Review of Economic Studies (1995) 62, 115-129
0034-6527/95/00060015$02.00

Learning, Matching and Growth

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First version received July 1992; *final version accepted July* 1994 (*Eds.*)

We examine an endogenous growth model in which market frictions are an integral part of the economic environment. Workers invest in education when young, which raises their productivity once employed. The *level* of schooling also acts as a key determinant of the *rate* of economic growth by influencing workers' ability to accumulate additional human capital on-the-job. Once schooling is completed, workers search for employment. The division of the surplus between vacancies and searching workers is characterized, as is the optimal level of education. The economy may display multiple steady-state growth paths.

1. INTRODUCTION

There is little doubt that a major source of sustained growth in *per capita* income stems from the accumulation of knowledge and its subsequent embodiment in either society's physical or human capital. There are many channels through which knowledge is acquired: formal schooling, primary research, on-the-job training, learning by doing, and product innovations (both vertical and horizontal), to name but a few. The primary contribution of the recent theory of endogenous growth, poineered by Romer (1986), Lucas (1988), and Rebelo (1991), has been to endogenize the process through which knowledge is accumulated and thereby the rate of economic growth itself. The results of this new literature have already led to substantive theoretical and empirical insights into the forces that drive sustained growth. A recurrent finding is that economic distortions which influence the *level* of output also affect the *rate* of economic growth.

To date much of the analysis has been conducted within a 'representative agent' framework, in which an integrated consumer-producer is conceived of as maximizing an intertemporal optimization problem subject only to various constraints imposed by technology and initial endowments. While this indubitably is a useful methodological starting point for analysing many issues, it has the effect of imposing a 'long-run' perspective on the economic growth process by ruling out potentially important 'short-run' market frictions and other such maculations right from the outset. Indeed, over two decades ago Solow remarked in his Radcliffe lectures: "(T)here is an . . . obvious need for someone to synthesize the theory of growth, which takes full employment for granted, with the shorter-run macroeconomics whose main subject is variation of the volume of employment" (ibid p. 14). There are potentially manifold benefits to be had from this integration. For instance, it may be possible to uncover mechanisms whereby contemporaneous macroeconomic events affect the rate of economic growth and to thus determine whether or not government stabilization policies (and other market interventions) may be useful for engineering sustained increases in the growth rate.

Accordingly, in this paper, we attempt to make a start at rectifying this lacuna, by examining the 'long-run' endogenous rate of growth in an economy in which short-run market frictions are an integral part of the trading environment. We utilize the 'matching and bargaining' framework promulgated in the seminal contributions of Diamond (1982*a*, 1982*b*, 1984), Mortensen (1982), and Pissarides (1984, 1985, 1987).[1] In this approach, the standard Walrasian assumption that trade is coordinated through the market is dropped and instead an explicit stochastic matching technology is posited that describes how unfilled vacancies and unemployed workers are brought together. The explicit differentiation of workers and firms dispenses with the need for a fictitious integrated consumer/producer, allowing for a more complete understanding of the role played by participants on each side of the market.

In Section 2 we describe the basic structure of the model. Our approach follows Romer (1986, 1989*a*), Lucas (1988), and Stokey (1991) in that we assume that the accumulation of human capital is the primary engine of economic growth.[2] Workers accumulate a proportion of the stock of public knowledge by investing in education when young. As in Rosen (1976) we assume that a worker's schooling effort influences both his/her initial productivity *and* its subsequent rate of growth once employed.

In order to solve for the steady-state equilibrium, we proceed through a process of backwards induction. We begin by treating the level of investment in education—and hence the rate of economic growth—as exogenous. Section 3 describes the matching and bargaining game. Once workers have completed their schooling, they enter the labour market and search for employment. Workers and vacancies are brought together pairwise by a stochastic matching technology and, upon meeting, bargain over the division of the surplus accruing to the match. Section 4 derives the model's steady-state properties with exogenous schooling effort. In Section 5 we complete the process of backwards induction by determining the level of education undertaken by workers in steady-state equilibrium. This enables us to calculate the endogenous rate of economic growth. Our results point to an intimate nexus between the extent of market development (measured by the severity of search/entry frictions) and the growth rate. We show, amongst other things, that an improvement in matching efficacy promotes economic growth by increasing the returns to education.[3]

Section 6 demonstrates that the economy may possess multiple steady-state growth paths. The intuition behind this result hinges on the positive feedback between the extent of market thickness and the returns to education: a 'thick' labour market encourages workers to invest in costly schooling, which raises the economic surplus from a successful match with a vacancy. In turn, this induces entry by new firms (spurred on by higher profits), which then goes to make for a thick-market. A 'thin-market' may also be an

1. Aghion & Howitt (1994) have recently constructed a Schumpeterian model of matching. In contrast to this paper, their research primarily considers how the rate of creative destruction (with concomitant sectoral reallocation of workers) impacts upon the natural rate of unemployment.

2. This view of the economic growth process is consistent with several recent empirical studies. For example, Jorgenson and Fraumeni (1991, p. 21) conclude that in the U.S. economy, "the accumulation of human and non-human capital accounts for the predominant share of economic growth". A similar finding is reported in Romer (1989*b*).

3. Phelps (1972, p. 79) was one of the first to point to the potential importance of short-run labour market conditions as a key determinant of training opportunities and therefore the level of output.

equilibrium for similar reasons. This result extends the theory of macroeconomic coordination failures which is typically conducted within a stationary environment, without economic growth. Finally, Section 7 offers some conclusions.

2. THE BASIC ENVIRONMENT

This section describes the economic environment. We utilize a continuous-time framework, in which the economy is populated by a continuum of identical infinitely-lived agents, each possessing an identical instantaneous discount rate over consumption of $\delta > 0$. Workers are endowed with a unit of labour which they can supply to firms without disutility from effort. The economy is comprised of two separate theatres of activity: an educational sector and a labour market in which unfilled vacancies and searching workers are brought together via a stochastic matching technology. As we explain below, the impetus behind sustained economic growth stems from the (endogenous) rate at which workers acquire additional human capital on the job.

The educational sector

Workers are born at the constant rate $\beta > 0$, whereupon they immediately enter the educational sector and choose a level of schooling effort $s \geqq 0$ (a description of schooling costs is deferred to Section 5).[4] Following Stokey (1991) we assume that schooling enables workers to absorb a proportion of the extant shock of public knowledge K_0, which raises their productivity. An individual's private stock of knowledge k is,

$$k(s, K_0) = K_0\phi(s) \qquad K_0 > 0, \tag{1}$$

indicating that the amount of knowledge acquired depends upon the available stock of public knowledge as well as upon the worker's schooling effort.[5] In what follows we shall use an increase in K_0 to model the effects of (for example) a significant discovery which adds to the body of public knowledge.

Assumption 1. The function $\phi: \mathbb{R}_+ \rightarrow [0, 1]$ is strictly increasing, strictly concave and twice continuously differentiable in s, satisfying: $\lim_{s\rightarrow 0} \phi(s) > 0$ and $\lim_{s\rightarrow\infty} \phi(s) \leqq \bar{\phi} < 1$.

Assumption 1 implies that there are diminshing returns to investment in education. The boundary condition $\bar{\phi} < 1$ rules out omniscience.

The labour market

Employers possess a fixed-coefficient production technology requiring a single worker. The cost of a unit of capital is denoted v_0. In practice, v_0 will depend upon a host of factors such as the extent of frictions in financial markets as well as the economy's tax structure which is often geared toward providing investment incentives.[6] We assume that

4. As explained below in footnote 11, allowing workers to choose the length of the schooling period would not alter our main findings.

5. It may be helpful to think of K_0 as the stock of knowledge available in public libraries and of $\phi(s)$ as the fraction of this knowledge that is accumulated by workers.

6. Bencivenga and Smith (1991) construct an endogenous growth model in which they explore the role of financial intermediation in the growth process. The important role played by the tax structure in determining investment decisions is discussed at length in Greenwood and Hercowitz (1991).

entry into the labour market is unrestricted: upon paying v_0, any number of firms can instantly enter search for unemployed workers.

Once workers have completed their schooling, they enter the labour market and search for employment. Unemployed workers (U) and unfilled vacancies (V) are brought together by a stochastic matching technology (as described in Section 3). Upon meeting, the surplus accruing to the match is equally divided amongst both parties according to a Nash bargaining rule; employment-tenure is granted, whereupon the filled vacancy (F)-employed worker (E) pair then exit the search market.[7] Once employed, workers accumulate additional human capital at the rate $\gamma(s)$.[8] Following Rosen (1976) we assume that $\gamma(s)$ depends positively upon the level of schooling, s. Thus, formal education provides workers with useful information (embodied in k) *and* an enhanced ability to acquire additional skills once employed. As in Lucas (1988) the rate at which human capital is accumulated is a crucial determinant of the rate of economic growth.

Assumption 2. The rate of growth of human capital $\gamma: \mathbb{R}_+ \to \mathbb{R}_+$, is strictly increasing, strictly concave, and twice-continuously differentiable in s, satisfying: $\lim_{s\to\infty} \gamma(s) < \delta$.

The condition, $\lim_{s\to\infty} \gamma(s) < \delta$ ensures that the present discounted value of a match is bounded. The human capital of a worker with an employment tensure of τ_E, who exerted schooling effort s is given by,

$$h(s, K_0, \tau_E) = k(s, K_0) \exp[\gamma(s)\tau_E]. \tag{2}$$

Normalizing units so that the instantaneous flow of output produced by a worker equals his or her stock of human capital, the total discounted value accruing to the match between a worker and a vacancy is:

$$a(s, K_0) = \int_0^\infty \exp[-\delta\tau_E] h(s, K_0, \tau_E) d\tau_E = K_0\{\phi(s)(\delta - \gamma(s))^{-1}\}. \tag{3}$$

From (3) it is immediate that $a(\cdot)$ is increasing with K_0 and from assumptions 1 and 2 that it is increasing, twice-continuously differentiable, and bounded above in schooling effort, s. In addition,

Assumption 3. The function $a: \mathbb{R}_+^2 \to \mathbb{R}$ is strictly concave in s.

The concavity assumption is made for convenience, since later we shall be interested in determining the optimal level of schooling effort.[9]

Throughout we assume that $a(0, K_0) > v_0$, which ensures a viable labour market for all levels of schooling. Finally, we assume that there are no direct instantaneous costs (or benefits) to either firms or workers from search activity.

7. Given the assumption of free entry, it is never optimal for an employer to fire an incumbent worker and to pay v_0 to re-enter the labour market, since profits are strictly positive only if the match continues.

8. The assumption that workers are infinitely lived is made for convenience. Instead, we could assume that workers die at the constant rate β and that they pass on a fraction of their human capital to their offspring. This formulation would ensure a constant population of employed workers and a positive rate of *per capita* economic growth in the steady-state.

9. For example, assumptions 1-3 are satisfied if $\phi(s) = \phi_0 - \phi_1 \exp[-s]$ and $\gamma(s) = \phi_1(1 - \exp[-s])$ [provided $0 < \phi_1 < \phi_0 < 1$ and $\phi_1 < \delta/2$].

3. MATCHING AND BARGAINING

In this section we describe the search environment and the (steady-state) division of the surplus between workers and firms.

Matching

Let $U(V)$ denote the mass of searching workers (vacancies). The flow probability that a worker (vacancy) locates a vacancy (worker) is $\mu(\eta)$. Since a vacancy is filled by one (and only one) worker, it follows that:

$$\mu U = \eta V. \tag{4}$$

Although μ and η are determined in equilibrium, both workers and firms treat them as parametric when making their decisions.

In order to complete the description of the model it is necessary to specify the *matching technology*, $m = m_0 M(U, V)$, which describes the instantaneous flow meeting rate between unfilled vacancies, V, and searching workers, U. Consider assumption 4,

Assumption 4. The matching technology, $M: \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$, (i) is strictly increasing, strictly concave and twice continuously differentiable (ii) exhibits constant returns to scale: $M(xU, xV) = xM(U, V), x > 0$ and (iii) satisfies the Inada conditions: $\lim_{W\rightarrow 0} \partial M/\partial W = \infty$, $\lim_{W\rightarrow 0} \partial M/\partial W = 0$, $W \in \{U, V\}$ and the boundary conditions: $M(0, V) = M(U, 0) = 0$.

An increase in the number of participants on either side of the market increases the instantaneous number of matches, but at a diminishing rate (part (i)). The constant-returns-to-scale assumption is made for convenience. The Inada boundary conditions ensure an interior steady-state solution. The extent of the search frictions in the labour market is conveniently parameterized by $m_0 > 0$: An increase in m_0 raises the instantaneous matching rate (for given values of V and U) and hence captures a reduction in the severity of market frictions.

The wage

We now turn to the determination of the wage offer function. We assume that workers and firms are risk neutral, which simplifies the analysis, and enables us to attribute our conclusions to the consequences of the search and matching process rather than the incompleteness of insurance markets (see the discussion in Diamond (1982*a*)).

The match between a searching worker and a vacancy will lead to a positive surplus to be divided between the two parties. We assume that all negotiations are instantaneous and conducted according to a symmetric Nash bargain, in which both parties receive an equal share in the surplus from a match (though nothing essential hinges on the symmetry assumption). It should be noted that even with an equal division rule, the returns accruing to either party from a match are endogenous and depend upon the matching technology as well as the other parameters of the model.

To solve for the outcome of the Nash bargain the surplus accruing from a match must be defined. Workers can be in one of two separate states: employed (E) or unemployed (U); similarly vacancies can either be filled (F) or unfilled (V). We next turn to the definition of the 'asset values' associated with being in these states. Consider the match between a worker and a vacancy. Let J_E and Π_F denote, respectively, the present-discounted values of income accruing from the match to the worker and vacancy. Similarly, let J_U denote the expected discounted value to a worker of continued search and let Π_V denote the corresponding asset value of an unfilled vacancy. The gains to a worker from accepting employment are: $J_E - J_U$; likewise, the value of filling a vacancy is $\Pi_F - \Pi_V$. The symmetric Nash bargaining rule implies that these must be equal, so that:

$$J_E - J_U = \Pi_F - \Pi_V \geqq 0. \tag{5}$$

Let $w(s, K_0, \Pi_V, \mu)$ denote the present discounted sum of wage income agreed under the Nash-bargain rule. In what follows we refer to w simply as 'the wage'.[10] The asset value functions are (see the appendix):

$$J_E = w(s, \cdot\,) \tag{6}$$

$$J_U = \{\mu/(\mu+\delta)\}w(s, \cdot\,) \tag{7}$$

$$\Pi_F = a(s, K_0) - w(s, \cdot\,) \tag{8}$$

$$\Pi_V = \{\eta/(\eta+\delta)\}\{a(s, K_0) - w(s, \cdot\,)\} \tag{9}$$

Proposition 1 summarizes the properties of the wage offer function: $w(s, K_0, \Pi_V, \mu)$.

Proposition 1. *The unique wage offer function, $w(s, K_0, \Pi_V, \mu)$, determined in the Nash bargain between workers and vacancies, is given by:*

$$w(s, \cdot\,) = \{(\mu+\delta)/(\mu+2\delta)\}\{a(s, K_0) - \Pi_V\}$$

and satisfies: $\partial w/\partial s > 0$, $\partial w/\partial K_0 > 0$, $\partial w/\partial \Pi_V < 0$, *and* $\partial w/\partial \mu > 0$.

Proof. All proofs are in the Appendix. ||

The comparative static properties of $w(s, \cdot\,)$ have intuitive interpretations. An increase in either schooling effort, s, or the stock of public knowledge, K_0, raises $a(s, K_0)$, which increases the size of the 'pie' to be divided between the two parties (without altering either sides *relative* bargaining position) and the wage w rises accordingly. An increase in Π_V, by directly raising the value of the firm's outside opportunity, lowers the wage. A more frequent matching rate of workers with vacancies, μ, makes *alternative* options more accessible to workers, thereby raising their wage offer.

4. STEADY-STATE ANALYSIS I: EXOGENOUS SCHOOLING

It is helpful to first characterize the properties of steady-state equilibrium, retaining the assumption that the level of schooling, s, is exogenously pre-determined. Consider,

Definition 1. A steady-state equilibrium with exogenous schooling, s, is a wage function $w(s, K_0, \Pi_V^*, \mu^*)$ and a quadruple $(\mu^*, \eta^*, U^*, V^*)$ satisfying the following conditions:

(i) (Symmetric Nash bargain): $J_E^* - J_U^* = \Pi_F^* - \Pi_V^* > 0$,
(ii) (Unrestricted entry): $\Pi_V^* = v_0$, and
(iii) (steady state)

$$\mu^* U^* = \eta^* V^* = m_0 M(U^*, V^*), \tag{10a}$$

$$\mu^* U^* = \beta. \tag{10b}$$

10. Our model is therefore one of lifetime earnings as in Rosen (1976). The only restriction we place upon the instantaneous stream of earnings is that their present discounted value equals $w(s, \cdot\,)$.

Condition (i) of the definition ensures that the wage $w^*(s, \cdot)$ is consistent with the equal-division rule. Part (ii) reflects the assumption of free entry into the labour market. Upon paying v_0 firms can instantly enter the search market and commence searching for workers. If $\Pi_V > v_0$, the expected value of entry exceeds v_0, leading to a large (instantaneous) influx of new entrants, which, in turn, lowers Π_V by making it more difficult (and therefore time consuming) for vacancies to locate workers. Similarly, $\Pi_V < v_0$ is impossible, since entry is not be profitable. Part (iii) of the definition provides necessary and sufficient conditions for constant populations of vacancies, V, and searching workers, U. Equation (10a) states that the instantaneous matching rate of vacancies and searching workers equals that determined by the matching technology $m_0M(U^*, V^*)$, while (10b) indicates that the instantaneous outflow of workers from the unemployment pool (through successful matches) must equal the birth rate of new entrants.

The model possesses a convenient recursive structure, which can be exploited to prove the existence of equilibrium and analyse its properties. First, the equilibrium values of the matching rates μ^* and η^* can be determined (as described below) from the unrestricted entry condition $\Pi_V^* = v_0$ and the steady-state conditions (10a) and (10b). Secondly, once μ^* and η^* have been determined, the wage w^* can then be calculated from Proposition 1. Finally, the equilibrium values of U^* and V^* can then be obtained directly from equations (10a) and (10b), since $U^* = \beta/\mu^*$ and $V^* = \beta/\eta^*$.

The equilibrium entry condition (the EE locus)

Utilizing Proposition 1 together with the definition of Π_V enables the *unrestricted entry condition*, $\Pi_V^* = v_0$, to be written as:

$$\Pi_V^* \equiv (\eta/(\eta+\delta))\{a(s, K_0) - w(s, \cdot)\} = v_0. \tag{11}$$

Equation (11) implicitly defines a function: $\eta = \eta^{EE}(\mu; s, K_0, v_0) \geqq 0$ (referred to as the '*EE*' locus in Figure 1), which gives the values of η and μ that equalize the costs and benefits to firms from entering the labour market.

Lemma 1 (*The EE locus*). *The function* $\eta = \eta^{EE}(\mu; s, K_0, v_0)$ *is linear in* μ *with a positive intercept* $\eta_0 \equiv 2\delta v_0/\{a(s, K_0) - v_0\}$ *and satisfies*:

$$\partial\eta^{EE}/\partial\mu > 0 \;\; \partial\eta^{EE}/\partial s < 0 \;\; \partial\eta^{EE}/\partial K_0 < 0 \textit{ and } \partial\eta^{EE}/\partial v_0 > 0.$$

An increase in the rate at which workers contact vacancies, μ, raises the wage (Proposition 1) and lowers profits. This discourages entry and raises η. An increase in either the stock of public knowledge, K_0, or schooling, s, raises $a(s, K_0)$ and consequently the value of employing a worker. This, in turn, increases the expected return from a match, which stimulates the entry of vacancies and lowers η. An increase in the entry fee, v_0, makes entry less attractive which lowers the number of vacancies and consequently raises η.

The steady-state matching condition (the SS locus)

The constant-returns-to-scale property of the matching technology (Assumption 4), implies: $m = m_0VM(U/V, 1)$. This, in conjunction with (10a) and the fact that $U/V = \eta/\mu$, yields $\eta = m_0M(\eta/\mu, 1)$, which implicitly defines the relationship: $\eta = \eta^{SS}(\mu; m_0)$ along which $\dot{U} = \dot{V} = 0$ (referred to as the '*SS*' locus in Figure 1). The properties of the *SS* locus follow directly from Assumption 4 are summarized for convenience in Lemma 2.

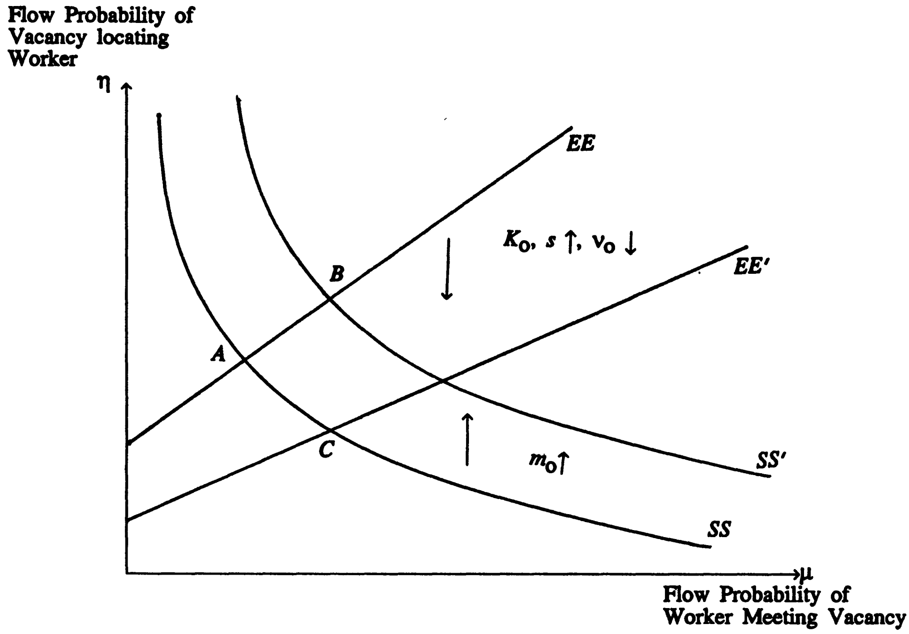

FIGURE 1

Steady state with exogenous schooling and comparative statics of the equilibrium

Lemma 2 (*The SS locus*). *Under Assumption* 4 *the function* $\eta = \eta^{SS}(\mu; m_0)$ *satisfies the following properties*:

(i) $\partial\eta^{SS}/\partial\mu < 0$ *and* $\partial\eta^{SS}/\partial m_0 > 0$ (ii) $\lim_{\mu\to 0} \partial\eta^{SS}/\partial\mu = -\infty$ *and* $\lim_{\mu\to\infty} \partial\eta^{SS}/\partial\mu = 0$.

Steady-state equilibrium

By exploiting the properties of the *EE* and *SS* loci it is straightforward to prove the existence of a steady-state equilibrium and to characterize its properties.

Proposition 2 (*Steady-state equilibrium with exogenous schooling*). *Under Assumptions* 1–4, *a unique steady-state equilibrium exists, which possessses the following properties.*

(i) *Matching rates:*
  (a) $d\mu^*/dm_0 > 0$, $d\mu^*/ds > 0$, $d\mu^*/dK_0 > 0$, *and* $d\mu^*/dv_0 < 0$
  (b) $d\eta^*/dm_0 > 0$, $d\eta^*/ds < 0$, $d\eta^*/dK_0 < 0$, *and* $d\eta^*/dv_0 > 0$

(ii) *Steady-state population of vacancies*, V^*, *and searching workers* U^*:
  (a) $dU^*/dm_0 < 0$, $dU^*/ds < 0$, $dU^*/dK_0 < 0$ *and* $dU^*/dv_0 > 0$
  (b) $dV^*/dm_0 < 0$, $dV^*/ds > 0$, $dV^*/dK_0 > 0$ *and* $dV^*/dv_0 < 0$.

(iii) *Wage offers*
  $dw^*/dm_0 > 0$, $dw^*/ds > 0$, $dw^*/dK_0 > 0$, *and* $dw^*/dv_0 < 0$.

(iv) *The* (*exogenous*) *rate of economic growth per-employee is*: $g = \gamma(s)$.

From Lemma 1 the *EE* locus begins at a positive finite value of η_0 and is linearly monotone increasing in the matching rate, μ. The *SS* locus begins at infinity and approaches zero asymptotically as μ approaches infinity. Since both functions are continuous, there must exist a unique point at which the two loci cross (depicted by A in Figure 1), which determines μ^* and η^*.

Referring to Figure 1, a reduction in the severity of search frictions, captured by an increase in m_0, shifts the *SS* curve to the right to *SS'*. This raises the equilibrium values of both μ^* and η^* and shifts the economy from point A to B. The increase in μ^* and η^* lower, respectively, the level of unemployment and the number of vacancies. An increase in schooling effort, s, or the stock of public knowledge, K_0, or a reduction in the entry fee, v_0, shifts the *EE* locus down to *EE'* by encouraging the entry of new firms (Lemma 1). The resulting increase in the number of unfilled vacancies, V^*, makes it easier for workers to find jobs (μ^* rises and U^* falls) and more difficult for vacancies to find workers (η^* falls) and the equilibrium moves from point A to point C.

A change in the level of schooling has two mutally re-enforcing effects on the equilibrium wage, w^*. First, an increase in s, raises workers' productivity directly and therefore the wage offer (Proposition 1). Second, the concomitant increase in the equlibrium value at which workers contact vacancies, μ^*, also raises the wage. The other comparative-static results reported in part (iii) follow similarly. Finally, by definition, $\gamma(s)$ is rate of growth of output per employee.

5. STEADY-STATE ANALYSIS II: ENDOGENOUS SCHOOLING EFFORT

We now consider the optimal choice of education by workers in the economic environment described in Sections 2–4, thus endogenizing the rate of economic growth.

Investment in education is beneficial to workers in two respects: it raises their initial stock of human capital and their ability to accumulate additional human capital once employed. We assume that schooling effort is costly, as a result of either a direct pecuniary cost or the disutility from scholastic effort. Let $c(s)$ denote the utility cost to workers from effort s.[11]

Assumption 5 (Schooling cost). The function $c:\mathbb{R}_+\rightarrow\mathbb{R}_+$, is strictly increasing, twice continuously differentiable, and convex in, s, and satisfies $dc(0)/ds=0$.

Each individual, taking as given Π_V and the matching parameters μ and η, is conceived of as solving the following programme (P):

$$\text{(P)} \qquad \tilde{J}\equiv \text{Max}_{s\geq 0}\,\{J_U-c(s)\}.$$

Since $a(s, K_0)$ is bounded above, the asset value, $J_U=[\mu/(\mu+2\delta)][a(s, K_0)-v_0]$, is also bounded. Programme (P) possesses a unique interior solution, which is characterized by the following first-order necessary and sufficient condition:

$$[\mu/(\mu+\delta)]\partial a(s, K_0)/\partial s-dc(s)/ds=0. \tag{12}$$

Individuals invest in education in order to increase their productivity and hence their wage once employed. Factors which influence wage income or matching probabilities, by changing the returns to education, alter workers schooling effort.

11. Our analysis can easily be amended, without altering the basic results, to the case in which education is time consuming by considering the programme: $\max_s \exp[-\delta s]J_U-c(s)$, where s is now the length of the schooling period.

Lemma 3. *The individual schooling effort function, $s = s(K_0, \mu)$ satisfies:*

$$\partial s/\partial K_0, \textit{ and } \partial s/\partial \mu > 0.$$

An increase in the stock of public knowledge, K_0, raises schooling effort by increasing the marginal returns to education (see equation 3). Similarly, an increase in the contact rate μ promotes schooling by raising the (expected discounted) marginal benefits from education: $\{\mu/(\mu+2\delta)\}\partial a/\partial s$. The definition of a steady-state equilibrium must be emended to allow for the optimal choice of s.

Definition 2. A steady-state equilibrium with endogenous schooling is a wage function $w(s^*, K_0, \Pi_V^*, \mu^*)$, a schooling function $s(K_0, \mu^*)$, and a quadruple $(\mu^*, \eta^*, U^*, V^*)$ satisfying the conditions set out in Definition 1 and, in addition, $s^* = \text{argmax}_{s \geqq 0}\{J_U - c(s)\}$, i.e., s^* solves Programme (P).

In order to analyse the properties of the model, we again exploit the model's recursive structure and first solve for μ^* and η^*. Since education is not an argument of the *SS* locus, its properties are identical to those described in Lemma 2. The *EE* locus is the set of points (μ, η) satisfying the free entry condition:

$$\{\eta/(\eta+\delta)\}\{a(s(K_0, \mu), K_0) - w(s(K_0, \mu), K_0, v_0, \mu)\} \equiv v_0. \tag{11'}$$

It may be observed in (11′) that the level of education chosen by workers optimally responds to changes in K_0 and μ. The *EE* locus possesses qualitatively similar properties with respect to K_0 and v_0 to those already described in Lemma 1. However, an increase in the matching rate μ now has two separate effects: (i) it raises the wage (Proposition 1) and (ii) it increases the level of education undertaken by workers (Lemma 3). The former of these effects tends to raise η by reducing the value to firms from successfully matching with workers (thus discouraging the entry of vacancies) while the latter effect tends to lower η for analogous (but opposite) reasons. It follows that the effect of an increase in μ upon η is potentially ambiguous and (as described in Section 6) that the *EE* locus may possess decreasing segments. However, the *EE* locus will increase with μ if the wage effect dominates the schooling effect. This will be so if an increase in μ induces a "small" change in the optimal level of education chosen by workers. Consider:

Condition U. Let $(a_s)^2/c_{SS} < (a(s, K_0) - v_0)$, for all $s \geqq 0$, where $a_s \equiv \partial a/\partial s$ and $c_{SS} \equiv \partial^2 c/\partial s^2$.

Lemma 4. *If condition U is satisfied, the EE locus is strictly increasing in μ.*

Condition U ensures that the level of education is relatively unresponsive to changes in μ, which is the case if either the returns to education (a_S) are 'small' or else the schooling cost function is relatively convex in s—i.e., c_{SS} is 'large'.

Proposition 3 (*Steady-state equilibrium with endogenous schooling s*). *Under Assumptions* 1–5 *and Condition U, a unique steady-state equilibrium with endogenous schooling exists.*

If Condition U is satisfied the EE locus is strictly increasing in μ (but not necessarily linear). It must therefore cut the (strictly) decreasing *SS* locus at a unique interior point under Assumption 4 (observe that Figure 1 remains a qualitatively valid representation of the equilibrium). This point of intersection determines the steady-state values of μ^* and η^*. The equilibrium values of $w(\cdot)^*$ and $s(\cdot)^*$ are then determined from Proposition 1 and Lemma 3.

Proposition 4 (*Characterization of the steady-state equilibrium with endogenous schooling*). *Around the unique steady-state equilibrium described in Proposition* 3, *the level of schooling* $s^* = s^*(K_0, m_0)$, *the rate of growth of output per employee* $g = \gamma(s)$, *and the populations of searching workers* U^* *and unfilled vacancies* V^* *satisfy,*

(i) $ds^*/dm_0 > 0$ $ds^*/dK_0 > 0$; $ds^*/dv_0 < 0$ $dg^*/dm_0 > 0$ $dg^*/dK_0 > 0$ *and* $dg^*/dv_0 < 0$.

(iia) $dU^*/dm_0 < 0$ $dU^*/dK_0 < 0$, *and* $dU^*/dv_0 > 0$.

(iib) $dV^*/dm_0 < 0$ $dV^*/dK_0 < 0$, *and* $dV^*/dv_0 > 0$.

The matching and entry friction parameters m_0 and v_0 are key determinants of the rate of growth of output per employee and the level of unemployment.[12] An increase in m_0 (reduction in v_0) raises the equilibrium value of μ^* (which lowers the equilibrium level of unemployment) and enhances the returns to education. In turn, the increase in education, s, promotes economic growth by improving workers ability to accumulate human capital on-the-job. These results suggest that the quality of an economy's 'infrastructure'—to the extent that it mitigates market frictions or lowers entry costs—might be a key determinant of both the *growth* rate and the *level* of economic performance. An innovation or discovery that raises the stock of public knowledge, K_0, enhances workers' incentives to undertake schooling, raising the growth rate. The comparative-static results relating to the populations of searching workers and vacancies are essentially the same as those reported in Proposition 2.

6. MULTIPLE EQUILIBRIA

In this Section, we establish:

Proposition 5 (*Multiple steady-state equilibria*). *With endogenous schooling effort there may exist multiple steady-state equilibria.*

Figure 2 depicts the possibility of multiple equilibria.[13] This occurs if the *EE* locus displays decreasing segments and cuts the *SS* locus at a number of points (see points *A*–*C*). At *A* the labour market is 'thick', in that there is a high contact rate of workers with vacancies, μ. This high contact rate encourages workers to invest in costly schooling which ensures a high life-time productivity upon employment. In turn, this stimulates entry by firms, which then makes for a thick market. Similarly, the equilibrium at point *C* exemplifies a 'thin market' with a low volume of trade. It may be further observed that since $g^* = \gamma(s^*)$ and that $U^* = \beta/\mu^*$, the equilibrium depicted at *A* also displays the highest rate of economic growth and the lowest level of unemployment. If the government can act as a 'market-maker' and can keep the economy away from the 'poor' equilibria *B* and *C* government policy may be a critical determinant of the steady-state growth rate.

12. It should be observed that Condition U is a *sufficient* condition for the comparative static results reported in Proposition 4 to hold (a necessary condition—which allows decreasing segments of the *EE* locus—is that in equilibrium: $\{\partial\eta^{EE}/\partial\mu - \partial\eta^{SS}/\partial\mu\} > 0$).

13. We have verified this possibility in a number of numerical simulations; an example is provided in the Appendix.

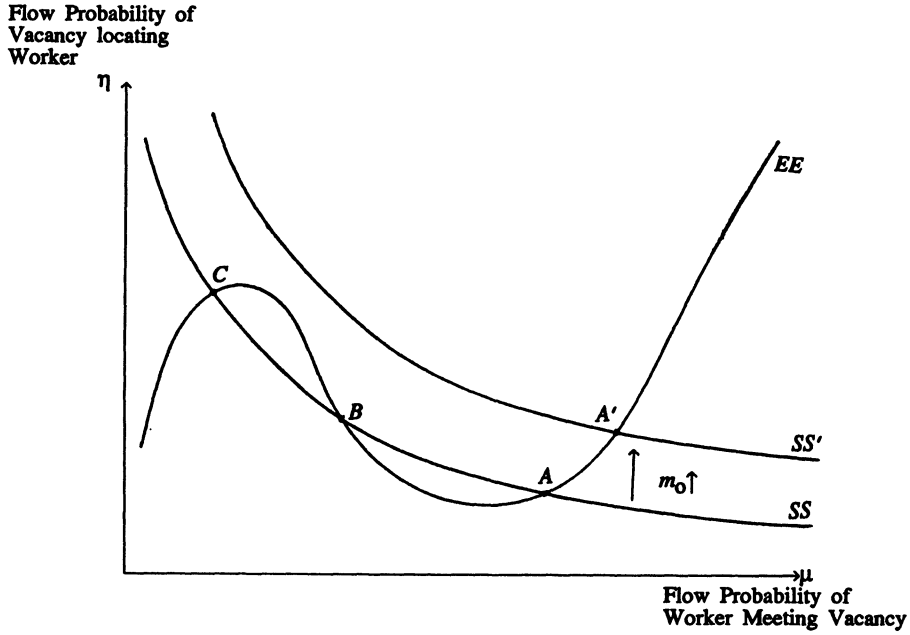

FIGURE 2
Multiple steady state growth paths

Given the presence of multiple equilibria, improvements in the quality of the economy's infrastructure (which raise either the matching rate, m_0, or reduce entry costs v_0) may lead to discrete jumps in the rate of economic growth. To see this, suppose that in Figure 2 the economy begins in the low growth equilibrium (C). Observe that an increase in m_0, which shifts the SS curve up to SS', leaves only the high *growth* equilibrium A' (a similar result pertains if v_0 falls and shifts the EE locus down). This result suggests that (potentially) small improvements in market efficacy may lead to a 'take-off' of sustained economic growth.[14]

7. CONCLUSIONS

This paper examines the endogenous economic rate of growth in an economy in which short-run market frictions are an integral part of the trading environment. The use of the search and bargaining framework allows the assumption of an integrated consumer-producer to be relaxed, permitting the explicit analysis of the interaction between workers and firms. We characterize the division of the surplus between vacancies and workers. The optimal level of investment in education is shown to depend upon both search frictions and entry costs in the labour market. A reduction in the extent of these frictions increases the returns to education, which induces workers to exert greater schooling effort. In turn, the increase in schooling promotes economic growth, by enhancing workers' ability to accumulate additional human capital on the job. The economy may exhibit multiple steady-state growth paths. This result extends the theory of macroeconomic coordination failures,

14. Murphy, Shleifer and Vishny (1991) formalize a 'big-push' argument to economic development, in which infra-structural improvements may lead the economy from low to high *levels* of equilibrium income.

which is typically conducted within a stationary environment without perpetual economic growth.

We believe that our model could be extended in a number of ways. For brevity we outline just a few possibilities. First, the inclusion of firms' learning-by-doing through external production spillovers, as in Stokey (1988) and Young (1991), may be of interest in addition to the accumulation of knowledge through worker's on-the-job training. This type of increasing returns to scale creates, of course, another possible source of multiplicity in equilibrium growth paths. Moreover, the interactions between market and production externalities may lead to very rich growth dynamics. Secondly, another extension is to analyse the case in which the stock of public knowledge continuously grows at a rate that depends upon the *average* level of human capital. This would allow for the study of potentially interesting educational externalities, presently absent from this paper.

APPENDIX

Proofs of Lemmas 1–4 and Propositions 1–5.

The *asset value functions* (equations (6)–(8)): $J_E = w(s,\cdot)$ and $\Pi_F = [a(s, K_0) - w(s,\cdot)]$ are immediate;

$$J_U = \int_0^\infty \mu \exp[-(\mu+\delta)\tau]w d\tau = \{\mu/(\mu+\delta)\}w;$$

$$\Pi_V = \int_0^\infty \eta \exp[-(\eta+\delta)(\tau - t)]$$

$$[a(s, K_0) - w(s,\cdot)][d\tau = [\eta/(\eta+\delta)][a(s, K_0) - w(s;\cdot)]. \quad \|$$

Proof of Proposition 1 (The wage offer function). Substituting equations (6)–(8) in (5) yields the wage function: $w(s;\cdot) = [(\mu+\delta)/(\mu+2\delta)]\{a(s, K_0) - \Pi_V\}$. Differentiation yields the reported comparative static results. ||

Proof of Lemma 1 (The *EE locus*). Manipulating (11) yields $\eta^{EE} = (2\delta+\mu)\{v_0/[a(s, K_0) - v_0\}$. The properties of the locus then follow immediately. ||

Proof of Lemma 2 (The SS locus). Totally differentiating the expression $\eta = m_0 M(\eta/\mu, 1)$ yields $(\mu - m_0 M_U)d\eta + m_0(\eta/\mu)M_U(\eta/\mu, 1)d\mu - \mu M(\eta/\mu, 1)dm_0 = 0$, implying that the signs of the derivatives hinge upon: $(\mu - m_0 M_U)$. From the Euler theorem it follows (using $\eta/\mu = U/V$): $\mu U = m_0 M(U, V) = m_0[M_U(\eta/\mu, 1)U + M_V(\eta/\mu, 1)V]$, which implies $\mu U - m_0 M_U = m_0 M_V(\mu/\eta) >$ proving the result. The limiting properties follow directly from the Inada conditions. ||

Proof of Proposition 2 (Steady-state equilibrium with exogenous schooling). Existence and uniqueness are immediate from Lemmas 1 and 2. The point of intersection determines a pair (μ^*, η^*) consistent with the entry and steady-state conditions given in parts (ii) and (iii) of Definition 1. Once μ^* and η^* are determined, the other values of the endogenous variables can be solved from Proposition 1 and equations (10*a*) and (10*b*).

(i) *Matching rates.* The equilibrium is characterized by a pair (μ^*, η^*):

$$\eta^* - \eta^{EE}(\mu^*; K_0, s, v_0) = 0, \tag{A1}$$

$$\eta^* - \eta^{SS}(\mu^*; m_0) = 0. \tag{A2}$$

Total differentiation of (A1) and (A2), in conjunction with Lemmas 1 and 2, yield the results reported in part (i) of the Proposition.

(ii) *Steady-state populations of searching workers, U^*, and vacancies, V^*.* (a) The steady-state level of unemployment is $U^* = \beta/\mu^*$. Hence, sign $(dU^*/d\lambda) = -\text{sign}\,(d\mu^*/d\lambda)$, where $\lambda = s, K_0, v_0, m_0$. (b) The steady-state population of vacancies is $V^* = \beta/\eta^*$. Hence, sign $(dV^*/d\lambda) = -\text{sign}\,(d\eta^*/d\lambda)$

(iii) *Wage offers.* Totally differentiating the wage offer function with respect to arbitrary λ yields

$$dw^*/d\lambda = (\partial w^*/\partial\lambda) + (\partial w^*/\partial\mu^*)(d\mu^*/d\lambda).$$

The results follow immediately from Proposition 1 and part (i) of Proposition 2.

(iv) Output produced by a successful match grows at the rate $\gamma(s)$, and hence $g = \gamma(s)$. ||

Proof of Lemma 3 (The individual schooling offer function). Each worker solves programme (P): $\tilde{J} = \text{Max}_{s\geq 0}\{J(0)_u - c(s)\}$. The unique solution to this programme is characterized by,

$$[\mu/(\mu+\delta)]\partial a/\partial s - dc(s)/ds = 0. \tag{A3}$$

Totally differentiation of (A3) with respect to s, K_0, v_0, and μ yields

$$\{[\mu/(\mu+\delta)](\partial^2 a/\partial s^2) - d^2c/ds\}ds + [\mu/(\mu+\delta)](\partial^2 a/\partial s\partial K_0)dK_0 + [(2\delta/(\mu+2\delta)^2](\partial a/\partial s)d\mu = 0.$$

Thus $\partial s/\partial K_0 > 0$ and $\partial s/\partial\mu > 0$ as claimed. ||

Proof of Lemma 4. With endogenous schooling, the *EE* locus must be modified in an obvious way as:

$$\eta = \eta^{EE}(\mu; K_0, s(K_0, \mu), v_0) \tag{A1$'$}$$

allowing schooling effort to optimally vary with μ. It is easily checked from (11′) that $\partial\eta^{EE}/\partial\mu > 0$ only if: $(\partial a/\partial s)(\partial s/\partial\mu) < \{a(s, K_0) - v_0\}/\{\mu + 2\delta\}$. However, from (12): $\partial s/\partial\mu = (2\delta a_S)/\{(\mu(c_{SS} - a_{SS}) + 2\delta c_{SS})(\mu + 2\delta)\}$, where $a_S \equiv \partial a/\partial s$ etc. This implies that, $\partial\eta^{EE}/\partial\mu > 0$ only if:

$$2(a_S)^2\delta/(\mu(c_{SS} - a_{SS}) + 2\delta c_{SS}) < \{a(s, K_0) - v_0\}.$$

However, condition U implies: $2(a_S)^2\delta/(\mu(c_{SS} - a_{SS}) + 2\delta c_{SS}) \leqq (a_S)^2/c_{SS} < \{a(s, K_0) - v_0\}$, which proves the results. ||

Proof of Proposition 3 (Steady-state equilibrium with endogenous schooling). Given Condition U, existence and uniqueness of the equilibrium is immediate from Lemma 2 and Lemma 4, since $0 < \lim_{\mu\to 0} \eta^{EE}(\mu; \cdot) < \infty$. ||

Proof of Proposition 4 (Characterization of the equilibrium). (i) Differentiating $s^*(K_0, v_0, m_0, \mu)$ with respect to $\lambda \in \{K_0, v_0, m_0\}$ yields:

$$ds^*/d\lambda = \partial s^*/\partial\lambda + (\partial s^*/d\mu)(\partial\mu^*/\partial\lambda). \tag{A4}$$

The signs of $\partial s^*/\partial\lambda$ are given in Lemma 3 ($\partial s^*/\partial K_0 > 0$, $\partial s^*/\partial v_0 = \partial s^*/\partial m_0 = 0$, and $\partial s^*/d\mu > 0$). It is necessary to determine $\partial\mu^*/\partial\lambda$. These derivatives are obtained by totally differentiating (A2) and (A1′). The results follow from Lemma 3. Notice that sign $\{dg^*/ds\} = $ sign $\{ds^*/d\lambda\}$ since $\partial\gamma/\partial s > 0$.
(iia) The steady-state level of unemployment is $U^* = \beta/\mu^*$. Hence, sign $(dU^*/d\lambda) = -$sign $(d\mu^*/d\lambda)$. (iib) Similarly the steady-state population of vacancies is $V^* = \beta/\eta^*$. Hence, sign $(dV^*/d\lambda) = -$sign $(d\eta^*/d\lambda)$. ||

Proof of Proposition 5 (Multiple steady-state growth paths). Consider the following example-economy: $k(s, K_0) = K_0\phi(s) = 0{\cdot}8 - 0{\cdot}5\exp(-s)$, $\gamma(s) = 0{\cdot}5[1 - \exp(-s)]$, $\delta = 1{\cdot}8$, $v_0 = 0{\cdot}13$, $m = 5{\cdot}751\, V^{0{\cdot}95}\, U^{0{\cdot}05,}$ and $c(s) = s^3$. Under this parametrization, there are three steady-state growth equilibria: $(\mu_1^*, \eta_1^*, s_1^*, g_1^*) = (1{\cdot}42\times 10^{-6}, 12{\cdot}731, 2{\cdot}92\times 10^{-4}, 1{\cdot}46\times 10^{-4})$, $(\mu_2^*, \eta_2^*, s_2^*, g_2^*) = (0{\cdot}35, 6{\cdot}656, 0{\cdot}129, 0{\cdot}06)$, *and* $(\mu_3^*, \eta_3^*, s_3^*, g_3^*) = (1{\cdot}0, 6{\cdot}301, 0{\cdot}188, 0{\cdot}086)$. ||

Acknowledgements. We wish to thank Mike Baye, Rick Bond, Keith Crocker, John Fender, Barry Ickes, and Will Roberds; seminar participants at Austin, The Atlanta Fed, Essex, Indiana, the LSE, Southampton, Sussex, The Econometric Society Meetings in Anaheim; and the editor of this Journal and two anonymous referees for helpful comments. Needless to say the usual disclaimer applies.

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