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Toward a general-equilibrium theory of a core–periphery system of cities

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Abstract

This paper constructs a general-equilibrium spatial model of a core–periphery system of cities, allowing for labor heterogeneity. While unskilled workers are specialized in food production in the periphery, skilled workers with heterogeneous characteristics manufacture a high-tech commodity in the core. Unskilled wages are determined competitively, whereas skilled wages are determined via a symmetric Nash bargain between high-tech firms and skilled workers. A system of local cities forms competitively, and a single metropoly emerges due to sufficiently large positive matching externalities. We show that such an equilibrium spatial configuration exists under some regularity conditions. Moreover, the determinants of income disparity between the core and the periphery regions are examined.

Keywords: Core–periphery; Bargaining; Matching

JEL classification: R13; C78

1. Introduction

Recent urbanization trends in some advanced and industrializing countries can be characterized by two main features: first, there has been a concentration of population and economic activities in a single metropolitan

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area (i.e. the core); second, there has been a widened income disparity between the core and the periphery (i.e. the local cities surrounding the metropolis).[1] These two features have shaped an urban hierarchical structure that is centered around a single dominant metropolitan area in which most of the high-tech, research and development (R&D), and business and financial services locate, leading to higher concentration of high-skilled, high-paid labor. As a consequence, local cities are populated by less-skilled and lower-paid workers and income inequality between the core and the periphery has widened significantly.

There has been some effort in the urban economic literature toward explaining the hierarchical structure of a system of cities as a result of the variation in cities' industrial composition (see Abdel-Rahman, 1988, 1990, 1994, and Abdel-Rahman and Fujita, 1993). Also, there has been some attempt using general equilibrium frameworks to explain the formation of specialized cities (see Helsley and Strange, 1990, and Henderson and Abdel-Rahman, 1991, among others). However, these previous studies have some drawbacks: (1) the system of cities generated in these models cannot provide appropriate interpretations for the core–periphery phenomenon mentioned above; (2) most of these models cannot capture the heterogeneous urban labor market structure as observed in reality; and, (3) these models cannot explain the observed geographical income disparity and consequent unequal utility.[2] Therefore, there is a call for a paper on a system of cities that can address these issues.

This paper can be regarded as a first attempt at constructing a general-equilibrium optimizing model that can explain the recent urbanization trends considered above. It is not the purpose of the present paper to generate, endogenously, income disparity through self-selection of different workers. Rather, we explore the forces underlying this observed hierarchical structure of a system of cities within the general equilibrium framework, given heterogeneous labor skill between the core and the periphery regions.[3] In particular, we obtain a single metropolis by allowing for sufficiently large positive search-and-match externalities for skilled workers, while examining how search-and-match parameters, as well as transportation and city formation costs, may affect income disparity in the spatial economy. We

[1] For example, see Fujita and Ishii (1990) for the case of Japan, Thailand and China.

[2] Helsley and Strange (1990) presented a model of a system of identical cities with heterogeneous labor. Under the symmetric equilibrium considered in their paper, all workers achieve the same utility level. Kim (1991) applied a symmetric Nash wage bargain to determine the size of an open city. However, we want to generate unequal utility levels in equilibrium.

[3] Our paper, therefore, contrasts with Wang (1990, 1993) where economic agents are heterogeneous in preferences.

derive a Nash bargaining wage schedule to establish spatial equilibrium for this core–periphery system, and then characterize an earned income inequality measure to highlight plausible sources of the observed income disparity.

In modeling a system of cities, we develop a two-sector (food and manufacturing), general-equilibrium spatial structure in a closed economy consisting of two groups of the labor force, 'unskilled' and 'skilled'. To simplify the analysis, unskilled workers are assumed homogeneous and their wages are determined competitively, whereas skilled workers are heterogeneous in their skill characteristics and their wages are determined within a symmetric Nash wage bargaining framework. There are two goods produced in the economy: one is a high-tech manufacturing commodity produced with the use of the skill-differentiated labor, the other is a food product produced by the unskilled labor. We establish conditions under which a two-level, completely specialized urban structure is obtained in which the manufacturing good is produced only in the metropolis and food is produced only in the periphery region.[4]

In the context of spatial economics, it is important to specify the tension within each city: (1) there are agglomerative forces leading to a concentration of population and economic activities, and (2) there are dispersive forces resulting in deconcentration. When these forces are balanced at the margin, the equilibrium size of each city in the core–periphery spatial system is then determined. In the present paper the agglomerative force in the periphery is due to the decreasing average cost of the infrastructure required for the intracity transportation system (which can be regarded as increasing returns to scale in the form of setup costs). In the metropolis, however, there is an additional agglomerative force when labor migration is nonselective. As a result of an increase in the mass of the skilled workers, the productivity of each firm will increase due to a better match with specialized skilled labor. The only dispersive force considered in both core and periphery regions is the associated high commuting costs resulting from the expansion of the physical size of the city. This single metropolis emerges when the marginal benefit of a larger city size resulting from better matching and reduction in per capital city formation cost exceeds the marginal cost resulting from an increase in transportation costs.

The organization of the remainder of this paper is as follows: Section 2 constructs a general-equilibrium spatial model allowing for labor hetero-

[4] See Christaller (1993) and Losch (1940) for a description of this type of spatial distribution of cities. The issues concerning agglomeration efficiency and income inequality in this spatial system, however, still remain unexplored.

geneity; Section 3 establishes and characterizes the spatial equilibrium; Section 4 examines the determinants of an earned income inequality between the core and the periphery; and Section 5 concludes the paper.

2. The model

Consider a closed economy with a system of cities spreading over a flat featureless plane. Two final goods, x and z, are produced and traded within the spatial system at zero transportation costs. Good x represents a food product made by unskilled workers of mass N, whereas good z is a high-tech product manufactured by a variety of skilled labor. While z is assumed to be homogeneous, each firm of type S on the circle with a unit circumference in the Z-industry has a different requirement in terms of skill characteristics.[5] Skilled workers of type $T \in [0, 1]$ are uniformly distributed on the unit circumference circle with density $M(T) = M$, where M is the mass of skilled labor in the economy. Due to the skill requirement, unskilled workers cannot manufacture the high-tech commodity, though food can be produced by both unskilled and skilled workers.

To simplify the analysis, we assume that labor migration is nonselective: if any firm desires to move out of an existing high-tech city to form a new one, it can only draw a group of skilled workers consisting of uniformly all types of skill characteristics in $[0, 1]$. In particular, if there is more than one high-tech city, then all types of skilled workers need to reside in each city. In this case the skill distribution remains uniform while the density of the skill workers become lower. This assumption is consistent with commonly used anonymous Nash games in the search-and-match literature: under the static framework with a large number of economic agents, collective bargaining or coalition formation are not allowed (see, for example, Diamond, 1982a, and Mortensen, 1982). Furthermore, the nonselective migration assumption is applicable when the new city to be formed is a large metropolis and no restriction is imposed on population movements within the system, i.e. an open migration policy unrestricted by pre-migration agreement. In other words, when the total mass of the skilled labor is sufficiently large such that the individual contract with a specific firm prior to the formation of the new city is not possible, this assumption is the valid one to use. On the other hand, the selective labor migration assumption is more applicable when the new city to be formed is a small company town, or a club, in which migration is restricted by a pre-migration contract. Although the assumption of nonselective labor migration rules out the possibility that

[5] It is assumed that the distribution of firms on the skill-characteristic space is given. Thus, our framework differs from the spatial competition model, where firms choose their location.

each high-tech firm can form a specialized city with all best-matched skilled workers, it highlights the role of positive matching externalities in urban agglomeration with diversified workers. However, we will discuss briefly the case in which migration is selective while a single metropolis still emerges.

Households in our model economy are assumed to have identical preferences. Each household/worker is endowed with one unit of labor and is perfectly mobile and free to reside in the city in which the highest level of satisfaction can be achieved. The formation of local cities and the metropolis require a public infrastructure (such as an intracity transportation system) which is produced using good z. The total costs for this public infrastructure in each city are assumed to be shared equally by its residents; the cost share (per capita) is decreasing in the population of each city, thus providing an incentive for the agglomerative behavior. Futhermore, it is assumed that each household owns an equal share of the land rent generated in the core or the periphery in which it resides.[6]

2.1. Households' behavior and city formation

Each household in a given local city or in the metropolis can only reside at one location and can have only a single job that requires commuting to the central business district (CBD) in which all firms locate. As in any monocentric city model, each location within a city can be characterized by its distance from the CBD. For simplicity, we postulate that each household consumes one unit of land, i.e. demand for land is perfectly inelastic.[7] In addition, all households in the economy are assumed to have an identical utility function of the following Cobb–Douglas form:

$$u = Ax^{\alpha}z^{(1-\alpha)}\,, \quad \alpha \in (0, 1)\,, \tag{2.1}$$

where A is a positive scaling constant.

We next specify the budget constraint facing a household residing at distance r from the CBD in a given local city (with subscript 'L') or in the metropolis (with subscript 'M'):

$$z_i(r) + Px_i(r) + R_i(r) = Y_i - tr\,, \quad i = \mathrm{L}, \mathrm{M}\,, \tag{2.2}$$

where P measures the (relative) price of good x (in units of good z); $R_i(r)$

[6] This assumption is common in general equilibrium models of a system of cities, since it avoids having different groups of people in the model, namely land owners, and at the same time considers the impact of the land rent generated in the city on household utility.

[7] This simplifying assumption is common in the literature on systems of cities. It circumvents the difficulty associated with general equilibrium models with continuous location and variable lot size as pointed out by Berliant et al. (1990) and Berliant and ten Raa (1991).

represents the unit land rent at distance r in city i; t is the amount of good z required to be spent for commuting a unit distance; and Y_i denotes the household income (in units of z).

From the first-order conditions of the maximization of (2.1) subject to (2.2), one can obtain the uncompensated demand for x and z in a city $i = \mathrm{L}, \mathrm{M}$:

$$x_i(r) = \alpha P^{-1}[Y_i - tr - R_i(r)] , \tag{2.3}$$

$$z_i(r) = (1 - \alpha)[Y_i - tr - R_i(r)] . \tag{2.4}$$

For simplicity we do not consider a separate landlord but, instead, assume public ownership of land, under which the household income can be specified as

$$Y_i = \begin{cases} W_{\mathrm{L}} + (ALR_{\mathrm{L}} - FL^{\epsilon})L^{-1} , & \text{if } i = \mathrm{L} , \\ W_{\mathrm{M}} + (ALR_{\mathrm{M}} - FM^{\epsilon})M^{-1} , & \text{if } i = \mathrm{M} , \end{cases} \tag{2.5}$$

where ALR_{L} and ALR_{M} are, respectively, the aggregate land rent in a local city and in the metropolis; FL^{ϵ} and FM^{ϵ}, with $\epsilon \in (0, 1)$ and $F > 0$, are, respectively, the infrastructure costs required for the formation of a local city and the metropolis; and, L is the size of a representative local city. Notably, F can be regarded as the shift parameter of the costs of the public infrastructure (such as an intracity transportation system) required for forming a city. Since $\epsilon < 1$, the (per capita) share of the infrastructure costs is decreasing in the city size.[8]

Recall that each household consumes exactly one unit of land. The total population of a representative local city can thus be derived as

$$L = \int_0^b 2\pi r \, \mathrm{d}r = \pi b^2 , \tag{2.6}$$

where b represents the urban fringe distance for each (identical) local city; the number of local cities in the spatial system is then $n = N/L$. Similarly, the population of the metropolis is

$$M = \int_0^1 M(T) \, \mathrm{d}T = \int_0^f 2\pi r \, \mathrm{d}r = \pi f^2 , \tag{2.7}$$

where f is the urban fringe distance for the metropolis. The first equality in

[8] With the infrastructure cost function specified above, our model economy, in effect, exhibits increasing returns in the context of city expansion. Nevertheless, the presence of transportation costs will balance out this cost-reducing benefit on the margin.

(2.7) is a direct consequence of the assumptions of the uniform density of skilled labor, as well as nonselective migration, while the second equality is a result of the case that the economy has only one metropolis in which all skilled workers reside. To guarantee the emergence of a single metropolis it is required to have sufficiently large positive matching externalities in the manufacturing industry; this condition will be discussed in Subsection 3.3 below (Assumption 4).

Locational equilibrium requires that all unskilled workers in a given local city achieve the same utility level. Hence, from (2.3) and (2.4) it must hold at each $r \leq b$ that $Y_L - tr - R_L(r) = Y_L - tb - R_L(b)$. For convenience, we normalize the opportunity cost of land to be zero so that $R_L(b) = 0$. Thus, $R_L(r) = t(b - r)$, which enables us to obtain the aggregate land rent in each local city:

$$ALR_L = \int_0^b R_L(r) 2\pi r \, dr = \lambda L^{3/2} , \tag{2.8}$$

where $\lambda \equiv t/(3\pi^{1/2})$. Similarly, under locational equilibrium, all skilled workers of the same type must achieve the same utility, implying that the land rent function is $R_M(r) = t(f - r)$. Thus, the aggregate land rent for the metropolis is

$$ALR_M = \int_0^f R_M(r) 2\pi r \, dr = \lambda M^{3/2} . \tag{2.9}$$

We next turn to examining the formation of the core–periphery system of cities. Notice that the unskilled labor will form more local cities until the nonwage income is driven to zero, i.e. $ALR_L - FL^{\epsilon} = 0$.[9] Using (2.8), we can then derive the equilibrium size of each local city as

$$L^* = [(3\pi^{1/2} F)/t]^{2/(3-2\epsilon)} . \tag{2.10}$$

Thus, the equilibrium size of a local city is decreasing in the unit commuting cost, t, and increasing in the shift parameter of the infrastructure costs required for forming a city, F. Intuitively the infrastructure costs are financed by the revenue from the aggregate land rent (ALR_L). As F increases, the total costs of city infrastructure required will increase and ALR_L must also increase, thus resulting in a larger city size. Conversely, from (2.8), an increase in t must be accompanied by a higher ALR_L, and,

[9] One can also think of the process of city formation as a perfectly competitive developer setting up cities. Notably, it is assumed that the population size of a city is relatively small compared with the total population of the economy.

hence, given that $\epsilon < 1$, the city size must decrease in order to equate ALR_L with the total infrastructure costs.

We are now prepared to derive the uncompensated demand for x and z by a representative household in a given local city:

$$x_L(r) = \alpha P^{-1}[W_L - R_L(r) - tr] , \tag{2.11}$$

$$z_L(r) = (1 - \alpha)[W_L - R_L(r) - tr] . \tag{2.12}$$

Similarly, the uncompensated demand for x and z by a representative household of type T in the metropolis can be written as

$$x_M(T, r) = \alpha P^{-1}[W(T) - R_M(r) - tr + \tau] , \tag{2.13}$$

$$z_M(T, r) = (1 - \alpha)[W(T) - R_M(r) - tr + \tau] , \tag{2.14}$$

where $\tau \equiv \lambda M^{1/2} - FM^{\epsilon - 1}$ denotes a lump-sum redistribution for any household residing in the metropolis.

2.2. Production and wage schedules

2.2.1. The food industry

The perfectly competitive food product, x, is made by unskilled workers via a simple constant-returns-to-scale technology:

$$X = \gamma L , \quad \gamma > 0 . \tag{2.15}$$

Thus, by production efficiency, a representative firm in the food industry will employ unskilled labor in the local cities at the wage rate of

$$W_L = \gamma P . \tag{2.16}$$

2.2.2. The manufacturing industry

The single manufacturing good, z, is produced with the use of skilled workers who are uniformly distributed on a circle with a unit circumference in which each point on the circumference represents a specific skill characteristic. Each firm in the Z-industry has a job requirement corresponding to a point on the skill-characteristics space. Recall the assumption that labor migration is nonselective. That is, if any firm desires to form a new high-tech city, it can only do so by drawing a group of skilled labor who uniformly represent the characteristic space. When a firm hires workers who possess a particular skill characteristic perfectly matching the job requirement, the productivity of each of such workers is $a > 0$. Later, a will be referred to as the maximal productivity. Perfect matching is, however, not always attained. In general, a manufacturing firm of type S may employ a skilled worker of type T with distance $\delta \equiv T - S$ away from the ideal skill requirement. The corresponding productivity of this employee will then be

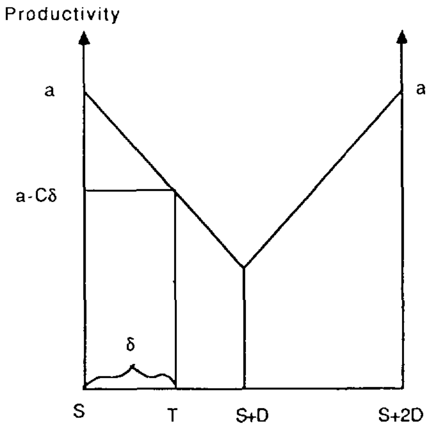

Fig. 1. Skill-characteristics space.

$a - C\delta$, where $C\delta$ measures the loss of productivity due to imperfect firm–worker match. Later, we shall use δ to indicate the particular type of skilled worker. Fig. 1 provides a graphical display of this productivity profile. Notice that there are positive matching externalities in this framework, in which an increase in the mass of skilled workers enables better matching. When such positive matching externalities are sufficiently large (see Assumption 4 below), a single metropolis where high-tech manufacturing firms locate will emerge.

Let D denote the maximal skill-characteristics distance acceptable to a representative firm. This distance will be determined endogenously using the free-entry condition. Thus, the output of a firm of type S is given by

$$Z(S) = 2 \int_0^D [(a - C\delta)M - K] \, \mathrm{d}\delta , \tag{2.17}$$

where K is the fixed factor required for each type of skilled worker, such as a personal computer or the office space necessary for skilled labor to engage in production. For simplicity, this fixed cost is treated as an entry fee that does not affect the bargaining process.

Let $W(\delta)$ be the wage rate for a skilled worker with a skill-characteristics match measured by distance δ upon employment. The profit of a type-S firm, $J(S)$, can then be expressed as

$$J(S) = 2 \int_0^D \{[(a - C\delta) - W(\delta)]M - K\} \, \mathrm{d}\delta , \tag{2.18}$$

where $W(\delta)$ will be determined through a Nash bargain pairwise between a firm and worker.

2.2.3. The Nash wage bargain

In principle, there could be a continuum of wage rules consistent with the Nash bargaining framework, depending on the relative bargaining power between the manufacturing firms and the skilled workers. To simplify the analysis we follow in the spirit of Diamond (1982a) to use his Raiffa bargaining solution. This solution specifies a symmetric Nash bargain, imposing an equal division of the matching surplus between the two parties involved in the bargaining process. A skilled worker indexed by a characteristics distance δ from the nearest adjacent firm has the best alternative (i.e. the threat point) as potential employment by the second most adjacent firm. The surplus of matching between this particular worker and the most adjacent firm is, therefore, the productivity differential between the ideal match and the best alternative. To ensure that working in the food industry is not the best alternative, we assume[10]

$$a - 2CD > \gamma P . \tag{2.19}$$

This requires a restriction on the preference and production parameters (see Assumptions 2 and 3 below). From Fig. 2, it can easily be shown that an equal division of the matching surplus yields a wage schedule independent of skill characteristics:

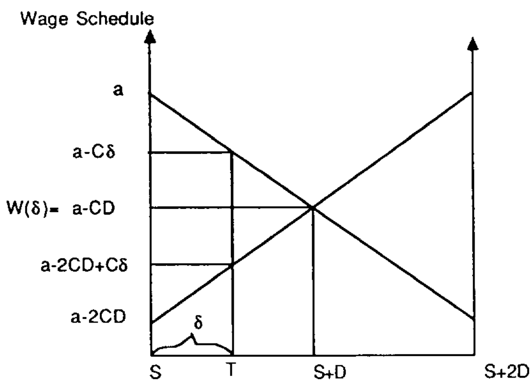

Fig. 2. Wage bargain.

[10] This type of assumption is standard in the search literature. By assuming (2.19), it directly implies the presence of an income disparity between the core and the periphery regions. However, it would still be interesting to examine the determinants of the 'magnitude' of such an income disparity. This will be discussed in Section 4 below.

$$W(\delta) = a - CD\,, \quad \text{for all } \delta \in [0, D]\,. \tag{2.20}$$

In the next section we establish and characterize the equilibrium under this simple wage-bargaining rule.

In a related paper, Kim (1991) applied a symmetric Nash wage bargain to determine the size of an open city. While the labor market in the core in our paper is parallel to that in Kim (1991), there are several important differences between the two models. First, Kim focused on an open city in which the utility level is pinned down exogenously, whereas our paper allows for the endogenous determination of the utility level attained in the core and periphery. Second, Kim's model is not a complete general equilibrium model as rents exit the system, contrasting with our general equilibrium setup. Third, unlike Kim's one-good economy, we consider a two-sector model so as to examine an important substitution effect through the relative price (between the high-tech and the food products). Finally, in Kim's work, scale economies are captured by a threshold measure (namely, a minimal efficiency scale parameter). In contrast, our model is built on the aggregate matching externalities via the mass of skilled workers, following the spirit of the search framework developed by Diamond (1982b).

3. Equilibrium

3.1. Equilibrium entry of high-tech firms

Free entry implies that each firm in the Z-industry achieves zero profit in equilibrium, i.e. $J(S) = 0$ for all S. Thus, from (2.18), the equilibrium area of skilled labor employment for a representative firm is $D^* = 2K/(CM)$ and the equilibrium number of firms in the metropolis can be derived as

$$m^* = CM(4K)^{-1}\,. \tag{3.1}$$

Straightforward differentiation shows that the equilibrium number of firms in the Z-industry is increasing in the mass of skilled labor, M, and in the marginal productivity loss due to mismatch, C, but decreasing in the fixed factor cost of production, K. Intuitively, an increase in M results in a more concentrated employment area with higher average productivity, leading to higher profit and entry. An increase in C penalizes a mismatch and, hence, raises average productivity over the employment area, thereby encouraging the entry of new firms. Finally, a reduction in K leads to higher profit and thus causes entry into the Z-industry.

Substituting (3.1) into (2.20) we can derive the equilibrium wage rate for skilled labor as

$$W_M^* \equiv W(\delta) = a - 2KM^{-1} . \tag{3.2}$$

To explain the observed urbanization trends, we shall find a sufficient condition such that the following inequality holds in equilibrium:

$$Y_M > Y_L = W_L . \tag{3.3}$$

This ensures that skilled workers are not willing to reside in local cities (see Assumption 2 in Subsection 3.3 below).

3.2. Determination of equilibrium price

In any local city within the spatial system, the locational equilibrium condition, together with (2.5), (2.6), (2.10), (2.11), (2.12), and (2.16), enables us to derive the demand for goods x and z by a representative unskilled worker as

$$x_L = x_L(r) = \alpha P^{-1}(\gamma P - \beta) , \tag{3.4}$$

$$z_L = z_L(r) = (1 - \alpha)(\gamma P - \beta) , \tag{3.5}$$

where $\beta \equiv [3Ft^{2(1-\epsilon)}\pi^{-(1-\epsilon)}]^{1/(3-2\epsilon)}$ represents the payment by each household for land rent and commuting costs. Analogously, from (2.5), (2.7), (2.13), (2.14), and (3.2) we obtain the demand for goods x and z by a skilled worker with skill index δ:

$$x_M = x_M(\delta, r) = \alpha P^{-1}\left[a - \frac{2K}{M} - 2\lambda M^{1/2} - FM^{\epsilon-1}\right] , \tag{3.6}$$

$$z_M = z_M(\delta, r) = (1 - \alpha)\left[a - \frac{2K}{M} - 2\lambda M^{1/2} - FM^{\epsilon-1}\right] . \tag{3.7}$$

The material balance condition for good x requires that total demand, $Nx_L + Mx_M$, be equal to total supply, γN. This yields the equilibrium price of good x:

$$P^* = \alpha[(1 - \alpha)\gamma N]^{-1}[aM - 2K - FM^{\epsilon} - 2\lambda M^{3/2} - \beta N] . \tag{3.8}$$

Observe that in order for $P^* > 0$, we need to impose an inequality restriction:

$$aM - 2K > FM^{\epsilon} + 2\lambda M^{3/2} + \beta N . \tag{3.9}$$

In effect, this implies that the aggregate output of the Z-industry is greater than the total expenditure on infrastructure and commuting costs in both the metropolis and the local cities (since both of these costs are assumed to be paid in good z). Of course, by Walras's law, the material balance condition for good x, together with the individuals' budget constraints, would induce the material balance condition for good z.

By differentiating (3.8), one can show that[11]

$$\frac{dP^*}{dN} < 0 \quad \text{and} \quad \frac{dP}{dM} \lesseqgtr 0 . \tag{3.10}$$

An increase in N enlarges the supply of good x, thus lowering the equilibrium price of x. The effect of the mass of skilled workers, M, on P^* is, however, ambiguous in sign. An increase in M, on the one hand, will result in higher aggregate productivity of the skilled labor; on the other hand, it will also lead to higher commuting and infrastructure costs in the metropolis. The former effect tends to increase the relative price of x by raising the supply of good z, whereas the latter suppresses P^* via an enlarged demand for z, which raises the relative price of z $(1/P^*)$. Therefore, the net effect will depend on the sign of $[a - (\epsilon F M^{\epsilon - 1} + 3\lambda M^{1/2})]$.

It is also straightforward, by totally differentiating (3.8), to characterize the equilibrium (relative) price schedule. The equilibrium price of good x is increasing in the maximal productivity of the skilled labor, a, and decreasing in the marginal commuting costs, t, in the shift parameter of the infrastructure costs required for forming a city, F, in the fixed factor used by the Z-producer, K, and in the productivity of the unskilled labor, γ.

3.3. *The existence of spatial equilibrium*

Now, we substitute (3.4) and (3.5) into (2.1) to obtain the equilibrium utility level for a household residing in a local city:

$$U_{\mathrm{L}}^* = A_0 (P^*)^{-\alpha} (\gamma P^* - \beta) . \tag{3.11}$$

where $A_0 \equiv A\alpha^{\alpha}(1-\alpha)^{1-\alpha}$ is a constant scaling factor. In order for this utility to exceed that with passive behavior (which is normalized to be zero), we impose $\gamma P^* > \beta$, restricting the equilibrium unskilled wage rate in the local cities to be large enough to cover the land rent and commuting expenses. Under this inequality, each consumer can afford to have a positive consumption of x and z in equilibrium. Utilizing (3.8), this inequality is met if and only if

Assumption 1.

$$\beta N < \alpha(aM - 2K - FM^{\epsilon} - 2\lambda M^{3/2}) .$$

[11] From (3.8) we can derive

$$dP^*/dN = -\alpha[(1-\alpha)\gamma N]^{-1}\beta - P^* N^{-1} < 0 .$$
$$dP^*/dM = \alpha[(1-\alpha)\gamma]^{-1} N^{-1}(a - 3\lambda M^{1/2} - \epsilon F M^{\epsilon - 1}) \gtreqless 0 .$$

Of course, under Assumption 1 the inequality of (3.9), guaranteeing a positive equilibrium relative price, will be automatically satisfied.

Similarly, substitution of (3.6) and (3.7) into (2.1) yields the equilibrium utility level for a skilled worker residing in the metropolis:

$$U_{\mathrm{M}}^{*} = A_0(P^*)^{-\alpha}M^{-1}[aM - 2K - FM^{\epsilon} - 2\lambda M^{3/2}], \tag{3.12}$$

which is unambiguously positive under Assumption 1. From (3.2) and (3.8), condition (3.3), ensuring that a skilled worker receives a higher income than an unskilled worker, can be rewritten as

$$(1-\alpha)N(a - 2K/M - FM^{\epsilon-1} - 2\lambda M^{1/2})$$
$$> \alpha M(a - 2K/M - FM^{\epsilon-1} - 2\lambda M^{1/2} - \beta N/M).$$

This condition will be met if the mass of unskilled workers is sufficiently large compared with the mass of skilled workers:

Assumption 2.

$$\alpha M < (1-\alpha)N.$$

Moreover, substitution of D^* and P^* into (2.19) yields the following inequality:

$$(1-\alpha)N(a - 4K/M) > \alpha M(a - 2K/M - FM^{\epsilon-1} - 2\lambda M^{1/2} - \beta N/M),$$

which, under Assumption 1, is guaranteed if the fixed cost of each high-tech firm is small:

Assumption 3.

$$2K < FM^{\epsilon} + 2\lambda M^{3/2} + \beta N.$$

In summary, while Assumption 2 is sufficient to ensure that skilled labor receives a higher income in equilibrium, Assumptions 2 and 3 together guarantee that food industry employment cannot be the best alternative for skilled workers.

Under the assumption that labor migration is nonselective, we still need to guarantee that only a single metropolis can emerge. Without loss of generality, we can establish a condition such that, given the same relative price structure, P^*, $\mathrm{d}U_{\mathrm{M}}^{*}/\mathrm{d}M > 0$:[12]

[12] It is assumed that the goods market is relatively large such that a change in M due to migration will not influence P. This is consistent with the price-taking behavior of the individual, given that the goods market is competitive.

Assumption 4.

$$2K > \lambda M^{3/2} - (1-\epsilon)FM^{\epsilon} .$$

This assumption implies that the marginal benefits from positive matching externalities and from per capita city formation cost reduction exceed the marginal cost of transportation when the city size gets bigger. It therefore ensures the existence of a single high-tech metropolis. Notice that this condition, in contrast to Assumption 3, sets a lower bound for the fixed cost of each high-tech firm (K). Furthermore, if there were no positive matching externalities, the wage for the skilled labor would not be affected by a change in M. In this case, to generate a single metropolis, i.e. $\mathrm{d}U^*/\mathrm{d}M > 0$, would require that $\lambda M^{3/2} - (1-\epsilon)FM^{\epsilon} < 0$, which is much stronger than Assumption 4 stated above.[13]

We are now ready to establish:

Theorem 1. Given any set of parameters $(\alpha, a, F, t, K, N, M)$ satisfying Assumptions 1–4, there exists a unique equilibrium system of cities under a symmetric wage bargain in which the core produces good z and all periphery cities produce good x.

Note that under a symmetric Nash bargain, households/workers in this spatial system achieve exactly two different utility levels in equilibrium. More specifically, by Assumption 3 and (2.16), (3.2) and (3.8), unskilled workers residing in the periphery cities attain a lower utility level than the skilled workers who reside in the core. The consideration of heterogeneous labor in our model economy therefore enable us to examine the determinants of income disparity between the core and the periphery regions.

4. Determinants of income inequality

For analytical convenience, we focus on an earned income inequality (i.e. wage inequality) within the core–periphery spatial system:[14]

[13] This condition is derived from $\mathrm{d}U^*/\mathrm{d}M > 0$ by holding P^* and W^*_{M} unchanged. If one takes a single firm as the discrete unit of change in the city size, then in order to ensure that no firm will have an incentive to form a new city, in the case of selective labor migration, the above condition should be replaced by:

$$\{[c/(4K)]^{1-\epsilon} - M^{\epsilon-1}\}F(2\lambda)^{-1} > M^{1/2} - (4K/c)^{1/2} .$$

[14] The difference between income inequality and wage inequality is the lump-sum redistribution, τ. Since such a redistribution is only made for convenience to close the model, we exclude it in our analysis of income disparity.

$$I \equiv W_M - W_L . \tag{4.1}$$

Utilizing (2.16) and (3.2), this income inequality measure can be derived as

$$I = a - (2K/M) - \gamma P^* . \tag{4.2}$$

Straightforward differentiation of (4.2) yields:

Theorem 2. Income inequality is increasing in the shift parameter of the infrastructure costs, F, and in the commuting cost, t. If the food elasticity of marginal utility, α, is sufficiently small, then an increase in the maximal productivity, a, or a decrease in the fixed factor of producing the manufacturing goods, K, widens income inequality. The productivity of the unskilled labor, γ, has no impact on income inequality.

Thus, the income disparity between the core and the periphery regions is found to depend on both the search and match parameters, including maximal productivity, entry cost, costs of forming the city, and commuting cost. The better the search-and-matching process in the high-tech manufacturing industry, or the higher the costs of forming a city, or commuting, the greater is the geographical income inequality.

5. Conclusions

This paper develops a two-sector, general-equilibrium spatial model of a system of cities allowing for a heterogeneous labor force. The model generates a core–periphery spatial structure, explaining the observed urbanization trends. The resulting equilibrium configuration is characterized by pure specialization in which the core produces only the high-tech manufacturing good while the periphery produces only the food product. We show that, under a symmetric Nash wage bargain between the high-tech firms and the skilled workers, the equilibrium utility level attained in the core is the same for all types of skilled workers. Moreover, we find that search-and-match parameters, as well as transportation and city formation cost parameters, are the main determinants of income disparity between the core and the periphery regions.

We should emphasize that this study is only a first step toward a general-equilibrium theory of a core–periphery system of cities. There are several possible extensions for future research. First, it would be interesting to generate endogeneously the two types of cities through self-selection of different workers. In this case the equilibrium will be of Tiebout-type where workers choose their locations depending on their types. As pointed out by Bewley (1981), the existence of such an equilibrium requires that there must be at least as many regions as types of consumers, even when public services are treated as essentially private goods. Under our setup of a continuum of types of skilled workers, it is therefore impossible to generate a spatial structure with a finite number of high-tech cities. To resolve this problem, we must properly refine the equilibrium concept; for example, one may use the so-called ϵ-core to define the self-selecting spatial equilibrium. Second, the intercity spatial structure is absent in the present paper. To allow for this, it is necessary to introduce intercity transportation cost in a fashion analogous to Samuelson's iceberg as in Abdel-Rahman (1994). This generality will likely require further simplification of the current framework in order to obtain an analytical equilibrium solution. Finally, considering variable lot size will be useful in understanding the intra-metropolis spatial structure. However, appropriate refinements of the equilibrium concept are again necessary in order to circumvent the nonexistence problem mentioned in Subsection 2.1 above.

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