ELSEVIER Regional Science and Urban Economics 26 (1996) 645-669

regional SCIENCE & urban ECONOMICS

Spatial agglomeration and endogenous growth

Theodore Palivos[a,b,*], Ping Wang[c]

[a]CentER, Tilburg University, P.O. Box 90153, 5000 Le Tilburg, Netherlands
[b]Department of Economics, Louisiana State University, Baton Rouge, LA 70803, USA
[c]Department of Economics, Pennsylvania State University, University Park, PA 16802, USA

Received 15 January 1996

Abstract

This paper constructs a dynamic general equilibrium model with spatial interactions in which a human capital externality is the centripetal force towards agglomeration. The resource cost of transportation is, on the other hand, the main centrifugal force, preventing a city from growing unboundedly. A central feature of our analysis is the dynamic interaction between (perpetual) economic growth and (bounded) city growth. We examine the socially optimal and the decentralized growth rates as well as city sizes. In the decentralized environment, individuals under-invest, whereas cities are under-populated. We show how public policies may enable a decentralized city to attain the socially optimal allocation.

Keywords: Urban growth; Optimal and decentralized city sizes; Endogenous growth

JEL classification: D90; O41; R11

1. Introduction

Why is spatial agglomeration or urbanization observed in the process of economic development?[1] Why do some cities rise while others

* Corresponding author. Theodore Palivos, Department of Economics, Louisiana State University, Baton Rouge, LA 70803, USA. Tel.: +504 388-3791; fax: +504 388-3807; e-mail: eopali@unix1.sncc.lsu.edu.

[1]For instance, the number of American cities (2,500 residents or more according to the old census definition) increased from slightly over 30 in 1800 to more than 4,000 in 1950. Over the same period, urban population as a percentage of total population increased from about 6% to almost 60%. The population of the largest city (New York), in particular, increased from approximately 0.1 million to 10 million.

PII S0166-0462(96)02139-4

fall?[2] What are the determinants of the size of cities and the rates of regional income growth? Can a city attain the socially optimal allocation in a laissez-faire environment? If not, what sorts of public policies can help the local economy achieve efficient intertemporal and spatial allocations? Although these questions are important, surprisingly little has been done in addressing them within a dynamic general equilibrium framework.

Traditionally, economists have analyzed urban and regional growth using models that do not explicitly allow for a spatial dimension.[3] Beginning with the pivotal work of Fujita (1976), there has been limited but increasing attention to the study of spatial agglomeration in a dynamic environment.[4] Owing to analytical convenience, most of the studies build on the Solow growth model, ignoring the optimal rate of economic growth. Kanemoto (1980) is among the first to construct a (somewhat modified) optimal growth model and to derive the optimal steady-state size of a city by maximizing the undiscounted sum of the difference between the household's instantaneous utility and optimal steady-state level. Anas (1992) studies the birth and growth of cities in a two-city perfect-foresight economy without reproducible capital in which inter-city migration depends on the difference in the utility levels achieved in the two locations. Krugman (1992) investigates the spatial dynamics of a geographical product-differentiation model without reproducible capital in which the share of the manufacturing labor force evolves over time in response to the real wage differential between locations. The existing literature has unquestionably produced interesting dynamics and has enhanced our understanding of the city development patterns. Nevertheless, spatial agglomeration in an endogenous growth framework with perpetually accumulated reproducible capital(s) has basically been ignored. A notable exception is a recent paper (Ioannides, 1994) which studies a system of cities specializing in the production of differentiated products, with product variety being the main engine of growth.

In the present paper we follow the spirit of Romer (1986), hypothesizing uncompensated spillovers of knowledge as the main engine of sustained growth. As pointed out by Jacobs (1969) and Lucas (1988), this type of externality is essential in analyzing agglomerative behavior.[5] People in Manhattan or downtown Chicago, for example, are paying economic rent

[2]According to the Census of Population (U.S. Bureau of the Census), there have been significant changes in the ranks of cities over the past century. Rising cities include, for example, Lexington, Los Angeles, and New Orleans, whereas falling cities include Albany (periphery to New York), Pittsburgh (iron industry) and St. Louis (fur-trading post). Only New York has retained its leading rank over the entire period.

[3]See, for example, Smith (1974).

[4]See the studies of Anas (1978), Henderson and Ioannides (1981), Miyao (1981), and Fujita (1982).

[5]See also the static models of Abdel-Rahman and Fujita (1990) and Krugman (1991) which emphasize the role of urbanization externalities in the formation of a city.

for being near other people. However, owing to the presence of a non-reproducible factor (land), we postulate the size of the monocentric city and its population/employment to be asymptotically bounded from above.[6] We formalize the hypothesis that human capital externalities play a central role in explaining agglomerative behavior (Lucas, 1988) by developing a simple dynamic general equilibrium framework which integrates residential and production models. In so doing, we follow the suggestive elaboration of Jacobs (1969) who defined a city as a 'settlement that consistently generates its economic growth from its own local economy' (p. 262). We also postulate that economic growth has generated cities, but the formation of cities has also created positive externalities that feed back to foster economic growth.[7] In other words, cities provide effective organization for economic activities in which uncompensated knowledge spillovers is the main centripetal force that encourages agglomeration.[8] To ensure bounded city population, we also allow for a centrifugal force, arising from the resource cost of transportation, which prevents the city from growing unboundedly. In particular, the transportation cost is postulated to be proportional to consumption so that it grows over time despite constant population and city size.

We examine the interactions between the rate of economic growth and the city/population size in two environments: (i) a socially planned economy in which a city developer dictates the consumption, capital accumulation, and migration patterns, and (ii) a decentralized economy in which individual households/producers determine the paths of their consumption and investment and the city developer decides upon the city/population size.[9] Under

[6]This setup, in which the economy grows without bound but the steady-state size of each city is constant, is, in general, consistent with the results obtained in Anas (1978) and Kanemoto (1980), as well as the empirical evidence presented in Madden (1956). In the exogenous neoclassical model and in the absence of (exogenous) technological advances, however, the sustained growth of a nation can occur only through growth in the number of cities; see the analysis in Henderson and Ioannides (1981).

[7]Note that for analytical convenience we adopt the one-sector framework of Romer (1986). Nevertheless, we are aware of the importance of external benefits across industries (owing to linkages through labor force, communications, and service input interactions) which have been found to be significant in the empirical studies of (i) Hansen (1990), indicating that mutual interdependence between manufacturing and service industries leads to higher productivity, and (ii) Glaeser et al. (1992), suggesting that diversified cities have experienced faster economic growth than specialized ones.

[8]Other centripetal forces identified in the literature include the provision of local public goods (Arnott and Stiglitz, 1979), internal economies of scale (Dixit, 1973, and Papageorgiou and Thisse, 1985), external economies of scale (Abdel-Rahman, 1990, and Krugman, 1992), combined externalities in consumption and production (Fujita, 1988, and Abdel-Rahman, 1988), and gains from trade with heterogeneous agents (Berliant and Wang, 1993, and Wang, 1993).

[9]In recent years many towns have limited population growth by means of exclusionary zoning, restrictions on the annual number of building permits issued, or other forms of migration control.

some regularity conditions, a social optimum and a decentralized equilibrium are shown to exist. We find that the growth rate of the planned economy is, however, greater than that of the decentralized one. The reason for this is the discrepancy between the private and the social rate of return on capital. Similarly to the static model in Abdel-Rahman (1990), but in contrast to most of the urban economics literature, the decentralized city is also found to be under-populated compared with the socially optimal city size. Moreover, although the decentralized economy initially exhibits a higher consumption path, the socially planned economy overtakes within a finite period. In order for the decentralized city to attain the social optimum, proper subsidies on behalf of the city government are required. These subsidies can be either to production, or to new capital stock (e.g. an education subsidy and/or an investment tax credit), or to the entire capital stock (e.g. an interest rate subsidy).

The remainder of the paper is organized as follows. Section 2 presents the spatial structure of the city, and Section 3 the structure of preferences and technology. Sections 4 and 5 derive and characterize the socially optimal and decentralized outcomes, respectively. Finally, Section 6 compares the two solutions and provides policy implementation to remedy the inefficiency of the decentralized equilibrium, while Section 7 concludes the paper.

2. The spatial structure

Consider a circular city in which production takes place in the central business district (CBD) and is assumed not to require any space. Two factors are postulated to account for city formation: (i) a population effect, i.e. greater population size enlarges the city's stock of knowledge and hence induces more significant positive externalities, and (ii) an endogenous growth effect, i.e. the growth of income encourages endogenous accumulation of human capital which consequently generates external benefits. We assume that learning spillovers are uncompensated and can only take place within a particular location. As a consequence, all individual producers wish to take advantage of this freely available knowledge, thus justifying why all business activities are located at the CBD. Throughout the paper we also assume that the location of the CBD is a priori determined by first nature.[10] These two assumptions, on knowledge spillover effects and on the location of the CBD, provide analytical simplicity. If knowledge is allowed to be disseminated to any location at some variable cost, depending on the distance from the source of knowledge, then the endogenous determination

[10]For a general equilibrium theory on the formation of the CBD, the reader is referred to Berliant and Wang (1993).

of the location of the CBD becomes a very complicated issue and requires an approach different from that of Berliant and Wang (1993) mentioned above. In particular, in the context of a system of cities, the location of the CBD, the size of each city and the spatial structure will all depend on the cost of learning interactions. This extension, however, calls for a different paper and is beyond the scope of the present study.

Let $b(t) > 0$ denote the city size at time $t \geq 0$ and consider (for simplicity) a continuum of identical households of mass N residing in the zone that stretches from 0 to $b(t)$. As a first approximation and in order not to complicate the analysis, we follow Fujita (1976) and assume that land demand is perfectly inelastic, so that each household occupies a land lot of fixed size $m > 0$.[11] Also let $L(z)$ denote the density function of land in the residential zone, where $z \in [0,b(t)]$. The amount of land available for residing between distance z and $z + \mathrm{d}z$ from the CBD is then equal to $L(z)\mathrm{d}z = 2\pi z\mathrm{d}z$, while the household distribution, $\xi(z)$, is given as:

$$\xi(z) = \begin{cases} (2\pi z/m) & \text{for } z \leq b(t)\,, \\ 0 & \text{for } z > b(t)\,. \end{cases} \tag{1}$$

Furthermore, population identity requires

$$N(t) = \int_0^{b(t)} \xi(z)\mathrm{d}z = \int_0^{b(t)} \frac{2\pi z}{m}\mathrm{d}z = \frac{\pi[b(t)]^2}{m},$$

and thus

$$b(t) = [mN(t)]^{1/2}\pi^{-(1/2)}. \tag{2}$$

Without loss of generality, the mass of households is treated as the population. We also use the words 'households' and 'agents' interchangeably.

Let $c(t)$ be per capita consumption and δ be the transportation cost of one unit of good per unit distance. At each point in time t, a household located at z must pay a transportation cost $T(z,t) = \delta c(t)z$. Furthermore, we assume what is known in the literature as 'publicly-owned' land. That is to say, the city government rents the land for the city from rural households. Landowners are assumed not to be able to obtain any monopoly power and thus they can only charge the rural rent, $R(z,t)$ for $z \geq b(t)$, which is normalized to zero. The city government, in turn, sublets the land to city residents at the competitively determined rent, $R(z,t)$ for $z < b(t)$.

The net revenue from the land rent is used to finance the necessary

[11]For studies on the role of variable land/housing, see Fujita (1982), Braid (1988), and Wang (1993).

expenditures on city formation, e.g. landscaping, sewerage, and communication and transportation infrastructure, which is increasing in the city size and in the consumption activities. This setup, which is different from the fixed city formation cost model of Abdel-Rahman (1990), is necessary in order to be consistent with perpetually balanced growth. Furthermore, for analytical convenience, these services provided by the government are assumed not to enter the representative household's utility or production function. Relaxing this assumption, following for example Barro (1990), is straightforward but unavoidably tedious.[12]

Given identical households and a fixed lot size, locational equilibrium requires that

$$J(z,t) \equiv T(z,t) + R(z,t) = J(t), \quad \forall z \in [0,b(t)],$$

and thus

$$J(t) = T(z,t) + R(z,t) = T(b,t) = \delta c(t)b(t) = \tau c(t)[N(t)]^{1/2}, \tag{3}$$

where $\tau \equiv \delta m^{1/2}\pi^{-(1/2)}$. Therefore, rent at location z is given by

$$R(z,t) = T(b,t) - T(z,t) = \delta c(t)[b(t) - z], \quad \forall z \in [0,b(t)]. \tag{4}$$

As expected, rent decreases as we move away from the CBD. Furthermore, utilizing (2), we can compute the total transportation cost (TTC) spent by all residents,

$$\begin{aligned} TTC(t) &= \int_0^{b(t)} \xi(z)T(z,t)\,\mathrm{d}z = (2/3)\delta^3\tau^{-2}c(t)[b(t)]^3 \\ &= (2/3)\tau c(t)[N(t)]^{3/2}, \end{aligned} \tag{5}$$

as well as the total land rent (TLR) collected by the city government,

$$\begin{aligned} TLR(t) &= \int_0^{b(t)} \xi(z)R(z,t)\mathrm{d}z = (1/3)\delta^3\tau^{-2}c(t)[b(t)]^3 \\ &= (1/3)\tau c(t)[N(t)]^{3/2}. \end{aligned} \tag{6}$$

[12] We may introduce government consumption into the utility function and/or the ratio of government spending to private aggregate capital in the production function. As long as the utility and/or the production function are homothetic, we can solve for the balanced growth equilibrium path, which will have similar properties to the one in our current model. We also compare our results with those obtained under alternative dispositions of land rents below (see the Remark in Section 6).

Finally, the resource constraint for this local economy can be written as

$$Ny = Nc + N\dot{k} + (2/3)\tau c N^{3/2} + (1/3)\tau c N^{3/2}, \tag{7}$$

where y and $\dot{k} \equiv \mathrm{d}k/\mathrm{d}t$ denote per capita output and per capita net investment, respectively.[13] Note that the sum of the last two terms in Eq. (7), which combines TTC and TLR, is increasing and convex in city size and city population. However, the average cost, $J(t)$, defined as $(TTC + TLR)/N(t)$, is linear in city size, b, and concave in population size; more specifically, $J = \tau c N^{1/2} = \delta cb$.

3. Preferences and technology

Consider a dynamic, general equilibrium model in which a homogeneous good is produced and consumed by each agent (household). The instantaneous utility function is assumed to take the constant elasticity of intertemporal substitution form (CEIS) and to depend on per capita consumption, c, i.e. $u(c) = c^{1-\sigma}/(1-\sigma)$, where $\sigma > 0$.

The technology available to each agent is described by the production function $y = F(k,K)$, where k and K denote per capita and aggregate capital, respectively. Therefore, $K = \int_0^N k(i)\mathrm{d}i$ and in equilibrium, since all agents are assumed to be identical, $K = Nk$. This production function, first utilized in Romer (1986), is designed to capture spillovers of knowledge or ideas from one producer to another. Thus, k should be broadly viewed as a composite of human and non-human capital. Producers benefit (learn) from every other producer and thus the aggregate capital, a measure of total knowledge, enters each individual production function. A higher number of producing units in the economy implies a scale benefit which raises the economy's productivity.[14] Note that this provides a reason for the existence of a city. Furthermore, the existence of a balanced growth equilibrium path, i.e. perpetual growth at a constant rate, requires that the rate of return on capital(s) is constant so that the incentive to invest never diminishes. For this to be possible, in the absence of exogenous technical progress, the

[13] We adopt the conventional notation according to which primes denote derivatives of functions with only one argument and a dot over a variable denotes its time derivative. Furthermore, for the sake of simplicity, we assume that there is no depreciation and that the broadly defined capital stock is embodied. Thus, capital accumulation in (7) is written in a per capita form without including population changes. Finally, henceforth we will drop the time index, t, whenever doing so will not result in confusion.

[14] For example, Caballero and Lyons (1990) and Jaffe et al. (1993) find empirical evidence for external economies of scale and knowledge spillovers.

production technology must exhibit constant returns to scale with respect to all factors that can be accumulated (capital stocks).[15] Thus, $F(k,K)$ or, in equilibrium, $F(k,Nk)$ must be linearly homogeneous in k. It is therefore essential that we assume the production function to take the following form: $y = Ak^aK^{1-a}$, where $a \in (0,1)$ and $A > 0$ denotes a (constant) productivity parameter.

In the following sections we determine and compare the socially optimal and the decentralized equilibrium growth rates of output, θ_s and θ_d, population sizes, N_s and N_d, and, consequently, using Eq. (2), city sizes b_s and b_d.

4. The social optimum

We consider first the socially optimal solution. The city developer determines the optimal growth path by selecting either simultaneously or sequentially the consumption, capital accumulation, and population paths, $\{c(t),k(t),N(t)\}$, $\forall t$. To accommodate the comparison with the decentralized equilibrium solution, we follow the sequential approach and obtain the social optimum in two steps. Given the initial capital stock, $k(0)$, the benevolent city developer maximizes the representative household's lifetime utility first with respect to per capita consumption, c, and second with respect to the 'number' of households, N. Consider first the following problem:

$$\max_c U = \int_0^\infty \frac{c^{1-\sigma}}{1-\sigma} \exp\{-\rho t\} \mathrm{d}t, \tag{8}$$

subject to

$$\dot{k} = y - [1 + (2/3)\tau N^{1/2}]c - G/N, \tag{9}$$

$$G = (1/3)\tau c N^{3/2}, \tag{10}$$

$$k(0) = k_0 > 0, \tag{11}$$

where $y = Ak^aK^{1-a} = AkN^{1-a}$, and ρ and G denote the rate of time preference and government consumption, respectively. Eqs. (9) and (10) specify the private and government budget constraints, respectively, while

[15]The constant returns to scale technology, the constant rate of time preference, and the constant elasticity of intertemporal substitution are necessary and sufficient conditions for the exictence of a balanced growth equilibrium path (see Palivos et al., 1997).

Eq. (11) provides the initial capital stock. Next, consider the current value Hamiltonian, H_s, for this problem:

$$H_s(k,c,\lambda_s,t) = \frac{c^{1-\sigma}}{1-\sigma} + \lambda_s[AkN^{1-a} - (1+\tau N^{1/2})c],$$

where λ_s is the shadow price of capital. Applying Pontryagin's Maximum Principle we find that the optimizing program can be described by the following conditions:

$$c^{-\sigma} = \lambda_s(1+\tau N^{1/2}), \tag{12}$$

$$\dot{\lambda}_s = \rho\lambda_s - \lambda_s A N^{1-a}, \tag{13}$$

$$\lim_{t\to\infty} \lambda_s(t)k(t)\exp(-\rho t) = 0, \tag{14}$$

together with (9)–(11).[16]

Eq. (12) has the usual interpretation, namely that at the optimum the marginal benefit of an increase in consumption, the left-hand side of (12), must be equal to the marginal cost, the right-hand side (note that $\lambda_s \tau N^{1/2}$ is the average transport cost per unit of consumption). Eq. (13) governs the rate of change of the shadow price, λ_s, of the capital stock. It can be interpreted as requiring that, at the optimum, the marginal benefit of investing in capital, which consists of the marginal product of capital (AN^{1-a}) and the capital gain ($\dot{\lambda}_s/\lambda_s$, be equal to the marginal cost, which is, in turn, captured by the rate of time preference. Finally, Eq. (14), known as the transversality condition, is required for the exclusion of unbounded growth paths.[17]

Definition. A solution path $\{y(t),c(t),k(t),G(t),\lambda_s(t),N(t)\}$, $t \geq 0$, is said to be a non-degenerate *balanced growth (equilibrium) path* if the growth rates of the first five variables are constant and strictly positive, while the population growth rate is zero.

[16]The analysis could instead be conducted in terms of present values. In this case, the first-order conditions (12)–(14) can be written as $\partial H_s'/\partial c = 0$, $\dot{\lambda}_s' = -\partial H_s'/\partial k$, and $\lim_{t\to\infty} \lambda_s'(t)k(t) = 0$, where $\lambda_s' \equiv \exp(-\rho t)\lambda_s$ and $H_s' \equiv \exp(-\rho t)H_s = \exp(-\rho t)u(c) + \lambda_s'[AkN^{1-a} - (1+\tau N^{1/2}c]$ denote, respectively, the present value co-state variable and the present value Hamiltonian (for further details, see Kamien and Schwartz, 1981, pp. 151–153).

[17]The transversality condition (14) can be thought of as the limit of the terminal condition $\lambda_s(T)k(T)\exp(-\rho T) = 0$, which has the simple interpretation that at the end of the horizon (T) either the capital stock, $k(T)$, is zero or the present value of its shadow price, $\lambda_s(T)\exp(-\rho T)$, is zero. For further details, see Kamien and Schwartz (1981) and Michel (1982).

Proposition 1. If $N(t) = N$, $\forall t \geq 0$, then a balanced growth equilibrium path exists with

$$\dot{y}/y = \dot{k}/k = \dot{c}/c = \dot{G}/G = \theta_s = (AN^{1-a} - \rho)/\sigma \tag{15}$$

and

$$\dot{\lambda}_s/\lambda_s = -\sigma\theta_s. \tag{16}$$

Proof. All proofs are presented in the appendix. □

The growth rate is higher the greater the rate of return on capital, $r_s = AN^{1-a}$, and the elasticity of intertemporal substitution, $1/\sigma$, and the lower the rate of time preference, ρ. As we would expect, a larger population size leads to a higher rate of return on capital and thus a higher growth rate.

Thus far we have treated the population size as being parametrically given. Next, we determine the optimal population size in this city. Note that a larger population size, on the one hand, results in a higher level of public knowledge but, on the other hand, it increases both the transportation cost and the land rent. To obtain a convenient expression for the lifetime utility, we use the following proposition.

Proposition 2. The model exhibits no transitional dynamics and hence an optimal spatial agglomeration occurs instantaneously. Furthermore, the balanced growth equilibrium path obtained above is unique.

This result follows from the constancy of the rate of return on capital, which is, in turn, implied by the linear homogeneity of the production function, $y = AkN^{1-a}$, with respect to k, the only (reproducible) factor of production. Fig. 1 presents a phase diagram which depicts the dynamic equilibrium.

Since there are no transitional dynamics, we can focus our attention on the balanced growth path and, using (8), obtain the following expression for the lifetime utility U_s:[18]

[18]To see this notice that, since there are no transitional dynamics, $c(t) = c(0)\exp(\theta_s t)$, $\forall t \geq 0$, and hence

$$\int_0^\infty [c(t)]^{1-\sigma}(1-\sigma)^{-1}\exp(-\rho t)\,dt = [c(0)]^{1-\sigma}(1-\sigma)^{-1}\int_0^\infty \exp\{[(1-\sigma)\theta_s - \rho]t\}\,dt$$
$$= [c(0)]^{1-\sigma}(1-\sigma)^{-1}[(1-\sigma)\theta_s - \rho]^{-1}[\exp\{[(1-\sigma)\theta_s - \rho]t\}]\Big|_0^\infty$$
$$= [c(0)]^{1-\sigma}(1-\sigma)^{-1}[(1-\sigma)\theta_s - \rho]^{-1}(-1),$$

which is the one given in (17). In calculating the integral we have used the condition $\rho > (1-\sigma)\theta_s$, which is required for the lifetime utility to be bounded.

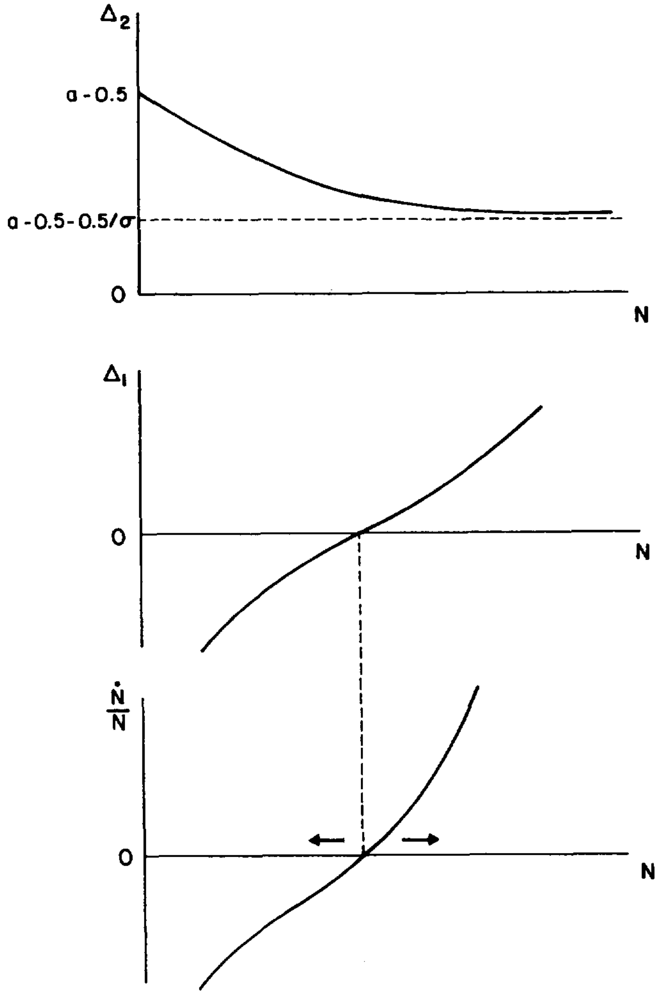

Fig. 1

$$U = (1-\sigma)^{-1}[\rho + (\sigma - 1)\theta]^{-1}[c(0)]^{1-\sigma}, \tag{17}$$

where

$$c(0) = k(0)\left[AN^{1-a} - \theta_s\right](1 + \tau N^{1/2})^{-1}, \tag{18}$$

after making use of (9). Substituting (11), (15), and (18) in (17) yields:

$$U_s(N) = \Lambda\{\rho + (\sigma - 1)AN^{1-a}\}^{-1}\left(\frac{\rho + (\sigma - 1)AN^{1-a}}{1 + \tau N^{1/2}}\right)^{1-\sigma}, \tag{19}$$

where $A \equiv \sigma^{\sigma}(1-\sigma)^{-1}(k_0)^{1-\sigma}$ is a constant.[19] Changes in population size, therefore, affect the representative agent's lifetime utility through the following two channels: the effective discount rate [or through the second term in (19)] and the initial consumption [or through the last term in (19)].[20] Next define

$$\Phi_s(N) \equiv \frac{(1-a)\sigma A N^{1-a}}{\rho + (\sigma - 1)AN^{1-a}}$$

and

$$\Omega(N) \equiv \frac{(1/2)\tau N^{1/2}}{1 + \tau N^{1/2}}.$$

Maximizing (19) with respect to N yields the following condition:

$$\Phi_s(N_s) = \Omega(N_s), \tag{20}$$

where N_s denotes the socially optimal population size. Eq. (20) requires that, at the optimum, the marginal social benefit of adding one more person to the community be equal to the marginal social cost. (The marginal social benefit (cost) of an additional resident is equal to $B(N)\Phi_s(N)$ $[B(N)\Omega(N)]$, where $B(N) \equiv \sigma^{\sigma}(k_0^{1-\sigma}[\rho + (\sigma-1)AN^{1-a}]^{-\sigma}[1+\tau N^{1/2}]^{\sigma-1} > 0.)$

Proposition 3. $\sigma > 1$ and $a \geq (\sigma+1)/2\sigma$ are necessary and sufficient conditions for the existence of an interior maximum.

In particular, if the positive externality in production, which is captured by $1-a$, is very strong, then the population goes to infinity and the lifetime utility does not reach a maximum.[21] Focusing on the interior solution, we can differentiate (20) to obtain $dN_s/d\tau < 0$, and hence $db_s/d\tau < 0$, and $d\theta_s/d\tau < 0$. Note that a decrease in τ, which will cause an increase in population and city sizes and will promote economic growth, corresponds, among other things, to an improvement in transportation technology, or a decrease in the energy (fuel) cost, or a decrease in the lot size (recall that $\tau = \delta m^{1/2}\pi^{-(1/2)}$).

[19]Looking at Eq. (19), we may be tempted to simplify it by combining the second term with the numerator of the third term. Nevertheless, the current form facilitates the interpretation as well as the comparison with the decentralized case (see Eq. (25) below).

[20]The effective discount rate is in general defined as $\rho + (\sigma - 1)\theta$, where θ denotes the growth rate of consumption.

[21]The assumption $\sigma > 1$ is supported by empirical evidence found in Hall (1988), which indicates that the intertemporal elasticity of substitution $(1/\sigma)$ is significantly less than one.

5. Decentralized equilibrium

In a decentralized economy agents determine the optimal path of their consumption and savings, taking as given the population size, which is determined by the city developer, and the aggregate capital stock, K. Each agent seeks to maximize her lifetime utility given in (8) subject to

$$\dot{k} = Ak^a K^{1-a} - (1 + \tau N^{1/2})c \tag{21}$$

and (11).

If we let λ_d denote the shadow price of capital in the decentralized economy, then the necessary conditions for this problem are (12)–(14), with λ_d replacing λ_s, (21), and

$$\dot{\lambda}_d / \lambda_d = \rho - Aak^{a-1}K^{1-a}. \tag{22}$$

To close the model we must take into account the government budget constraint given in (10) and the equilibrium condition $K = Nk$.

Proposition 4. If $N(t) = N$, $\forall t \geq 0$, then a balanced growth equilibrium path exists also in the decentralized economy, with

$$\dot{y}/y = \dot{k}/k = \dot{c}/c = \dot{G}/G = \theta_d = (AaN^{1-a} - \rho)/\sigma \tag{23}$$

and

$$\dot{\lambda}_s / \lambda_s = -\sigma\theta_d. \tag{24}$$

Similarly to the social optimum, the growth rate in the decentralized environment is given by the product of the difference between the (private) rate of return on capital and the rate of time preference, $r_d - \rho = AaN^{1-a} - \rho$, and the elasticity of intertemporal substitution, $1/\sigma$.

To determine the optimal population growth, the city developer next maximizes the lifetime utility of the representative agent, taking into account the equilibrium consumption path given by Eq. (12). That is, she maximizes (17), where in this case θ_d replaces θ_s. Substituting (18) and (23) in (17) yields:[22]

$$U_d(N) = \Lambda\{\rho + (\sigma - 1)AaN^{1-a}\}^{-1} \left(\frac{\rho + (\sigma - a)AN^{1-a}}{1 + \tau N^{1/2}} \right)^{1-\sigma}. \tag{25}$$

Similarly to (19), changes in the second term reflect changes in the effective rate of time preference, while changes in the last term reflect changes in the initial consumption level, $c(0)$.

[22] We can show that there are no transitional dynamics in the decentralized economy as well. The proof is similar to the one given in the appendix for the socially planned economy.

We define

$$\Phi_d(N) \equiv \frac{(1-a)(\sigma-a)AN^{1-a}}{\rho+(\sigma-a)AN^{1-a}} + \frac{(1-a)aAN^{1-a}}{\rho+(\sigma-1)AaN^{1-a}}.$$

The optimal population size in this decentralized economy (N_d) is then given by the following equation:

$$\Phi_d(N_d) = \Omega(N_d), \tag{26}$$

which requires that the sum of marginal private benefits of an additional resident be equal to the sum of marginal private costs. (The sum of marginal private benefits (costs) of an additional resident is equal to $\Gamma(N)\Phi(N)$ $[\Gamma(N)\Omega(N)]$, where $\Gamma(N) \equiv \sigma^{\sigma}(k_0)^{1-\sigma}[\rho+(\sigma-a)AN^{1-a}]^{1-\sigma}[1+\tau N^{1/2}]^{\sigma-1}$ $[\rho+(\sigma-1)aAN^{1-a}]^{-1} > 0$.) The limiting behavior of Φ_d is exactly the same as that of Φ_s and the signs of its first and second derivatives are the same as those of Φ_s' *and* Φ''_s (see the appendix). A sufficient condition for the existence of a maximum is $\Phi_d'(N_d) < \Omega'(N_d)$, while the upper bound of the external effect is again given by $1-a \leq (\sigma-1)/2\sigma$.

6. Comparison between social optimum and decentralized equilibrium

In this section we compare the allocations attained in each environment. Let s denote the savings rate, where $s \equiv \dot{k}/y$. The following proposition compares the population and city sizes, the growth and savings rates and the welfare levels.

Proposition 5. The socially optimal population size, city size, growth rate, savings rate and welfare level are all greater than those attained in the the decentralized environment, i.e. $N_s > N_d$, $b_s > b_d$, $\theta_s > \theta_d$, $s_s > s_d$, *and* $U_s > U_d$.

The positive production externality, stemming from knowledge spillovers, is internalized only in a socially planned environment and hence each individual 'contributes' more than in the decentralized economy; hence $\Phi_s(N)$ is always greater than $\Phi_d(N)$. This, in turn, implies that the socially optimal population and city sizes, N_s and b_s, are greater than the optimal sizes in the decentralized economy, N_d and b_d (see Fig. 2(a)). Our result is in contrast to the existing urban economics literature where almost invariably equilibrium cities are over-populated. This traditional result is due to: (a) the failure of competitive markets to internalize traffic congestion (a negative externality), or (b) the presence of economies of scale (in production) and of urban–rural migration, or (c) the instability of formation of new cities, unless they are formed at a critical size, which causes uncoordinated individuals to miss the optimal time to form a new city and to continue the expansion of an existing one. The first two cases are discussed in several standard urban economics textbooks (see, for example, Kanemoto, 1980, and Fujita, 1989), while the last is established in Anas (1992). An exception to the existing literature is Abdel-Rahman (1990), where the equilibrium city is under-populated. This occurs because individuals tend to ignore the marginal external benefit of an incremental resident, resulting from a reduction in the fixed city formation cost.

Moreover, θ_s is unambiguously higher than θ_d, in analogy to Romer (1986) and Lucas (1988). This occurs because in a private equilibrium, in determining their consumption and investment (savings) level, agents take into consideration the private but not the social benefit. In particular, they do not realize that an increase in their investment level increases the community's aggregate capital stock and thus its productivity. Put differently, the private rate of return on capital, $r_d = AaN_d^{1-a}$, is lower than the corresponding social rate, $r_s = AN_s^{1-a}$ (see Figs. 2(b) and 2(c)). This, in turn, implies that the socially optimal savings ratio is greater than the equilibrium one (see Fig. 2(d)).

It should also be noted that the output growth rate (optimal or equilibrium) can be either positive or negative. The savings ratio and hence the growth rate in each environment will be positive if the private rate of return on capital is greater than the rate of time preference. This, therefore, can provide an explanation as to why some cities fall while other rise.

Proposition 6. The decentralized economy initially exhibits a higher consumption path, but within a finite period the socially planned economy overtakes, i.e. $c(t)_d \gtrless c(t)_s$ *for* $t \lesseqgtr t'$, *where* $t' \in (0,\infty)$.

That is to say, owing to the higher savings ratio, the decentralized economy initially enjoys a higher consumption path than the socially optimum; nevertheless, since the latter grows at a higher rate, it overtakes within a finite period.

Remark. The main points of the paper do not change if the local government redistributes the land rents equally among all residents (equal internal ownership). Eqs. (9) and (21) become $\dot{k} = y - [1 + (2/3)\tau N^{1/2}]c$ and $\dot{k} = y - [1 + \tau N^{1/2}]c + T$, respectively, where T denotes the per household transfer and, in equilibrium, $T = TLR/N = (1/3)\tau N^{1/2}c$. Although this modification may alter the initial levels of consumption, it will affect neither the discrepancy between the social and the private rates of return, nor the growth rates, thus leaving our results unchanged.

In order for the decentralized economy to attain the social optimum (θ_s, N_s), the local government would have to implement one of the following three policies: (i) subsidize production; (ii) subsidize the stock of capital; or (iii) subsidize the accumulation of capital via an education subsidy and/or an investment tax credit. These subsidies can be financed by lump-sum taxation or, in the absence of labor–leisure choice, by a tax on consumption. Next we determine the optimal subsidy rates for the three alternative policies.

Consider first the case of a production subsidy. If we let μ_1 denote the subsidy rate, then the social and private rates of return on capital are AN_s^{1-a} and $(1+\mu_1)AaN_d^{1-a}$, respectively. In order to equalize these two rates of return the government must set $\mu_1^* = (1/a) - 1$. If instead the government chooses to subsidize the level of capital, then the private rate of return becomes $AaN_d^{1-a} + \mu_2$. Social optimality therefore dictates that $\mu_2^* = A(1-a)N_s^{1-a}$. Finally, in the case of an education subsidy or an investment tax credit, the private rate of return is $[1/(1-\mu_3)]AaN_d^{1-a}$ and the required subsidy rate is $\mu_3^* = 1 - a$.

7. Conclusions

We have examined population agglomeration and output growth of a monocentric city, in a centrally planned and in a decentralized environment. We have shown that the socially optimal output growth rate and city size are unambiguously greater than the decentralized one. This result occurs because in a decentralized environment agents do not take into account the positive spillover effect that an increase in their human/physical capital has on the community as a whole.

Our study is a start, and by no means an end, to a long-term research agenda. For further work along this line, it would be interesting to extend our monocentric city setup and consider a system of cities. By utilizing a two-city, differentiated-intermediate-service setup, one may study the dynamic effects of specialization versus diversification on economic growth. This can lend theoretical support to the empirical findings of Glaeser et al. (1992).

Moreover, by modifying the Fujita (1982) model of spatial dynamics, one may address the problem of durable housing in the context of a dynamic system of cities so as to investigate the effects of housing and zoning policies on city and economic growth. In particular, in a dynamic perfect foresight framework, it is optimal to mix differentiated activities in the same city if there are linkages among activities. The optimal mix in any given city will vary over time, depending on changes in the state of the spatial economy. Thus, given the durability and the costly adjustment of housing, it is necessary to reserve land to accommodate future space-requiring activities. The Fujita (1982) framework can be used to explain how land should be reserved for future uses when all agents are assumed to commute to the same (predetermined) CBD.[23] However, proper modifications are required in the context of a dynamic system of cities which allows for the endogenous formation of new cities.[24]

Acknowledgements

We are grateful to Hesham Abdel-Rahman, Alex Anas, Marcus Berliant, Eric Bond, John Boyd III, Janet Kohlhase, Geoffrey Turnbull, Henry Wan and two anonymous referees for helpful comments and suggestions. We have also greatly benefited from comments on previous drafts by seminar participants at the 1992 North American Meetings of the Regional Science Association International, the Fall 1993 Midwest Mathematical Economics Meetings, the Federal Reserve Banks of Atlanta and Dallas, and Purdue University. Needless to say, the usual disclaimer applies.

Appendix

Proof of Proposition 1. Eq. (13) implies that $\dot{\lambda}_s/\lambda_s = \rho - AN^{1-a}$. Furthermore, differentiating (12) we obtain:

$$-\frac{1}{\sigma}\frac{\dot{\lambda}_s}{\lambda_s} = \frac{\dot{c}}{c} \equiv \theta_s, \tag{A1}$$

which combined with (13) yields:

$$\theta_s = \frac{AN^{1-a} - \rho}{\sigma}, \tag{A2}$$

while the equality $\dot{c}/c = \dot{G}/G = \theta_s$ follows from differentiating (10). Next, divide both sides of (9) by $k(t)$ and utilize (10) to obtain:

$$\frac{\dot{k}}{k} = AN^{1-a} - (1 + \tau N^{1/2})\frac{c}{k}. \tag{A3}$$

Differentiating both sides of (A3) with respect to time and making use of (A2) and (A3) yields:

[23] Braid (1991) extends the Fujita (1982) framework to allow for multiple income groups and the deterioration of housing quality.

[24] We thank an anonymous referee for useful insights on this point.

$$\ddot{k} - \{\frac{AN^{1-a} - \rho}{\sigma} + AN^{1-a}\}\dot{k} + \frac{AN^{1-a} - \rho}{\sigma} AN^{1-a} k = 0, \tag{A4}$$

where $\ddot{k} \equiv d^2k/dt^2$. Eq. (A4) is a second-order linear differential equation with constant coefficients and accepts two solutions: $k_1 = k(0)\exp\{[(AN^{1-a} - \rho)/\sigma]t\}$ and $k_2 = k(0)\exp\{AN^{1-a}t\}$. Note, however, that the second solution violates the transversality condition (14) and, thus it must be rejected. Finally, $\dot{y}/y = \theta_s$ follows from differentiating the production function, $y = AkN^{1-a}$. Hence,

$$\frac{\dot{y}}{y} = \frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{\dot{G}}{G} = \theta_s = \frac{AN^{1-a} - \rho}{\sigma}$$

and

$$\frac{\dot{\lambda}_s}{\lambda_s} = -\sigma\theta_s = \rho - AN^{1-a},$$

which constitutes a balanced growth path. □

Proof of Proposition 2. The optimality conditions of the social planner's problem, specified in Section 4, taking into account the transition period, are

$$(1 - a)Ak(t)N_s(t)^{-a} = (1/2)\tau N_s(t)^{-0.5}c(t), \tag{A5}$$

which results by differentiating the current value Hamiltonian (H_s) with respect to N, together with (9)–(14).

Differentiating (12) with respect to time and using (13) yields:

$$\nu(t) = \frac{AN_s(t)^{1-a} - \rho}{\sigma} - \frac{1}{\sigma}\frac{0.5\tau N_s(t)^{-0.5}}{1 + \tau N_s(t)^{0.5}}\dot{N}, \tag{A6}$$

where ν denotes the growth rate of consumption along the transition path. The resource constraint [Eq. (9)] also implies

$$\eta(t) = AN_s(t)^{1-a} - [1 + \tau N_s(t)^{0.5}]\frac{c(t)}{k(t)},$$

or, by using (A5),

$$\eta(t) = (2a - 1)AN_s^{1-a} - [2(1 - a)/\tau]AN_s^{0.5-a}, \tag{A7}$$

where η denotes the growth rate of capital along the transition. As shown in Section 4, Proposition 1, along the balanced growth path $\nu(t) = \eta(t) = \theta$. Finally, differentiating (A5) yields

$$\eta(t) = (a - 0.5)[\dot{N}_s(t)/N_s(t)] + \nu(t). \tag{A8}$$

The differential equations (A6)–(A8) completely characterize the dynamics of the model. Combining these three equations we obtain:

$$\frac{\dot{N}_s(t)}{N_s(t)} = \Delta_1(N_s)[\Delta_2(N_s)]^{-1}, \tag{A9}$$

where

$$\Delta_1(N_s) \equiv [2a - 1 - (1/\sigma)]AN_s^{1-a} - [2(1-a)/\tau]AN_s^{0.5-a} + (\rho/\sigma)$$

and

$$\Delta_2(N_s) \equiv a - 0.5 - \frac{1}{\sigma}\frac{0.5\tau N_s(t)^{0.5}}{1 + \tau N_s(t)^{0.5}}.$$

It can be easily shown that

$$\lim_{N\to 0}\Delta_2(N_s) = a - 0.5,$$

$$\lim_{N\to 0}\Delta_2(N_s) = a - 0.5 - \frac{0.5}{\sigma} \geq 0, \quad \Delta_2' < 0, \quad \forall N_s \in (0,\infty).$$

Eq. (A9) is a first-order non-linear differential equation describing the evolution of the agglomerative dynamics of city population. As can be seen from Fig. 1, the unique critical point of this equation is the only initial value of N_s, which will put the local economy on the balanced growth path; hence, there are no transitional dynamics. □

Proof of Proposition 3.

(a) *Necessity*. It is straightforward to show that

$$\lim_{N\to 0}\Phi_s(N) = 0, \qquad \lim_{N\to\infty}\Phi_s(N) = (1-a)\sigma/(\sigma - 1),$$

$$\lim_{N\to 0}\Phi_s'(N) = \infty, \qquad \lim_{N\to\infty}\Phi_s'(N) = 0,$$

$$\Phi_s'(N) > 0, \quad \forall N \in (0,\infty) \quad \text{and} \quad \Phi''_s(N) < 0, \quad \forall N \in (0,\infty);$$

$$\lim_{N\to 0}\Omega(N) = 0, \qquad \lim_{N\to\infty}\Omega(N) = 1/2,$$

$$\lim_{N\to 0}\Omega'(N) = \infty, \qquad \lim_{n\to\infty}\Omega'(N) = 0,$$

$$\Omega'(N) > 0, \quad \forall N \in (0,\infty) \quad \text{and} \quad \Omega''(N) < \forall N \in (0,\infty).$$

A necessary condition for the existence of an interior maximum is $0 < \lim_{N\to\infty}\Phi_s(N) \leq \lim_{N\to\infty}\Omega(N)$, or equivalently,

$$\sigma > 1 \quad \text{and} \quad a \geq \frac{\sigma + 1}{2\sigma}. \tag{A10}$$

(b) *Sufficiency*. Given the properties $\Phi(\cdot)$ and $\Omega(\cdot)$, a sufficient condition for the existence of a maximum, is $\Phi_s'(N_s) < \Omega'(N_s)$.

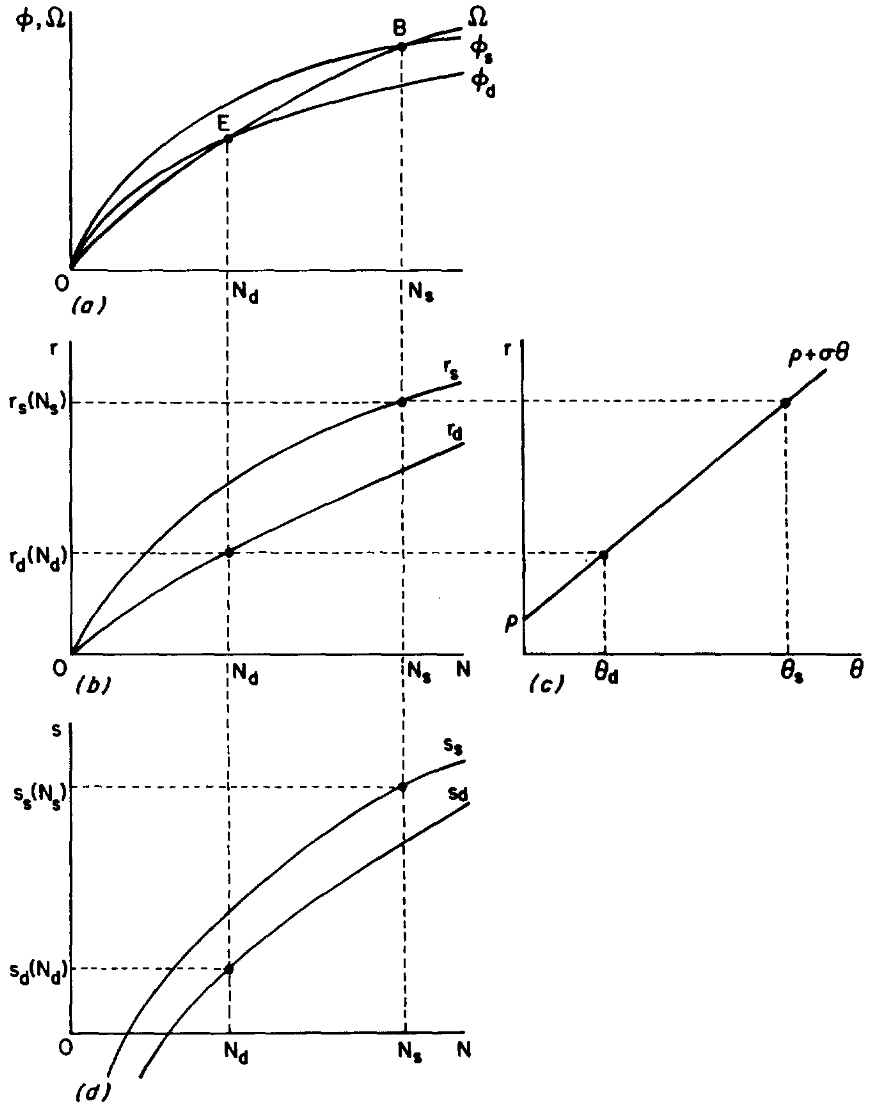

Fig. 2

Since $\Phi_s'(N) = \rho(1-a)^2\sigma A N^{-a}/[\rho + (\sigma - 1)AN^{1-a}]^2$ and $\Omega'(N) \equiv (1/4)\tau N^{-1/2}/[1 + \tau N^{1/2}]^2$, use of (20) yields the equivalent condition

$$\frac{\rho(1-a)}{\rho + (\sigma - 1)AN_s^{1-a}} < \frac{0.5}{1 + \tau N_s^{1/2}}. \tag{A11}$$

Manipulating (A11) yields $\rho(1-a)+\rho(1-a)\tau N_s^{1/2}<0.5\rho+0.5(\sigma-1)AN_s^{1-a}$. If $a\geqslant(\sigma+1)/2\sigma>0.5$, then $\rho(1-a)<0.5\rho$ and (A11) is implied by the following stronger condition:

$$N_s^{a-0.5}<N_u^{a-0.5}\equiv\frac{0.5(\sigma-1)A}{\rho(1-a)\tau}, \tag{A12}$$

which imposes an upper bound (N_u) on city population and also provides a convenient way to test whether the sufficient condition (A11) is met. To see this, define $\Delta_1(N)\equiv[2a-1-(1/\sigma)]AN^{1-a}-[2(1-a)/\tau]AN^{0.5-a}+(\rho/\sigma)$. Eq. (20) can then be rewritten as

$$N_s^{0.5}\Delta_1(N_s)=0.$$

Obviously, $N_s=0$ is a degenerate solution of (20). The other solution is the root of $\Delta_1(N)$. As shown in the lemma below, if $a\geqslant(\sigma+1)/2\sigma>0.5$ and $\sigma>0$, then $\Delta_1(N)$ has only one root. Furthermore, this root is greater than zero but less than N_u. Therefore, if $a>0.5$ and $\sigma>1$, then the sufficient condition (A12) is always satisfied for any non-degenerate solution of (20). □

Lemma. (a) If $a\geqslant(\sigma+1)/2\sigma$ and $\sigma>0$, then the function $\Delta_1(N)=[2a-1-(1/\sigma)]AN^{1-a}-[2(1-a)/\tau]AN^{0.5-a}+(\rho/\sigma)$ has one and only one root which is positive. (b) Moreover, this root is less than N_u, where

$$N_u\equiv\left(\frac{0.5(\sigma\text{-}1)A}{\rho(1-a)\tau}\right)^{1/(a-0.5)}.$$

Proof. (a) Note that $\Delta_1(N)$ is continuous and

$$\lim_{N\to0}\Delta_1(N)=-\infty,\qquad \lim_{N\to\infty}\Delta_1(N)=\infty\quad\text{and}\quad\Delta_1'>0,\quad\forall N>0.$$

By Bolzano's Intermediate Value Theorem and the fact that $\Delta_1'>0$, $\forall N>0$, it follows that there exists one and only one value of $N\in(0,\infty)$ such that $\Delta_1(N)=0$.

(b) Since $\mathrm{d}\Delta_1/\mathrm{d}N>0$, it suffices to show that $\Delta_1(N_u>0$, or

$$\rho/\sigma>[2(1-a)/\tau]AN_u^{0.5-a},$$

or, by substituting N_u,

$$\rho/\sigma>\frac{4(1-a)^2\rho}{(\sigma-1)},$$

or, since $a\geqslant(\sigma+1)/2\sigma$,

$$\frac{1}{\sigma} > \frac{\sigma - 1}{\sigma^2},$$

which follows from $\sigma > 0$. □

Proof of Proposition 4. The proof is almost identical to the proof of Proposition 1 and hence we only sketch it. Substituting $K = Nk$ in (25) yields:

$$\dot{\lambda}_d / \lambda_d = \rho - AaN^{1-a}. \tag{A13}$$

Next, combine (A13) with (12) to obtain $\theta_d = (AaN^{1-a} - \rho)/\sigma$, and use (24) to show that $\dot{k}/k = \dot{c}/c = \theta_d$. Finally, by differentiating (10) and the production function, we can show that $\dot{G}/G = \dot{y}/y = \theta_d$.

Proof of Proposition 5. Using the expressions for Φ_s and Φ_d, it is straightforward to show that $\Phi_s > \Phi_d$, $\forall N > 0$. Then given the properties of Φ_s, Φ_d, and Ω, it follows that $N_s > N_d$. Since $N_s > N_d$, $b_s > b_d$ follows from Eq. (2), while $\theta_s > \theta_d$ follows immediately by comparing (15) and (23). Consider next the savings ratio (s) in the two environments:

$$s_s = \frac{\dot{k}_s}{y_s} = \frac{\dot{k}_s}{k_s}\frac{k_s}{y_s} = \theta_s \frac{k_s}{Ak_s^a K_s^{1-a}} = \theta_s \frac{1}{AN_s^{1-a}} = \frac{1}{\sigma} - \frac{\rho}{A\sigma N_s^{1-a}}$$

and

$$s_d = \frac{\dot{k}_d}{y_d} = \frac{a}{\sigma} - \frac{\rho}{A\sigma N_d^{1-a}}.$$

Since $1 > a$ and $N_s > N_d$, it follows that $s_s > s_d$. Finally, we show that that $U_s(N_s) > U_d(N_d)$. We accomplish this in two steps. First we show that $U_s(N) > U_d(N)$, $\forall N > 0$. Manipulating the expressions for U_s and U_d (Eqs. (19) and (25)), we can show that $U_s(N) > U_d(N)$ *iff*

$$X(N) \equiv \left(\frac{\rho + (\sigma - 1)AN^{1-a}}{\rho + (\sigma - a)AN^{1-a}}\right)^{\sigma} > Y(N) \equiv \frac{\rho + (\sigma - 1)AaN^{1-a}}{\rho + (\sigma - a)AN^{1-a}}, \quad \forall N > 0,$$

which is always satisfied since $X(0) = Y(0) = 1$, $X'(N) < 0$, $Y'(N) < 0$ and $|Y'(N)| > |X'(N)|$. Second, note that

$$\frac{dU_d(N_s)}{dN} = \frac{\sigma^{\sigma}(k_0)^{1-\sigma}N_s^{-1}}{\rho + (\sigma - 1)AaN_s^{1-a}} \left(\frac{\rho + (\sigma - a)AN_s^{1-a}}{1 + \tau N_s^{1/2}}\right)^{1-\sigma} \{\Phi_d(N_s) - \Omega(N_s)\} < 0,$$

since $U_d(N)$ attains its maximum at $N = N_d$ and $N_s > N_d$. Combining these two steps, we can conclude that $U_s(n_s) > U_d(n_d)$. □

Proof of Proposition 6. Recall from Eq. (18) that

$$c(0)_i = k(0)[AN_i^{1-a} - \theta_i](1 + \tau N_i^{1/2})^{-1}, \quad i = \mathrm{s,d}.$$

Substituting (15) and (23) implies that $c(0)_d > c(0)_s$ *iff*

$$\frac{\rho + (\sigma - a)AN_d^{1-a}}{1 + \tau N_d^{1/2}} > \frac{\rho + (\sigma - 1)AN_s^{1-a}}{1 + \tau N_s^{1/2}}.$$

Since $a < 1$ and thus $(\sigma - a) > (\sigma - 1)a$, Eq. (26) implies

$$\frac{\rho + (\sigma - a)AN_d^{1-a}}{1 + \tau N_d^{1/2}} > \frac{(1 - a)\sigma AN_d^{1-a}}{(1/2)\tau N_d^{1/2}}.$$

Eq. (20), on the other hand, can be written as

$$\frac{\rho + (\sigma - 1)AN_s^{1-a}}{1 + \tau N_s^{1/2}} = \frac{(1 - a)\sigma AN_s^{1-a}}{(1/2)\tau N_s^{1/2}}.$$

Taking into account that $N_s > N_d$ and $a \geqslant (\sigma + 1)/2\sigma > 0.5$, we have

$$\frac{\rho + (\sigma - a)AN_d^{1-a}}{1 + \tau N_d^{1/2}} > \frac{(1 - a)\sigma AN_d^{1-a}}{(1/2)\tau N_d^{1/2}} > \frac{(1 - a)\sigma AN_s^{1-a}}{(1/2)\tau N_s^{1/2}}$$
$$= \frac{\rho + (\sigma - 1)AN_s^{1-a}}{1 + \tau N_s^{1/2}}.$$

Next we write $c(t)_s = c(0)_s \exp\{\theta_s t\}$ and $c(t)_d = c(0)_d \exp\{\theta_d t\}$. It follows then that $c(t)_d \gtreqless c(t)_s$ for $t \lesseqgtr t'$, where

$$\infty > t' = \frac{\ln[c(0)_d] - \ln[c(0)_s]}{\theta_s - \theta_d} > 0. \quad □$$

References

Abdel-Rahman, H.M., 1988, Product differentiation, monopolistic competition and city size, Regional Science and Urban Economics 18, 69–86.

Abdel-Rahman, H.M., 1990, Agglomeration economies, types, and sizes of cities, Journal of Urban Economics 27, 25–45.

Abdel-Rahman, H.M. and M. Fujita, 1990, Product variety, Marshallian externalities, and city sizes, Journal of Regional Science 30, 165–183.

Anas, A., 1978, Dynamics of urban residential growth, Journal of Urban Economics 5, 66–87.

Anas, A., 1992, On the birth and growth of cities: Laissez-faire and planning compared, Regional Science and Urban Economics 22, 243-258.

Arnott, R.J. and J.E. Stiglitz, 1979, Aggregate land rents, expenditure on public goods, and optimal city size, Quarterly Journal of Economics 93, 471-500.

Barro, R.J., 1990, Government spending in a simple model of endogenous growth, Journal of Political Economy 98, S103-S125.

Berliant, M. and P. Wang, 1993, Endogenous formation of a city without agglomerative externalities or market imperfections: Market places in a regional economy, Regional Science and Urban Economics 23, 121-144.

Braid, R.M., 1988, Uniform spatial growth with perfect foresight and durable housing, Journal of Urban Economics 23, 41-59.

Braid, R.M., 1991, Residential spatial growth with perfect foresight and multiple income groups, Journal of Urban Economics 30, 385-407.

Caballero, R.J. and R.J. Lyons, 1990, Internal versus external economies in European industry, European Economic Review 34, 805-830.

Dixit, A., 1973, The optimal factory town, Bell Journal of Economics and Management Science 4, 637-651.

Fujita, M., 1976, Spatial patterns of urban growth: Optimum and market, Journal of Urban Economics 3, 209-241.

Fujita, M., 1982, Spatial patterns of residential development, Journal of Urban Economics 12, 22-52.

Fujita, M., 1988, A monopolistic competition model of spatial agglomeration: Differentiated product approach, Regional Science and Urban Economics 18, 87-124.

Fujita, M., 1989, Urban economic theory (Cambridge University Press, Cambridge).

Glaeser, E.L., H.D. Kallal, J.A. Scheinkman and A. Shleifer, 1992, Growth in cities, Journal of Political Economy 100, 1126-1152.

Hall, R., 1988, Intertemporal substitution in consumption, Journal of Political Economy 96, 339-357.

Hansen, N., 1990, Do producer services induce regional economic development?, Journal of Regional Science 30, 465-476.

Henderson, J.V. and Y. Ioannides, 1981, Aspects of growth in a system of cities, Journal of Urban Economics 10, 117-139.

Ioannides, Y.M., 1994, Product differentiation and endogenous growth in a system of cities, Regional Science and Urban Economics 24, 461-484.

Jacobs, J., 1969, The economy of cities (Random House, New York, NY).

Jaffe, A.B., M. Trajtenberg and R. Henderson, 1993, Geographic localization of knowledge spillovers as evidenced by patent citations, Quarterly Journal of Economics 108, 577-598.

Kamien, M.I. and N.L. Schwartz, 1981, Dynamic optimization (North-Holland, New York, NY).

Kanemoto, Y., 1980, Theories of urban externalities (North-Holland, Amsterdam).

Krugman, P., 1991, Increasing returns and economic geography, Journal of Political Economy 99, 483-499.

Krugman, P., 1992, A dynamic spatial model, NBER Working Paper 4219.

Lucas, R.E., Jr., 1988, On the mechanics of economic development, Journal of Monetary Economics 22, 3-42.

Madden, C.H., 1956, On some indications of stability in the growth of cities in the United States, Economic Development and Cultural Change 4, 236-252.

Michel, P., 1982, On the transversality condition in infinite horizon optimal problems, Econometrica 50, 975-985.

Miyao, T., 1981, Dynamic analysis of the urban economy (Academic Press, New York, NY).

Palivos, T., P. Wang and J. Zhang, 1997, On the existence of balanced growth equilibrium, International Economic Review, in press.

Papageorgiou, Y.Y. and J.F. Thisse, 1985, Agglomeration as spatial interdependence between firms and households, Journal of Economic Theory 37, 19–31.

Romer, P., 1986, Increasing returns and long-run growth, Journal of Political Economy 94, 1002–1037.

Smith, D.M., 1974, Regional growth: Interstate and intersectoral factor reallocation, Review of Economics and Statistics 61, 353–359.

Wang, P., 1993, Agglomeration in a linear city with heterogeneous households, Regional Science and Urban Economics 23, 291–306.