v_t,$$

$$p_{1t} c_{1t} \le m_t,$$

$$v_{t+1} \le (v_t - m_t - b_{t+1}) + (m_t - p_{1t} c_{1t}) - p_{2t} c_{2t} - p_{1t} x_t + p_{1t} w_t n_t + p_{1t} r_t k_t + (1 + R_{t+1}) b_{t+1} + T_t,$$

$$k_{t+1} \le (1 - \delta) k_t + x_t,$$

where **v = nominal wealth, T = money transfer (M_{t+1} - M_t),**
R = nominal interest rate, w, r = real factor prices
p_i = nominal price of i (1 & 2 are cash & credit good)
p_i = p if both goods are produced

- **Firm's optimization**

max $\pi_t = p_{1t}c_{1t} + p_{2t}c_{2t} + p_{1t}x_t - p_{2t}r_tk_t - p_{1t}w_tn_t$

s.t. $c_{1t} + c_{2t} + x_t \leq F(k_t, n_t)$

- **FOCs**

$$\frac{u_1(t)\,F_2(t)}{u_3(t)} = 1 + R_{t+1},$$

$$\frac{u_1(t)}{u_1(t+1)} = \frac{(1+R_{t+1})}{(1+R_{t+2})}\beta\,[1-\delta+F_1(t+1)]$$

$$\frac{u_1(t)}{u_2(t)} = 1 + R_{t+1},$$

$$1 + R_{t+2} = \frac{p_{t+1}}{p_t}[1-\delta+F_1(t+1)],$$

and the CIA holds for equality in equilibrium

- **Asymptotic BGP equilibrium**

(A1) $F = Ak + Dk^{\alpha}n^{1-\alpha}$, **with** $\beta(1-\delta+A) > 1$

(A2) $u = [(c_1^{-\lambda} + \eta c_2^{-\lambda})^{-1/\lambda}(1-n)^{\psi}]^{1-\sigma}/(1-\sigma)$, **with** $\sigma > 1$, $\lambda \geq -1$

$$\gamma^{\sigma} = \beta[1-\delta+A]$$

$$\pi\gamma = \mu$$

$$1 + R = \pi[1-\delta+A]$$

$$c_2/c_1 = [\eta(1+R)]^{1/(1+\lambda)}$$

where $\mu = M_{t+1}/M_t$, $\pi = p_{t+1}/p_t$

Thus, $r = A - \delta$ **and** $\gamma = [\beta(1+r)]^{1/\sigma} = \mu/\pi$ **(growth rate of m/p)**

- **Comparative statics**
  - **money is superneutral in the narrow sense (γ independent of μ)**
  - **higher μ leads to higher π, higher R, and higher c_2/c_1 (the only source of nonsuperneutrality in the broad sense)**

b. Two-Sector Endogenous Growth Model of Money

- **Consumer's optimization:**

$$\max \sum_t \beta^t u(c_{1t}, c_{2t}, 1 - n_t)$$

s.t. $m_t + b_{t+1} \leq v_t,$

$$c_{1t} p_t \leq m_t,$$

$$v_{t+1} \leq (v_t - m_t - b_{t+1}) + (m_t - p_t c_{1t}) - p_t c_{2t}$$
$$- p_t x_{kt} - p_t x_{ht} + p_t w_t n_t h_t + p_t r_t k_t$$
$$+ (1 + R_{t+1}) b_{t+1} + T_t,$$

$$k_{t+1} \leq (1 - \delta_k) k_t + x_{kt},$$

$$h_{t+1} \leq (1 - \delta_h) h_t + x_{ht},$$

where physical/human capital investments are credit goods and the final good production function is given by, $F(k, nh) = Ak^{\alpha}(nh)^{(1-\alpha)}$, implying $k/h = \alpha/(1-\alpha)$ if $\delta_k = \delta_h$

- **Firm's optimization stays the same (except modified production)**
- **Asymptotic BGP equilibrium with $\delta_k = \delta_h$**

2 x 2 system in (γ, n):

(modified GR) $\gamma^{\sigma} = \beta[1 - \delta + \alpha A n^{1-\alpha}[(1-\alpha)/\alpha]^{1-\alpha}]$

(labor tradeoff) $\gamma = 1 - \delta + B n^{1-\alpha}\left(1 - \dfrac{(1-\alpha)(1-n)}{\psi n f(\mu, \gamma)}\right)$

where $f(\mu, \gamma) = 1 + \dfrac{\mu\beta^{-1}\gamma^{(\sigma-1)} - 1}{1 + \eta^{1/(1+\lambda)}(\mu\beta^{-1}\gamma^{(\sigma-1)})^{1/(1+\lambda)}}$

- **Comparative statics**
- **money is generally nonsuperneutral even in the narrow sense**
- **for $\lambda > 0$ (c_1, c_2 complements): higher μ reduces c_1 and c_2, lowering n and γ**
- **for $\lambda < 0$ (c_1, c_2 substitutes): higher μ reduces c_1 but raises c_2, lowering n and γ if η is sufficiently small (CIA binds for almost all purchases)**

D. Finance and Growth - A First Look: Becsi and Wang (1997)

(i) Key: Add a banking sector to the AK-model of endogenous growth

(ii) A Benchmark AK-model without the Financial Sector:

a. optimization:

max $$U = \int_0^\infty \frac{c^{1-\alpha} - 1}{1-\alpha} e^{-\rho t} dt$$

$$s.t.\quad \dot{k} = Ak - \eta k - c, \quad k(0) = k_0 > 0.$$

b. Key relationships without a banking sector:

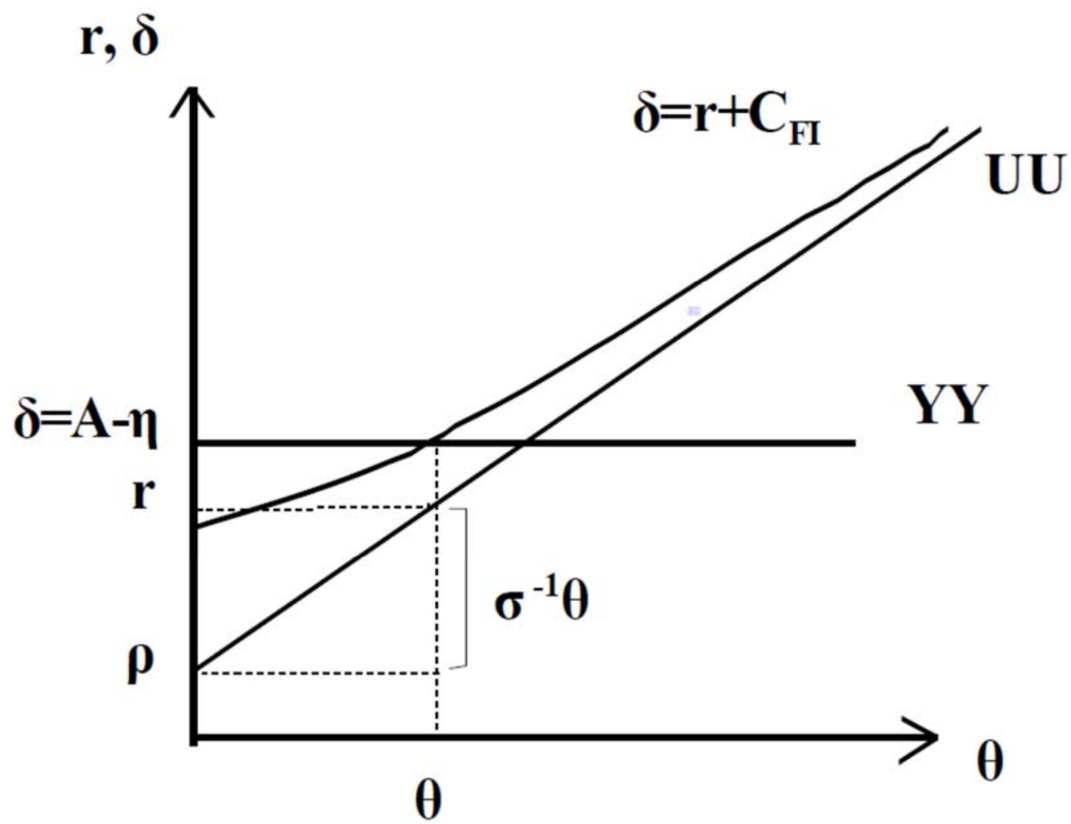
- **Keynes-Ramsey equation:** $\theta = \frac{\dot{c}}{c} = \frac{r - \rho}{\alpha} \Rightarrow r = \rho + \alpha\theta$ **(UU)**
- **Production efficiency:** $\delta = A - \eta$ **(YY)**
- **In the absence of an active banking sector:** $r = \delta$

(iii) Incorporation of the Financial Sector into the AK-Model

- **A key ingredient is to recognize the loan-deposit interest differential. With active banking, deposits are transformed into loans, but such operations are not costless.**
- **In the absence of reserve requirement, loanable funds equilibrium implies that deposits equal to loans, denoted by x (in real values)**
- **Denote the unit financial intermediation cost as C_{FI}, which is decreasing as an economy develops (i.e., $\partial C_{FI}/\partial\theta < 0$; see Lehr and Wang 1999 for empirical documentation).**
- **By competitive banking (perfectly competitive or monopolistically competitive), banks must reach zero profit: profit = $\delta x - rx - C_{FI}\, x = 0$, or, $\delta = r + C_{FI}(\theta)$.**
- **The financial markup can be derived as: $\mu = \delta - r = C_{FI}(\theta)$, which depends negatively on the stage of economic development measured by the rate of growth θ.**

(v) BGP Equilibrium

- **Along a BGP, the endogenous growth rate must be pinned down by the loan rate and the production technology, whereas the preferences determines the deposit rate.**
- **The BGP equilibrium (θ, δ) is determined when the YY locus intersects with the markup locus, which can then be used, in conjunction with UU, to pin down equilibrium r.**



- **Comparative statics:**
  - **production innovation: A ↗ => δ ↗, r ↗, μ ↘, θ ↗**
  - **banking innovation: exog. C_{FI} ↘ => δ unchanged, r ↗, μ ↘, θ ↗**
  - **annuity innovation: ρ ↘ => δ, θ ↗, r ↗, μ ↗**
  - **effective monitoring: A ↗ and C_{FI} ↘ => δ ↗, r ↗, μ ↘, θ ↗**
  - **technological and annuity innovation: A ↗ and ρ ↘ => δ ↗, r ? (↘ if direct effect dominates), μ ↗, θ ↗**
  - **limited bank entry: more local market power => μ ↗, θ ? (Smith vs. Schumpeter)**

E. Liquidity Management, Financial Intermediation and Growth: Bencivenga-Smith (1991)

- **The role of financial intermediation: liquidity management.**
- **Liquid investment is not as productive as illiquid investment. To accommodate illiquid investment and possible withdrawals, banks hold liquid reserves. However, should there be unexpected withdrawals, banks may face a illiquidity problem.**
- **The model generalizes Diamond-Dybvig (1983) by incorporating liquidity management into an endogenous growth framework.**

a. The Model

- **3-period overlapping-generations (pop = 1), supplying 1 unit of labor only when young and consuming when middle-aged and old**
- **Production:** $y = \bar{k}^{1-\theta} k^{\theta} L^{1-\theta}$ **(Romer)**
- **Labor demand per entrepreneur: MPL =** w

- **Utility:** $U = -(c_2 + \varphi c_3)^{-\gamma} / \gamma$ **,** $\gamma > -1$ **since** $\sigma = 1/(1+\gamma)$

  where $\varphi = 0$ **with probability 1-π (early withdrawers)**

  $\varphi = 1$ **with probability π (entrepreneurs)**
- **Investment returns:**
  - **liquid investment: return = n > 0 (safe return)**
  - **illiquid investment:**
    - **return after 1 period = x ε [0, n) (liquidated scrap value < n)**
    - **return after 2 period = R > n (LT investment return > n)**
- **Labor market equilibrium:** $\pi L = 1$ **(young's labor supply)**
- **Factor prices:**
  - $w = (1-\theta)k\pi^{\theta}$
  - $r_k = \theta\pi^{\theta-1}$
- **With financial intermediation, all wages are deposited in banks.**

- **Banks:**
  - **have asset management portfolio of {z, q}, choosing a fraction z in liquid investment and q in illiquid investment, where z + q = 1**
  - **have liabilities, paying**
    - **r_1 to 1-period deposits**
    - **r_2 to 2-period deposits without withdrawals (capital)**
    - **r_0 to liquidated 2-period deposits (scraped for consumption).**
- **Banks' resources constraints (payments = revenues):**
  - **1-period:** $(1-\pi)r_1 = \alpha_1 nz + \alpha_2 xq \qquad (\alpha_1 + \alpha_2 = 1)$
  - **2-period:** $\pi r_2 = (1-\alpha_2)Rq$
  - **2-period scraped:** $\pi r_0 = (1-\alpha_1)nz$
- **Gurley-Shaw's bank (in the interest of the depositors, i.e., banks as coalitions formed by the young): choose $\{q, z, \alpha_1, \alpha_2, , r_1, r_2, r_0\}$ to:**

max $$EV = (1-\pi)\left[-\frac{(r_1 w)^{-\gamma}}{\gamma}\right] + \pi\left[-\frac{\left(\theta\pi^{\theta-1} r_2 w + r_0 w\right)^{-\gamma}}{\gamma}\right]$$

s.t. $\alpha_1 + \alpha_2 = 1$, **z + q = 1 and 3 bank resources constraints**

b. Results

- **Equilibrium decisions: with $r_k R = \theta\pi^{\theta-1} R > n$,**
  - **$\alpha_1 = 1$ (1-period reserves always liquidated)**
  - **$\alpha_2 = 0$ (no pre-mature liquidation of capital)**
  - **$r_0 = 0$ (paying nothing to liquidated consumption)**
- **Financial intermediation emerges with *rate-of-return dominance***
  - **it requires large γ (or small intertemporal substitution)**
  - **intuitively, small intertemporal substitution is equivalent to more risk aversion intertemporally, thus giving a stronger role for banks to form.**
- **Key finding: the rate of growth with financial intermediation is higher than without it if x is sufficiently small.**

Remark: **Although the current model assumes forced savings (no value of period-1 consumption), main results are robust to such an extension.**

F. Finance, Human Capital Investment, and Growth: Tssidon (1992)

- Key: credit market imperfections can cause under-invest in human capital and low-growth trap

a. The Model

- Introducing Jaffee-Russell (1976)'s credit rationing model into a 3-period OLG model with human-capital based growth
- Production (time-to-educate): $Y_t = F(k_t, E_{t-1})$
- 3-period Individual Decisions:
  - period-1: borrow to finance risky investment in human capital with returns at rate R^j, with two types $j \in \{r, s\}$
  - period-2: work and save
  - period-3: consume

- **Investment in Human Capital:**
  - **investment loan =** L**, with two types** $j \in \{r,\ s\}$
  - **expected return:** $ER^j = (1 - p^j)(1 + R^j) + p^j \cdot 1$
  - **assumptions:** $ER^r > ER^s,\ p^r > p^s$ **(high risk, high returns)**
  - **individual's choice of** j **is unobservable to banks**

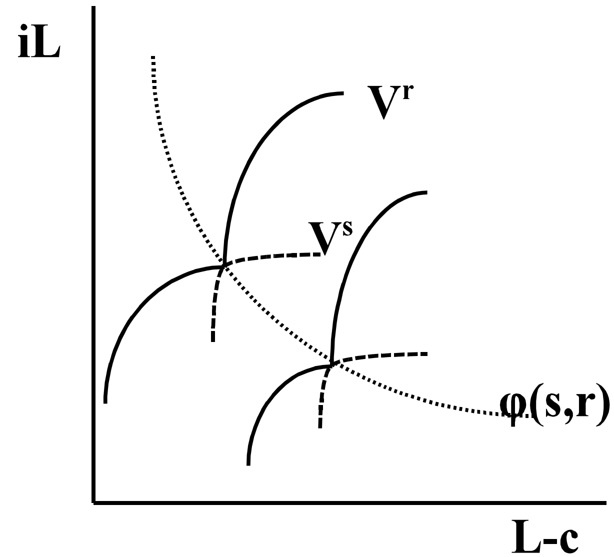
- **Expected Utility** $(j = r, s)$**:**

$$V_t^j = (1 - p^j)u((1 + \rho_{t+2})[\left(1 + R^j\right)W_{t+1} - (1 + i_{t+1})L]) + p^j u((1 + \rho_{t+2})(W_{t+1} - c_{t+1}))$$

  - ρ **= deposit rate**
  - c **= collateral**
  - **W = wage income**

- **The idea can be best illustrated by a figure in (L-c, iL) space:**
  - **iL = costs of borrowing**
  - ***L*-*c* = net benefit of borrowing (limited liability)**

(1) IC upward-sloping:
benefit$\uparrow \Rightarrow$ cost$\uparrow$ to keep V constant



(2) $V\uparrow$ towards southeast

(3) $c\downarrow$ (or L-$c\uparrow$) matters more to investment in r
$\Rightarrow$ need $iL\uparrow$ more to compensate $c\downarrow$
$\Rightarrow$ IC steeper for r

(4) $c\uparrow$ (L-$c\downarrow$) $\Rightarrow$ risk averters prefer *s*
$\Rightarrow$ upper envelope of IC^j

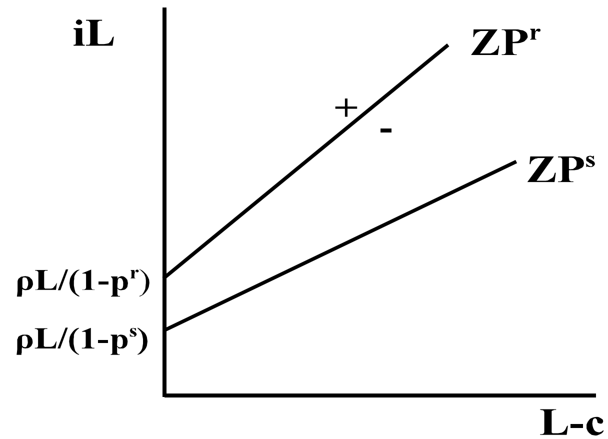
(5) $V^s = V^r \Rightarrow \phi$ (the locus is convex under CRRA utility)

(6) Key: non-convexity in IC $\Rightarrow$ multiple equilibrium

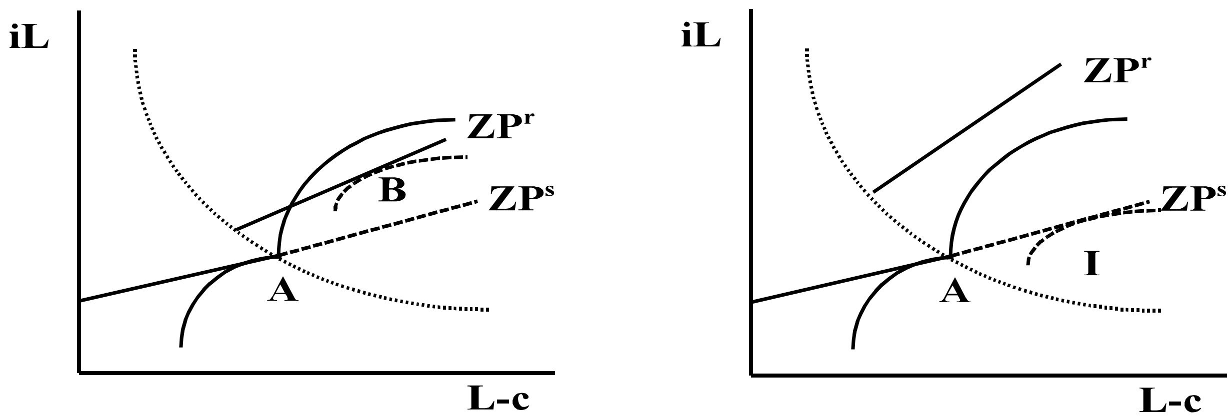
- **Banks' zero profit conditions:**

  $(1-p_j)i_t L - p^j(L-c) = \rho_t L$

  - **upward sloping lines**
  - **risky type with higher failure rate p^j => steeper ZP with higher intercept**



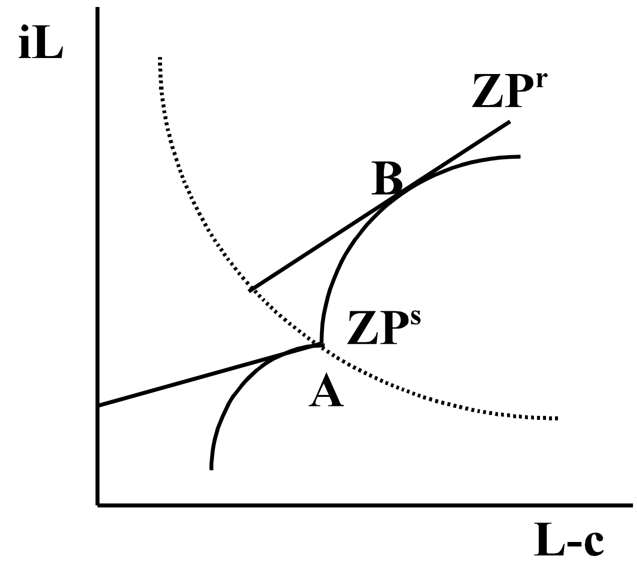
b. Temporary Equilibrium (given ρ, W)



- **A: constrained equilibrium due to moral hazard**
- **B: unconstrained; I: infeasible (negative profit)**

c. Full Dynamic General Equilibrium (DGE)

- **In DGE, ρ and W can adjust freely s.t.**
  - $WE + F_k K = F$
  - $F_k = 1 + \rho$
- **While W only affects expected utility *V*, ρ affects both *V* and ZP**
- **Adjustment: Look at the benchmark case with B more preferred to A**
  - **all choose B and risk higher**
  - **so expected profit goes down**
  - **funds must be constrained and ZP must shift up**
  - **the full dynamic equilibrium is reached with (ρ, W) adjusted accordingly (lower K, higher ρ and lower W)**



d. Main Findings

- **Moral hazard $\Rightarrow$ credit rationing $\Rightarrow$ equilibrium at A:**
  - **low human capital**
  - **low MPK (or ρ)**
  - **low-growth trap**

- **Policy Prescription:**
  - **provision of education loans with better monitoring or law enforcement**
  - **this will reduce moral hazard problems, thus promoting investment in high risk but high return higher education**
  - **as a result, it enhances economic growth, avoiding the low human capital-low growth trap**

G. Finance, Investment, and Growth: Aghion-Bolton (1997)

- **The Aghion-Bolton model can be regarded as an extension of Baneree-Newman (1993) by allowing full dynamics of wealth evolution with:**
  (i) endogenous occupational choice
  (ii) credit market imperfections
  (iii) nonstationary distribution (cf. Hopenhyne-Prescott)

a. The Model

- **1-period lived agents of with unit mass with bequest motive, one unit of time endowment, and heterogeneous initial wealth w ~ $G_t(w)$**

Occupational choice:

- ○ **Home production: return n > 0 (small)**
- ○ **Entrepreneurial activity:**

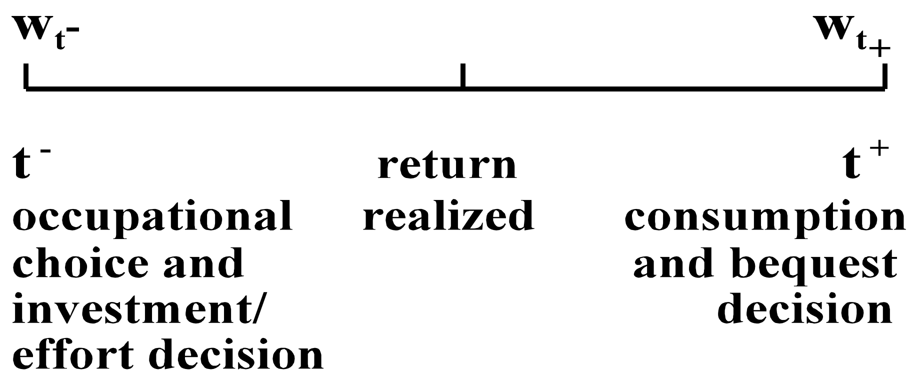
$$F(k,1) = \begin{cases} \text{r} \quad \text{for k} \geq 1 \text{(fixed cost), with prob.} = p \\ 0 \quad \text{for k} \geq 1\text{, with prob.} = 1 - p \\ 0 \quad \text{for k} < 1 \end{cases}$$

where p = effort with effort cost $C(p) = \dfrac{rp^2}{2a}, \ \ \boldsymbol{a} \in (0,\ 1]$

- ○ **Mutual fund deposit: safe return $A_t w_t$ (no labor input)**

- **Preferences: Leontief in consumption and bequest**
  $U = [\delta(1-\delta)]^{-1} \min\{(1-\delta)c, \delta b\} - C(p)$
- **Budget Constraint:** $c + b = w$

- **Timing**



b. Optimization

- **Consumption and bequest:**
  $(1-\delta)c = \delta b$ and BC $\Rightarrow b = (1-\delta)w,\ c = \delta w$

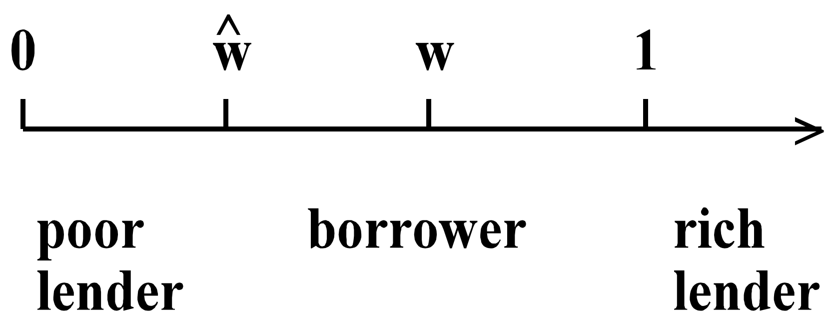
  $$\Rightarrow\ w_{t+1^-} = b_{t+1} = (1-\delta)w_{t^+}$$

- **Assumptions:**
  - **(A1) (mean wealth)** $\int w dG_0(w) < 1$
  - **(A2) (repayment)** $\forall w < 1$

  $$R(w) = \begin{cases} (1-w)\rho(w) & \text{with prob.} = p \\ 0 & \text{with prob.} = 1 - \text{p} \end{cases}$$

- **Occupational choice and investment decision:**
  - **Potential borrower (w <1): max$_p$** $\quad pr - p(1-w)\rho(w) - C(p)$,

    implying $\quad p(w) = a[1 - (\frac{1-w}{r})\rho(w)]$
  - **Rich lender** $(w \geq 1)$**: max$_p$** $\quad pr - C(p)$, $\qquad$ **implying** $\quad p(w) = a$



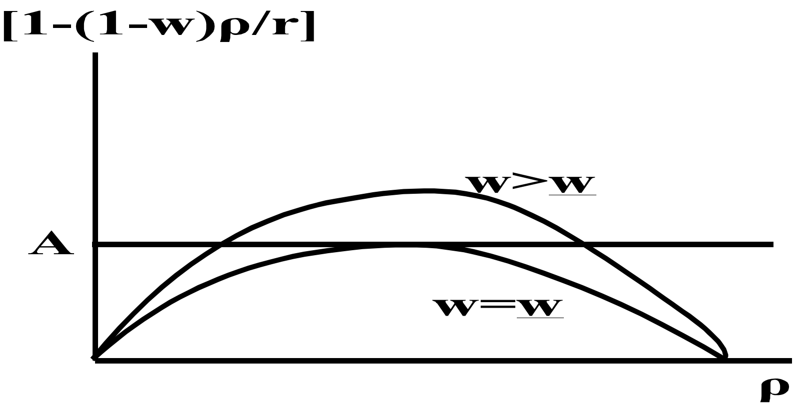
loan = 1-w

c. Atemporal Equilibrium

- **Rate of return equalization to lenders:**

$$p(w)\rho = A \quad \Rightarrow \quad \rho(w) = \frac{r}{2(1-w)}\left[1-\sqrt{1-\frac{4(1-w)A}{ar}}\right] \quad \forall\, w \in [\underline{w},\, 1]$$

where $\underline{w} = 1 - ar/(4A)$ with $\rho' > 0$ $(p' > 0)$



$w \in [0,\, \underline{w}) \Rightarrow$ no loan supply $\Rightarrow$ unable to borrow

- **Rate of return equalization to borrowers**

$$p(w_C)r - p(w_C)\rho(w_C)(1-w_C) - C(p(w_C)) - 1 = Aw_C + n$$

=> willing to borrow if $w > w_C$ or $A < (3/8)ar - n - 1$

- **Condition CR: $\exists\ \underline{w} > 0$ and $w > w_C$, or, $ar/4 < A < (3/8)ar - n - 1$**
- **Under condition CR, we have: $\forall\ w \in (w_C, \underline{w})$, they are willing to borrow but unable to obtain loan => equilibrium credit rationing**

d. Full Equilibrium

- **Wealth evolution:** $$w_{t+1} = \begin{cases} (1-\delta)(\mathrm{A}w_t + n) & \text{for } w_t \in [0, \hat{w}_t] \\ g_t(w_t, \theta_t) & \text{for } w_t \in [\hat{w}_t, \infty) \end{cases}$$

where $\hat{w}_t = \max\{\underline{w}, w_C\}$ **, θ_t = indicator function for success, and**

$$g(w,1) = \begin{cases} (1-\delta)[r - (1-w)\rho(w)] & w < 1 \\ (1-\delta)[r + (w-1)\mathrm{A}] & w \geq 1 \end{cases} \qquad g(w,0) = \begin{cases} 0 & w < 1 \\ (1-\delta)(w-1)\mathrm{A} & w \geq 1 \end{cases}$$

$$prob.(\theta = 1 | w) = h(w) = \begin{cases} p(w) & w < 1 \\ a & w \geq 1 \end{cases}$$

- **Distribution converges in weak* topology in Polish space**
- **Additional assumptions:**
  - **(A3) (incentive to lend)** $\frac{ar}{4}(1-\delta) > 1$
  - **(A4) (rapid accumulation)** $\frac{3}{8}ar(1-\delta) > 1+n$
- **Safe rate of return:**
  - $A_t \leq \frac{1}{1-\delta} \quad \forall\, t \geq T$ **(ow, unbounded wealth=>excess fund supply)**
  - **under (A3), CR does not exist for** $A_t \in [1,\ 1/(1-\delta)]$
  - **under (A3) and (A4),** $A_t \to 1$ **in finite time**
- **Trickle-down:**
  - **CR exists in early stage of development when A_t is high**
  - **As w_t ↑ over time, A_t falls in** $[1,\ 1/(1-\delta)] \to 1$ **=> no CR**
  - **Intuition: the rich trickle down increasing supply of loan and enabling the poor to borrow and invest by lowing capital cost** A_t