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Money demand and the relative price of capital goods in hyperinflations

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Abstract

We investigate dynamic interactions between relative price movements and money demand behaviors during hyperinflations, viewing relative price changes as resulting primarily from real disturbances. We develop a general equilibrium model with heterogeneous consumption and capital goods to illustrate how monetary shocks may produce real effects through the relative price channel. This motivates the design of long-run restrictions to identify a structural vector autoregression, employing data from the post-WWI Germany and the post-WWII Chinese hyperinflationary episodes. The empirical results support the contention that both real and nominal shocks have important effects on the relative price and money demand during hyperinflations.

Key words: Hyperinflation dynamics; Real and nominal interactions

JEL classification: E31; E41; N10

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1. Introduction

Hyperinflations provide a fertile area for research topics because there remain so many unanswered questions surrounding these phenomena. Past research has been unable to examine completely some fundamental issues, such as whether economic fluctuations during hyperinflations are similar, whether money growth produces real effects, or whether real shocks have a significant impact on money demand beyond inflation expectations.

Cagan (1956), in his pivotal work, models money demand in an adaptive expectations framework, in which an increase in the expected rate of inflation raises the cost of holding money and thus reduces real balances.[1] In a recent article, Taylor (1991) employs cointegration techniques to reexamine the Cagan hyperinflation study and finds that the traditional money demand specification is not supported by the German data.[2] We infer that these results imply that variables in addition to expected inflation have significant impact on money demand.

We hypothesize that real activities have an important bearing on the behavior of money demand even in a hyperinflationary environment. In previous studies of hyperinflation, real variables have generally been excluded from the estimated money demand regression because of the absence of adequate output measures at a monthly frequency. In contrast to previous work, we use additional data measures to indicate real economic factors.

Our study examines and compares two hyperinflationary episodes, post-World War I Germany and post-World War II China, both of which experienced the highest inflation with the longest sample and richest reliable data. Most previous work on hyperinflations assumed that all prices increased equiproportionately. In contrast, we examine the relative price movements measured by the ratio of the wholesale price to the cost-of-living index.[3] These two countries, to our knowledge, are the only ones experiencing hyperinflation to have separate indexes for consumer and wholesale prices. We use this price ratio

[1] Sargent (1977) modifies Cagan's approach by allowing individuals' expectations to be rational, while Frenkel (1977) implements the analysis using forward premium as a proxy for expected inflation. Abel, Dornbusch, Huizinga, and Marcus (1979) find that forward premium has significant explanatory power for money demand in addition to inflation expectations.

[2] Taylor shows that for the Cagan model to hold, real money demand and expected inflation must be cointegrated. For certain data samples, most notably for the post-World War I German hyperinflation (1920–1923), the null hypothesis of noncointegration cannot be rejected for these series.

[3] Garber (1982) first used this measure as a proxy for the relative price of capital to consumption goods because of the absence of a capital goods price index. As Garber noted, the proxy measure understates the actual relative price movement of capital goods because the wholesale price index contains prices for some final goods in addition to primary inputs and capital goods.

to proxy for the price of capital relative to consumption goods, a real variable. Interestingly, the movements of these two price indexes diverged at times in these two hyperinflations. In Germany, the relative price ratio increased from 1.17 in April 1920 to 1.94 in November 1922 and then declined to 1.49 in July 1923. Similarly, the relative price ratio in China rose from 0.94 in March 1946 to 2.0 in December 1948 and then dropped to 1.64 in March 1949.

This paper develops a dynamic general equilibrium model that enables us to study the dynamic interactions between the real and the monetary sectors in a hyperinflationary environment.[4] We introduce money into a competitive firm–consumer model via a modified cash-in-advance constraint that is altered so that the frequency of transactions is widely different from the frequency of consumption, capturing a stylized feature of hyperinflations. Utilizing a simple capital storage technology, we consider consumption and capital goods as heterogeneous, thereby generating a well-defined relative price ratio (measured by the capital good price in units of consumption good). We impose asymmetric liquidity constraints on the purchase of the consumption versus the capital good. Our motivation for this distinction is that capital goods purchases can be financed more often than consumption goods purchases. The asymmetric liquidity constraint places a wedge between the prices of goods that are restricted by a cash-in-advance constraint and the prices of those goods that are less constrained. Disproportionate (consumption and capital good) price movements, therefore, create a plausible channel through which we can study the dynamic interactions between real and nominal variables.[5]

Our main model implications suggest that money growth shocks decrease the demand for real money balances, but also increase the relative price of capital, a real effect. Real (Harrod-neutral) productivity shocks increase the output of consumption per unit of capital input, lowering the price of consumption relative to capital and raising the relative price of capital. When the productivity shock is multiplicative, its effect on real money demand is positive to the same degree as for the relative price.

The theoretical predictions allow us to impose necessary long-run restrictions to identify a structural vector autoregressive (VAR) model in a fashion similar to

[4] Policano and Choi (1978) examine relative price effects on money demand in a static, partial equilibrium model. In contrast, we allow the relative price and the inflation rate to be determined endogenously in a dynamic, general equilibrium framework.

[5] There is an existing literature that examines the relationship between inflation and the dispersion of the relative price. Implicitly, given that the relative price reflects real activity, this line of research investigates whether there are real effects of inflation. This relationship arises from either incomplete information, as in Hercowitz (1981), or from costly price adjustment, as in Sheshinki and Weiss (1977). In contrast to these studies, our paper focuses on the level of the relative price ratio rather than its variance. Also, we extend our investigation to study the dynamic interactions between the real and nominal sectors.

Blanchard and Quah (1989), King, Plosser, Stock, and Watson (1991), and Ahmed, Ickes, Wang, and Yoo (1993). We do not impose a structure on the short-run interactions that may be controversial, especially in a chaotic hyperinflationary environment. Rather, our approach allows the data to determine the short-run dynamics, while using the theoretical model to provide a structural interpretation of the fundamental disturbances driving the economy we analyze.

We estimate a system consisting of three variables (money growth, the money demand–relative price ratio, and the relative price) and three fundamental orthogonal disturbances (money growth, transactions interval or negative velocity, and real or productivity shocks). We use impulse response functions to display the short-run reaction of each variable to each unit shock and perform variance decompositions to assess quantitatively the important sources of fluctuations in money demand and relative price. Moreover, we provide sensitivity analysis considering alternative VAR models with plausible causal orderings to verify the robustness of the empirical evidence.

Our results support the general conclusion that there are significant effects of both real and nominal shocks on money demand and the relative price in hyperinflations. There are some differences across the two samples: for the German data about one third of the variance in money demand changes is associated with real variables, whereas for the Chinese data real variables appear related to two thirds of the variance in money demand changes.[6] Despite the contrasting results, the evidence implies that there is a significant role for real variables in the analysis of money demand in hyperinflations. The typical measures of welfare loss from inflation that use the Cagan money demand specification will overlook the impact of real distortions from nominal disturbances and thus underestimate their true cost.

The remainder of the paper is as follows. Section 2 develops the model and derives the implications. Section 3 describes the empirical methodology and the data. Section 4 presents the estimation results, with emphasis on the comparison of the two hyperinflationary episodes. Section 5 offers conclusions.

2. The model

The theoretical framework attempts to go one step further than the Cagan money demand model by studying a general equilibrium dynamic optimization problem for consumption, capital accumulation, and real money holdings. The model is designed specifically to incorporate real economic activities into the

[6] The contrasting results are consistent with institutional facts that offer explanations for the distinct results.

analysis of hyperinflationary dynamics. We develop a simple two-sector model of consumption and capital goods that allows the relative price between these goods to fluctuate.

We introduce money into the model economy using a generalized cash-in-advance (CIA) constraint. From Cassella and Feinstein (1990, p. 2) and Campbell and Tullock (1954, p. 243), the evidence suggests that even during hyperinflation, consumers employed domestic currency (cash) for transactions. Also, we assume that capital goods purchases are free of the liquidity constraint.[7] Evidence in Garber (1982) and Holtfrerich (1986) describes how in the German hyperinflation investment loan contracts were denominated in units of foreign currency, to insulate the lender from the substantial depreciation of the domestic currency. In addition, the German government offered subsidies and direct loans to support capital investment, further releasing capital goods from the domestic cash constraint. This differential cash-in-advance constraint is essential for generating the nonsuperneutrality result that money growth affects the real sector through the relative price measure.[8]

We depart from the conventional cash-in-advance model (e.g., Lucas, 1980; Stockman, 1981) by allowing velocity to vary in order to capture a major feature of hyperinflations, that is, the dramatic difference between consumption and transactions frequencies. Hypothetically, velocity movements during hyperinflations can be perceived as involving two components: the first one is the traditional concept that responds endogenously to alterations to expected inflation/money supply growth, the second element reflects autonomous shifts due to, for example, perceived instability of the social/political structure. We refer to this second component of velocity as an autonomous transactions frequency shock. More specifically, it is used to capture that component of velocity *not* directly caused by monetary expansion.[9] The political sources of this shock are especially relevant for our case studies of hyperinflation, given that Germany was in the midst of post-World War I reconstruction and reparations and that China was experiencing ongoing civil war. The traditional concept of velocity can be included into our analysis so that velocity will respond to money growth. This, however, will only reinforce the nonsuperneutrality result mentioned above. Thus, for the sake of simplicity, our analysis focuses on the autonomous transaction frequency shock.

[7] If a fraction of capital goods is subject to a CIA constraint, the main results will still hold.

[8] In a one-sector model where consumption and capial goods are homogeneous, money is superneutral when the capital good is not subject to the cash-in-advance constraint (see Stockman, 1981). The superneutrality result fails to hold when we allow for a fluctuating relative price of capital to consumption goods in a two-sector framework.

[9] The empirical identification specified in (13) is consistent with this concept of velocity shock.

Let M and P represent the (beginning-of-period) nominal money stock and the price level (in units of consumption goods), respectively. Let $m_t = M_t/P_t$ denote (beginning-of-period) real money balances and v refer to the aforementioned autonomous movements in transactions frequency. We define c as the nonstorable final consumption good. Because the capital good is not subject to the cash-in-advance constraint, it is convenient to define the transactions frequency measure as the ratio of consumption (rather than total output) to real money balances: $c_t = v_t m_t$.

We outline below the simple two-sector model that provides the framework for the analysis. The consumption good is produced using an intermediate capital good, x, owned by individual consumers. Let y denote the representative firm's final good production and q represent the relative price of the capital to consumption good. Therefore, the firm's maximization problem is

$$\max_{x_t} \phi_t = y_t - q_t x_t = a_t x_t^{\alpha} - q_t x_t, \tag{1}$$

where a is a positive Harrod-neutral technological factor and $(1 - \alpha)$ measures the degree of diminishing returns of the production technology. The first-order condition implies:

$$q_t = \alpha a_t x_t^{\alpha - 1}. \tag{2}$$

Without loss of generality, we assume that the consumer-supplied intermediate capital good is produced through a simple storage technology analogous to McCallum (1983). Let z denote the (beginning-of-period) capital stock. We specify the storage technology as

$$z_{t+1} = \gamma(z_t - x_t), \tag{3}$$

where $\gamma > 0$ is the (net-of-depreciation) growth factor.[10] This equation can be thought of as the production function of the capital good with the current capital goods resource constraint imposed implicitly.[11]

Define the inflation rate from period t to $t + 1$ as π_{t+1} [$= (P_{t+1}/P_t) - 1$]. Real balances at the end of period t, M_{t+1}/P_t, can be expressed as $(1 + \pi_{t+1})m_{t+1}$. Given the redistribution of the firm's profit, ϕ, and the lump-sum real money transfer from the government, τ, the representative consumer faces the following budget contraint:[12]

$$(1 + \pi_{t+1}) m_{t+1} = q_t x_t + \phi_t - m_t(v_t - 1) + \tau_t. \tag{4}$$

[10] For simplification, we only focus on cases in which the nonnegativity constraint on z, $z_t \geq 0 \;\forall\; t$, is not binding.

[11] We thank an anonymous referee for suggesting this interpretation.

[12] We have applied $c_t = v_t m_t$ to the derivation of the following equation.

We assume that the consumer's utility is time-additive with a constant discount factor β and with a stationary, logarithmic instantaneous utility function. The consumer's optimization problem is then

$$\max_{x_t, z_{t+1}, m_{t+1}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \ln c_t = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(v_t m_t), \tag{5}$$

subject to (3) and (4).

Let $\lambda_{1,t}$ and $\lambda_{2,t}$ be the Lagrange multipliers associated with (3) and (4), respectively. The first-order conditions of the consumer's problem are

$$-\gamma\lambda_{1,t} + \lambda_{2,t} q_t = 0, \tag{6}$$

$$-\lambda_{1t} + \gamma\lambda_{1,t+1} = 0, \tag{7}$$

$$\mathrm{E}_{t-1} \frac{1}{m_{t+1}} \{\beta^{t+1} - m_{t+1}[(1 + \pi_{t+1})\lambda_{2,t} + (v_{t+1} - 1)\lambda_{2,t+1}]\} = 0. \tag{8}$$

Notably, Eq. (8) ensures intertemporal consumption efficiency and no arbitrage opportunities between the two assets, capital and money, neither intertemporally nor contemporaneously.

To close the model, we specify the government's money supply process as $\tau_t = \mu_{t+1} m_t$. Under money market equilibrium, it is useful to note that $m_{t+1}/m_t = (1 + \mu_{t+1})/(1 + \pi_{t+1})$. The goods market clearing condition ensures that $c_t = y_t$. Utilizing (2), (6)–(8), equilibrium conditions, and the money supply process, we can derive the following relationships (see Appendix):

$$\frac{m_t}{q_t} = \frac{x_t}{\alpha v_t}, \tag{9a}$$

$$q_t = \alpha^\alpha a_t \left[\frac{v_t m_t}{q_t}\right]^{(\alpha - 1)}, \tag{9b}$$

$$\beta\gamma = \mathrm{E}_{t-1}\left[\frac{\left(\frac{m_t}{q_t}\right)(1 + \mu_{t+1})\gamma + (v_{t+1} - 1)\left(\frac{m_{t+1}}{q_{t-1}}\right)}{\left(\frac{m_{t-1}}{q_{t-1}}\right)(1 + \mu_t)\gamma + (v_t - 1)\left(\frac{m_t}{q_t}\right)}\right]. \tag{9c}$$

Eqs. (9a) and (9b) establish linear relationships between the money demand to relative price ratio (m/q) and (i) the capital good (x) and (ii) the relative price ratio (q). Eq. (9c) specifies that there is no arbitrage intertemporally implying that the marginal benefit of consuming today is equal to the marginal opportunity cost of not investing in the capital good. The key insight from this equation, though, is the independence of the money demand to relative price ratio from the multiplicative technological factor (a). This independence allows us to identify the empirical model described in the next section below.[13] Thus, the comparative static results from the model center on the analysis of this money demand to relative price ratio rather than the typical money demand variable (m).

By characterizing the steady-state equilibrium, the model predicts that an increase in the money growth rate will increase the relative price of the capital good.[14] Intuitively, higher money growth will increase the opportunity cost of holding money, thereby reducing real balances and limiting the demand for the cash-constrained consumption good. The capital good serves as a store of value; thus, in the midst of higher money growth, the demand for the capital good relative to the consumption good increases. As a consequence, the relative price ratio of the capital to consumption good rises. Further, the resulting decline in real balances combined with the increase in the relative price leads unambiguously to a lower money demand to relative price ratio.

An increase in the transactions frequency has two opposing effects on the relative price. On the one hand, given a fixed consumption frequency (normalized to one per period), an increase in the transactions frequency allows agents to facilitate higher consumption purchases for a given level of real cash balances. Hence, the increased relative demand for the consumption good leads to a decrease in the relative price of capital to consumption good. This direct effect can be seen from Eq. (9b) in which a higher transactions frequency (v) lowers the marginal valuation of capital in units of consumption (relative price). On the other hand, a rise in the transactions frequency affects the store-of-value role for capital. Eq. (9a) combined with the intertemporal no-arbitrage Eq. (9c), suggests that increased transactions frequency leads to a decrease in the marginal benefit of the capital goods investment as a store-of-value. Specifically, a given level of consumption requires less real cash balances, reducing the total cost of money holding and thereby diminishing the store-of-value role of capital. As a consequence, consumers hold a lower level of capital and thus supply less to the consumption good producers, which increases the return to capital under diminishing returns ($\alpha < 1$). This implies a higher relative price of capital to consumption good. Therefore, the effects of the transactions frequency on the relative price measure involves two contrasting forces: the direct liquidity

[13] We transform the money demand measure used in the theoretical model into the money demand to relative price ratio because the independence result mentioned in the text enables us to separate the technological (output) shock from shocks to money supply growth and transactions frequency in the empirical model.

[14] In the Appendix, we derive the model implications that provide support for the intuitive descriptions in the text. Rather than explore the technical aspects of the model we concentrate on highlighting the testable implications in the discussion.

constraint effect and the positive store-of-value effect. We are unable to determine which effect dominates. Nevertheless, a higher transactions frequency reduces the demand for real balances because fewer real balances are necessary to support a given level of consumption. This effect on real money demand overpowers the ambiguous net effect of transactions frequency on the relative price ratio. Thus, the net effect of an increase in transactions frequency on the money demand–relative price ratio is unambiguously negative.

Finally, Eq. (9b) indicates that a technological improvement (an increase in a) raises the supply of final consumption goods given the same level of capital good input. Thus, the larger supply of consumption leads to a lower price of the consumption good, thereby raising the relative price of the capital to consumption good. Notably, the right-hand side of Eq. (9c) is independent of a, so that the multiplicative technology shock has no effect on the money demand–relative price ratio. Thus, any multiplicative Harrod-neutral technological disturbance will not affect the money demand–relative price ratio. These two results imply that the positive effect of a on money demand exactly offsets its positive effect on the relative price ratio, leaving the money demand–relative price ratio unchanged.

3. Empirical methods and the data

The theoretical model derived in Section 2 provides implications on the long-run relationships between the variables of interest and the fundamental disturbances. Applying the structural vector autoregression (VAR) method developed by Blanchard and Quah (1989), King et al. (1991), and Ahmed et al. (1993), we utilize these long-run relationships to identify the system and interpret the shocks.[15] By imposing only long-run restrictions based upon the theoretical model, we are able to retrieve the structural disturbances while allowing the data to determine the short-run dynamics.

3.1. Empirical methodology

Proponents of identification via long-run restrictions offer it as an alternative to methods that impose restrictions on the short-run dynamics. Economists generally feel more confident in their knowledge of long-run relationships than their understanding of short-run interactions, so that constraints on the long-run responses appear less objectionable and more economically justifiable.

[15] We emphasize that the order of the system is based on implications from a theoretical model in contrast to Sims (1980) and in our case only impacts in the long run. Unlike Bernanke (1986), we provide a direct interpretation of the structural shocks.

Let ξ_t represent the (3×1) vector of structural disturbances, X_t represent the (3×1) vector of variables in stationary form, and $C(L)$ represent a nonsingular matrix of moving average coefficients, where L is the lag operator. The structural model is then

$$X_t = C(L)\,\xi_t, \qquad \mathrm{var}(\xi_t) = \Sigma. \tag{9}$$

The variance–covariance matrix (Σ) is diagonal, provided all fundamental shocks are orthogonal. The long-run moving average matrix, $C(1)$, is assumed lower triangular. Then, the structural model can be rewritten as

$$A(L)\,DX_t = BX_{t-1} + \xi_t, \tag{10}$$

where the first-difference of $A(L)$ is $C^{-1}(L) - C^{-1}(1)L$, B is $-C^{-1}(1)$, and D denotes difference operator.

The estimated reduced form of the system is

$$F(L)\,DX_t = GX_{t-1} + u_t, \qquad \mathrm{var}(u_t) = \Omega, \tag{11}$$

where u_t are the reduced form errors. To link the reduced form to the structural form, we transform the above equation as

$$HG^{-1}F(L)\,DX_t = HX_{t-1} + HG^{-1}u_t, \tag{12}$$

where H is the inverse of the Cholesky factor of $[G^{-1}\hat{\Omega}(G^{-1})']$, and by construction $\mathrm{var}[HG^{-1}u_t] = \Sigma$. By comparing the reduced form with the structural form, the estimated long-run moving average matrix is then $-H^{-1}$. Notice that the Cholesky factor is unique up to the sign of the diagonal elements of the $C(1)$ matrix.

3.2. The data

We employ two hyperinflation data sets: Germany from January 1920 to July 1923 and China from January 1946 to March 1949. The German data are taken from Holtfrerich (1986), which is based upon *Statistisches Reichsamt*. For China, we employ data translated from *The Shanghai Price Index Collection Before and After the Civil War* (in Chinese).

In each country, a wholesale price index (WPI) measures prices for capital goods, whereas a cost-of-living index (CLI) measures prices for consumption goods. We take 1913/14 = 1.00 as the base year for the German price indexes, and for China the base year is 1937 = 1.00. We then compute the ratio of the wholesale to the cost-of-living index as the relative price measure. The price level is measured by the cost-of-living index.

The money supply measure (MS) for Germany is the monetary base, whereas for China we use official currencies and notes.[16] Both money stock variables are measured mid-month using a simple (geometric) moving average. The German money supply is in billions of marks, while the Chinese money supply is in billions of CNCs.[17] Money demand (MD) is therefore defined as the nominal money stock deflated by the cost-of-living index.

3.3. *Identification*

From the empirical methodology discussion above, we know that the identification method requires lower triangularity of the long-run moving average representation in addition to orthogonality of the structural shocks. The structural shocks are by definition orthogonal in our framework; to identify the system, we need to transform the variables so that there exists a long-run causal ordering that generates the lower triangularity of the $C(1)$ matrix. We use the theoretical results above to justify the long-run causal ordering used in the empirical identification.

In our case, the design of the long-run causal ordering involves two primary assumptions. In the first one, we assume that the first variable in the system is predetermined in the long run. Secondly, we require that the second variable be independent of the third shock.

For the first assumption, we assume that money growth is the first variable in the long-run ordering. We justify this assumption by noting that it is possible to place the money growth measure first, second, or third in the 3×3 structural estimation. In principle, the shock associated with the variable placed first in the long-run ordering has a greater likelihood of explaining the variations in the data than if the variable is placed later in the ordering. The main purpose of the paper is to investigate the role of macroeconomic factors, *in addition to monetary expansion*, in explaining movements in money demand and the relative price of the capital good in hyperinflations.[18] By allowing money growth to be first in the long-run causal ordering, we impose a restriction that, in its empirical content, is consistent with existing evidence supporting the growth rate of

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[16] Holtfrerich (1986) notes that the monetary base best captures the money supply measure because the reserves held in the Reichsbank were a substantial proportion of the total stock. Such a figure is unavailable in China, but banking reserves in China were less essential to the monetary system.

[17] We make an adjustment to the money supply data for the revaluation of the Chinese currency, the failed monetary reform, in August 1948 to keep the series consistent.

[18] In a hyperinflation, we believe that the monetary and fiscal authorities are not independent (see Cagan, 1956; Tanzi, 1977). Thus, we refer to the money growth shock as a combined fiscal and monetary shock. As empirical support for this interpretation of the shock, we find that the effect of this shock on the money demand is negative.

money as the most important variable for explaining money demand behavior during hyperinflations.[19] Therefore, to explore the importance of *nonmonetary* factors, it would be most conservative if we rank the monetary variables first, and this is our benchmark case in the paper. By placing monetary growth first, we ignore the potential feedback effect from the macroeconomy to the decisions by the monetary authority. Nevertheless, this would only underestimate the importance of other real macroeconomic factors, as compared to monetary disturbances.[20] In doing this, we can minimize Type I error with regard to a null hypothesis that real economic variables do not help account for movements in money demand and the relative price of capital goods during hyperinflations. We relegate the discussion regarding alternative orderings to the end of Section 4.

For the second assumption, we have chosen the next two variables to be the money demand–relative price ratio followed by the relative price ratio. The unusual data transformation, the money demand–relative price ratio, has been selected based upon its independence from the multiplicative Harrod-neutral productivity shock derived from the theory. By placing this transformed variable second in the ordering, we allow it to respond to the money growth shock as well as to its own shock. From the theory, its own shock may be interpreted as the transactions frequency shock that is discussed in the theory section. The third variable, the relative price ratio, is associated with the productivity shock. We can identify this shock because from theory it has no effect on the money demand–relative price ratio nor on the predetermined money supply growth process. Therefore, in the structural VAR system below, we impose the long-run ordering starting with the money growth rate, followed by the money demand–relative price ratio, and then the relative price ratio, in conjunction with the related shocks.

To implement the empirical study, we transform the raw data to obtain the following series:

MGR = money supply growth rate, $D\ln(MS)$,

MRP = money demand − relative price ratio, $\ln(MD) - \ln(WPI/CLI)$,

RP = relative price, $\ln(WPI/CLI)$.

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[19] For example, see Cagan (1956), Holtfrerich (1986), and Webb (1989).

[20] Using the structural VAR approach, it is well-known that one may not completely identify a shock. The associated shocks (for example, 'aggregate supply' shocks associated with output in Blanchard and Quah, 1989) could pick up the effects of a persistent shock in 'aggregate demand' (associated with unemployment) that are otherwise assumed to have short-lived shocks. In estimations, we desire identifying restrictions that inflict the least damage to the underlying basis of the model.

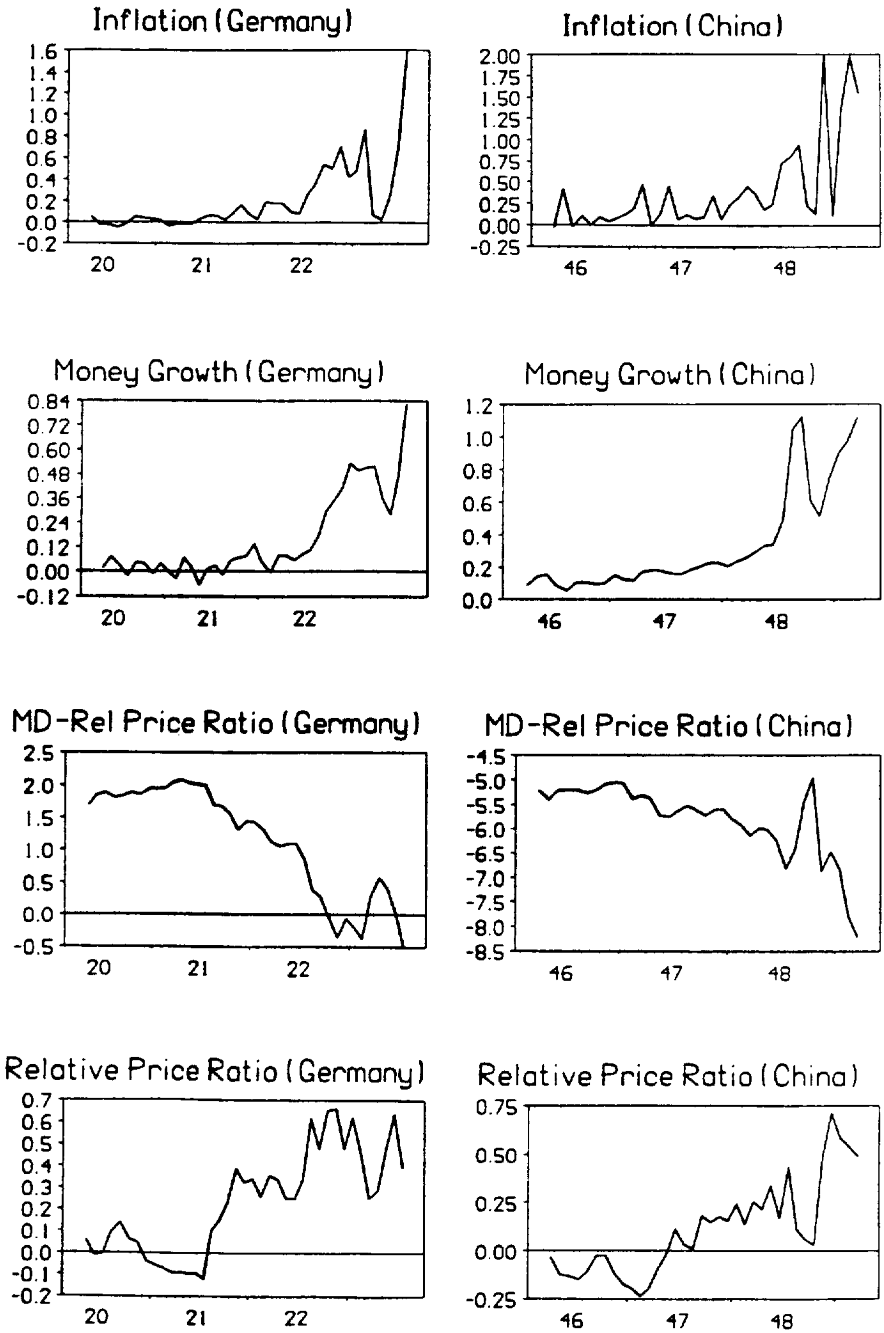

Fig. 1. Graphical plots of the data: Germany and China.

Table 1
Descriptive statistics

Germany	1/1920–6/1922	7/1922–7/1923	1/1920–7/1923
π (per month)	0.06 (0.013)	0.52 (0.111)	0.21 (0.057)
μ (per month)	0.04 (0.0009)	0.41 (0.051)	0.16 (0.033)
$\ln(m/q)$	1.69 (0.064)	0.10 (0.111)	1.17 (0.131)
$\ln(q)$	0.11 (0.032)	0.49 (0.039)	0.23 (0.038)
China	**1/1946–8/1948**	**9/1948–3/1949**	**1/1946–3/1949**
π (per month)	0.26 (0.046)	1.06 (0.328)	0.41 (0.087)
μ (per month)	0.22 (0.034)	0.86 (0.091)	0.34 (0.053)
$\ln(m/q)$	− 5.59 (− 0.079)	− 6.65 (− 0.440)	− 5.79 (− 0.122)
$\ln(q)$	0.04 (0.032)	0.42 (0.099)	0.11 (0.039)

We define the variable MRP as the ratio of money demand to relative price in order to identify the model using long-run restrictions. In the estimation, we employ first differences of the three transformed variables and denote them by $DMGR$, $DMRP$, and DRP, respectively. We summarize the univariate statistics for both countries in Table 1.[21]

In summary, apart from the lag dynamics, the structural VAR system in its moving average form can be written as

$$\begin{bmatrix} DMGR \\ DMRP \\ DRP \end{bmatrix} = \begin{bmatrix} \mu_0 \\ m_0 \\ a_0 \end{bmatrix} + \begin{bmatrix} c_{11} & 0 & 0 \\ c_{21} & c_{22} & 0 \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} SM \\ ST \\ SP \end{bmatrix}, \tag{13}$$

where μ_0, m_0, and q_0 capture constant drifts for the levels of the three transformed variables. We have explained the justification for the lower triangular structure of $C(1)$ in the beginning of Section 4, based on the long-run monetary growth model.

To ensure that the structural shocks can be retrieved uniquely from the estimation, we must fix the sign of the diagonal elements of the $C(1)$ matrix by theory. For example, the money growth shock affects money growth positively, the transactions frequency shock affects the money demand–relative price ratio negatively, and the productivity shock impacts the relative price ratio positively. For ease of interpretation of the impulse responses, it is convenient to redefine

[21] In Fig. 1, we present graphical display of the data series. We present a plot of the inflation rate, INF, followed by plots of money growth, MGR, real money demand–relative price ratio, MRP, and the relative price, RP, for Germany and China.

the second shock to be the inverse of the transaction frequency shock, which we will call the transactions interval shock, by normalizing c_{22} to be positive. Thus, the fundamental disturbances are constructed such that all diagonal elements of $C(1)$ are positive, specifically money growth (SM), transactions interval (ST), and productivity (SP) shocks.

The theoretical results obtained in Section 3 above imply the following signs for the (nonzero) off-diagonal elements. These implications are not required for the identification of the model, and therefore are empirically testable from the estimations. Higher money growth implies a decrease in the money demand–relative price ratio, so that $c_{21} < 0$. The relative price ratio increases in response to a positive money growth shock, implying $c_{31} > 0$. A positive transactions interval shock (a negative transactions frequency shock) involves two counteracting effects on the relative price, so the model does not offer an unambiguous implication. Thus, we cannot offer an unambiguous prediction for the sign of c_{32}.

4. Empirical results

Our analysis deviates from traditional investigations of money demand in hyperinflation that rely on partial equilibrium frameworks. Prior studies, most notably Cagan (1956), Frenkel (1977), and Abel et al. (1979), focus on data measures of expected inflation without addressing the role of any real macroeconomic aggregate. This is not surprising given that their theoretical paradigm concentrates only on expectations of aggregate price changes and that real measures are usually unavailable at high enough frequency for estimation. Our empirical method, however, allows us to identify both nominal and real disturbances. Therefore, we can quantitatively assess the important sources of money demand fluctuations and the dynamic interactions between the real and the nominal variables.

In recent time-series empirical work, researchers often address the issue of data stationarity by employing various statistical tests indicating integration, cointegration, or nonintegration of the time series. These statistical tests often require numerous data points in order to generate test statistics with the desired properties. In our empirical work, we perform some analysis of the stationarity properties of the data indicating that the relevant series are integrated of order one.[22] However, we will not emphasize these statistics because our sample of less than forty observations is insufficient for the test procedures.

[22] Results are available on request.

Using the cointegration test established in Engle and Granger (1987) and the critical values reported in Engle and Yoo (1987), we find no evidence of cointegration among the variables, money supply growth, money demand–relative price ratio, and relative price.[23] Therefore, the variables can be estimated by the VAR method described above because the moving-average coefficient matrix is nonsingular. Since all the structural shocks are fundamental, the covariance matrix is diagonal when the long-run restrictions are imposed. The evidence of no cointegration implies that there are three stochastic trends in the VAR system; also, the shocks that drive the system in the long-run dynamics are the same as those propagating the short-run dynamics. Our estimation procedure allows the data to determine short-run dynamics and identifies the model using weaker economic assumptions than alternative methods.

The Akaike information criterion suggests that the lag length for the VAR is two and three, respectively, for Germany and China. Overall, the estimated long-run responses conform with the theoretical predictions. For Germany, the estimated long-run responses of first differences of money growth, money demand–relative price ratio, and relative price to a unit shock in money supply growth are 0.84 (c_{11}), -2.4 (c_{21}), and 0.22 (c_{31}), respectively. Similarly, the estimated long-run responses for the Chinese case are 0.22, -0.21, and -0.07, respectively. For both cases, the negative response of the money demand–relative price ratio to the money growth shock is consistent with the theoretical prediction. In the German case, the positive effect of money growth shocks on the relative price ratio is supported. Such a long-run effect in the Chinese case appears negligible.

In response to the transactions interval shock, the money demand–relative price ratio and the relative price reflect a permanent change of 0.31 (c_{22}) and -0.24 (c_{32}), respectively, in Germany's case, while the corresponding estimates for China are 0.113 and -0.02. The long-run negative response of the relative price ratio to the transactions interval shock indicates that the store-of-value effect on the capital good overpowers the liquidity constraint effect on the consumption good in response to the shock. Finally, a unit shock in productivity results in an increase of 0.79 and 0.72 in the relative price ratios for Germany and China, respectively.

Since our main interest concerning the hyperinflationary phenomena is in the short-run dynamics, the above long-run responses are used only to check the consistency of the estimation with the theory. The remainder of the text will focus on the impulse response functions (IRFs) and the variance decompositions (VDCs) of money demand and the relative price. The IRFs show the estimated response of each variable to a one-standard-deviation impulse in the fundamental shock, while the VDCs account for the percentage of the forecast error

[23] Results are available on request.

variance of each variable explained by the particular shock.[24] Ultimately, we are interested in the responses of money demand and relative price individually to the hypothesized shocks. To obtain the desired responses for money demand, we combine the estimates from both the money demand–relative price ratio and the relative price ratio.

4.1. The German case

We plot the impulse responses for Germany in Fig. 2a. Concentrating on the reactions of money demand to shocks, which are displayed in the top two panels of the figure, we find that the money growth shock reduces money demand significantly in the short-run, consistent with the predictions of the theoretical model.[25] We also examine the effect of the disturbance on the level of money demand. The graph of this effect is hump-shaped, but persistently negative; specifically, money demand's reaction to the shock reaches a trough in the third month after the shock and then retraces to a smaller negative number. In contrast, money demand appears to have no significant reaction to the transactions interval shock until the fourth month after the disturbance. The cumulative reaction to the shock is positive, consistent with the theoretical model implications, but small, remaining so throughout the rest of the forecast horizon. The productivity shock has a positive and significant impact on money demand for the first two periods after the shock, and the level response is hump-shaped and also persistently positive in accord with theory.

In Table 2, we present the results of the variance decompositions for selected forecast horizons (1, 3, and 24 months). On impact, the money growth shock accounts for about one-third of the forecast variance of the rate of change of money demand and the productivity shock accounts for approximately the other two-thirds. After the first month, the effect of the productivity shock diminishes to less than one-third while the money growth shock becomes more influential. Interestingly, throughout the entire forecast horizon, the transactions interval shock explains only an insignificant fraction of the forecast variance of money demand changes. Although the money growth shock is the predominant driving force in money demand behavior, the productivity shock explains a nontrivial portion of money demand.

We next examine the impulse responses of the relative price variable. Consistent with our model implication, shocks to money growth increase significantly

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[24] We generated results from an alternative specification and a different identification technique (available upon request). The results suggest that our main conclusion is robust to these alternatives.

[25] We compute simulated standard errors using 1,000 replications of the system. Following Shapiro and Watson (1988), we employ one-standard-error bands in the impulse responses to imply significance.

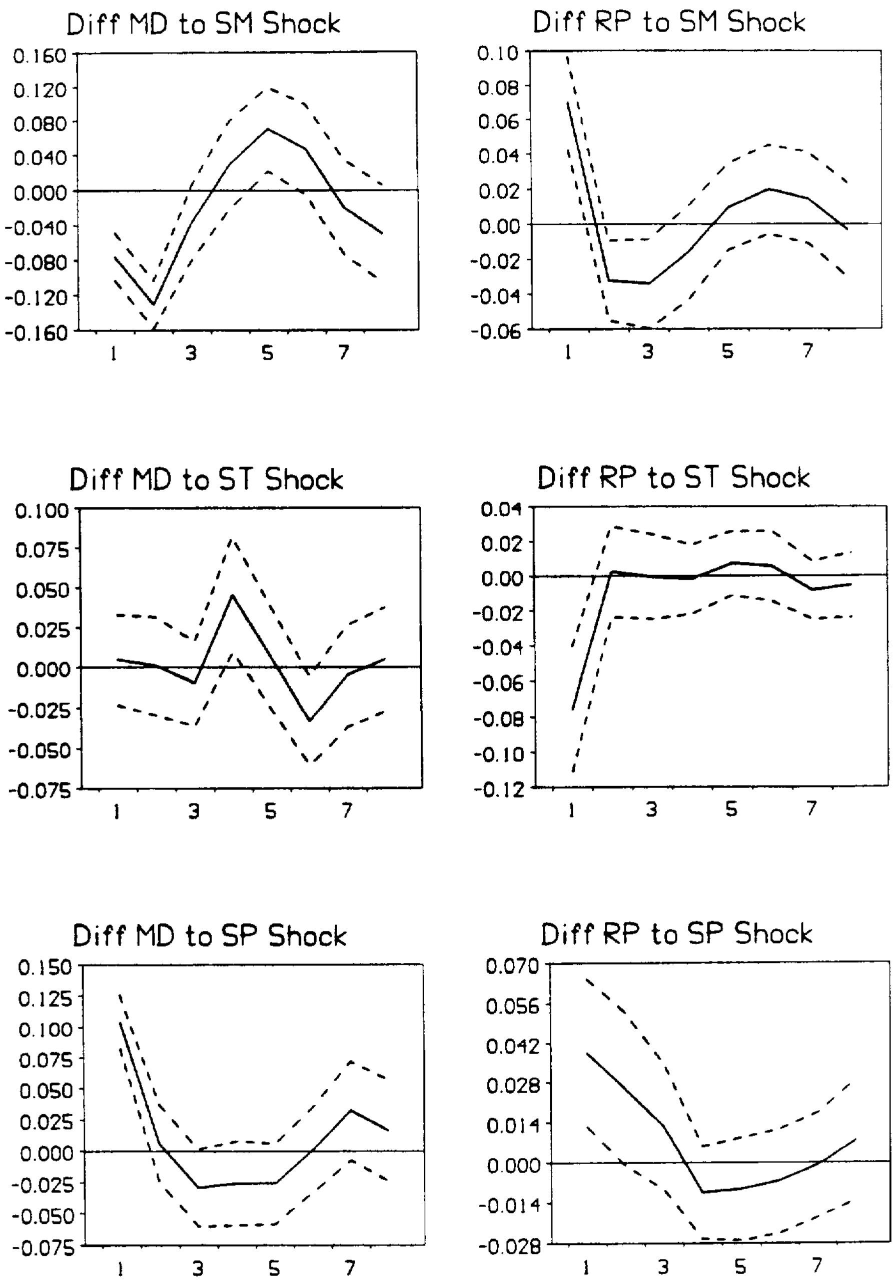

Fig. 2a. Impulse responses for Germany - *Differences* (*SM* = money supply growth rate, *ST* = transaction interval, *SP* = productivity (real), *MD* = money demand, *RP* = relative price ratio, *Diff* = first differences).

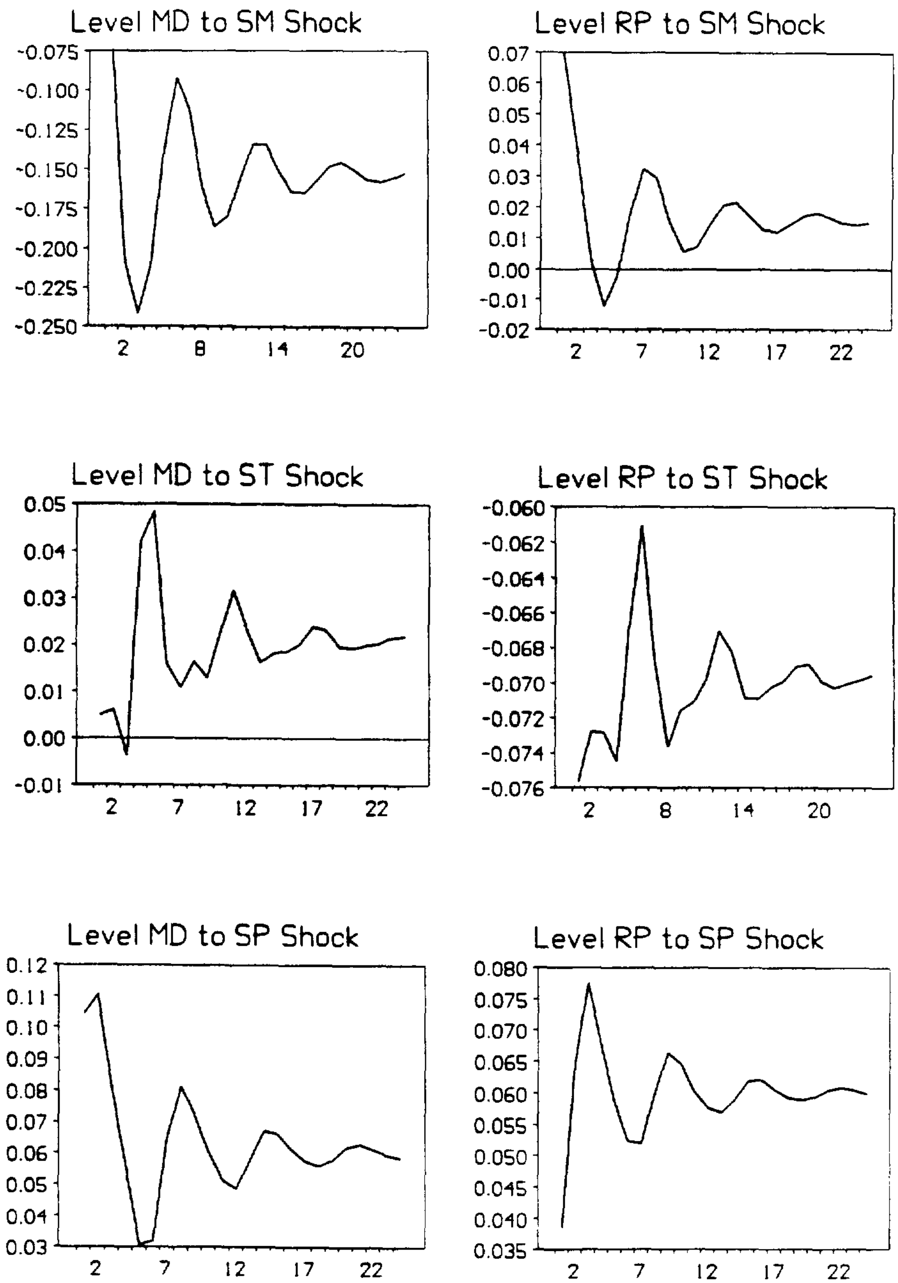

Fig. 2a. Impulse responses for Germany – *Levels* (SM = money supply growth rate, ST = transaction interval, SP = productivity (real), MD = money demand, RP = relative price ratio, *Diff* = first differences).

Table 2
Structural VAR variance decomposition

Variable	Money supply growth	One-standard-deviation shocks to transactions interval	Productivity
German hyperinflation: 1/1920–7/1923			
$\Delta\ln(q)$	40.45/46.77/49.60 (24.4)/(15.6)/(16.2)	47.16/37.76/34.55 (21.2)/(15.6)/(15.7)	12.38/15.46/15.84 (16.1)/(11.6)/(11.5)
$\Delta\ln(m)$	34.06/66.55/66.49 (19.7)/(15.4)/(16.2)	0.15/0.34/6.65 (7.9)/(7.1)/(10.0)	65.79/33.10/26.86 (21.3)/(14.1)/(12.6)
Chinese hyperinflation: 1/1946-3/1949			
$\Delta\ln(q)$	13.89/23.79/20.83 (14.2)/(9.0)/(18.0)	33.95/25.72/41.31 (22.3)/(13.5)/(13.2)	52.16/50.48/37.86 (23.9)/(14.6)/(14.9)
$\Delta\ln(m)$	75.66/44.27/35.76 (20.7)/(12.9)/(18.1)	3.66/42.07/44.35 (13.2)/(11.0)/(13.5)	20.67/13.66/19.88 (19.6)/(11.8)/(15.5)

Statistics reported are percent of variance in the rate of change of the relative price, $\Delta\ln(m)$, in response to each shock at 1/3/24 month forecasting horizon, respectively. Simulated standard errors from 1,000 replications are reported in parentheses.

the relative price ratio, although the cumulative response appears short-lived. Changes in the transactions interval lead to a significant negative impact effect on the relative price, indicating again that the store-of-value role of capital goods is crucial in driving relative price movements. Finally, in accord with theory, the productivity shock has a significant positive effect on the relative price ratio on impact, an effect that appears persistent from the cumulative impulse response graph.

More than 40 percent of the forecast error variance of the rate of change in the relative price ratio are explained by the money growth shock, suggesting that the nonsuperneutrality of money through the relative price variable should not be ignored. The transactions interval shock accounts for 47 percent of the variance of the rate of change in the relative price ratio in the first month, declining to 35 percent at the 24-month horizon. About 15 percent of the relative price forecast error variance is explained by the productivity shock.

4.2. *The Chinese case*

The impulse responses for China are presented in Fig. 2b. The negative effect of money growth shocks on money demand appears significant in the first month. Two months after the transactions interval disturbance, money demand displays a significant increase in response. The cumulative effect of the impulse is positive and appears persistent throughout the remaining forecast horizon. Money demand displays a significantly positive response on impact to the productivity shock, but such an effect appears to diminish over time. It is notable that the responses of the variables in levels are comparable to those in the German case.

The variance decompositions for China are also displayed in Table 2. In the first month of the forecast horizon, more than 75 percent of the forecast error variance of the money demand growth rate is explained by the money growth shock. Following the initial period, both money growth and transactions interval shocks explain about 40 percent of the forecast error variance of money demand. In contrast to the German case, the role of the transactions interval shock appears more influential for the behavior of money demand in China. Throughout the entire horizon, the productivity shock explains approximately 20 percent of money demand variance.

The relative price reaction to money growth shocks is positive but not very significant. In response to the transactions interval shock, the relative price ratio declines only for the short-run horizon. However, the productivity shock appears to play a very important role in driving the relative price. The level response achieves a peak five months out, and the cumulative effect is fairly persistent.

Money growth explains only about 20 percent of the forecast error variance of the rate of change in the relative price ratio, only about half the percentage explained by the money growth shock in the German case. Although the influence of the transactions interval shock increases and that of the productivity shock diminishes over the forecast horizon, each shock accounts for approximately 40 percent of the relative price variance. Whereas the transactions interval shock explains a comparable amount of the relative price variance, the productivity shock accounts for a much greater percentage than in the German case.

4.3. Discussion

In discussing the empirical results, it is useful to review and compare the historical experience of each hyperinflation episode. Prior to discussing the empirical results, we review and compare briefly the historical record of the German and Chinese hyperinflation to provide a backdrop for similarities and differences in their respective experiences.

For Germany over the period April 1920 to July 1923, the price level (measured by a cost-of-living index) increased by a factor of 3750 and inflation averaged 21 percent per month. The hyperinflation, however, exploded from a relatively moderate average rate of 6 percent per month in the period up to June 1922 to an average rate of 52 percent over the remainder. In China, the price level (in comparable measures) skyrocketed by a factor of 2.6 million

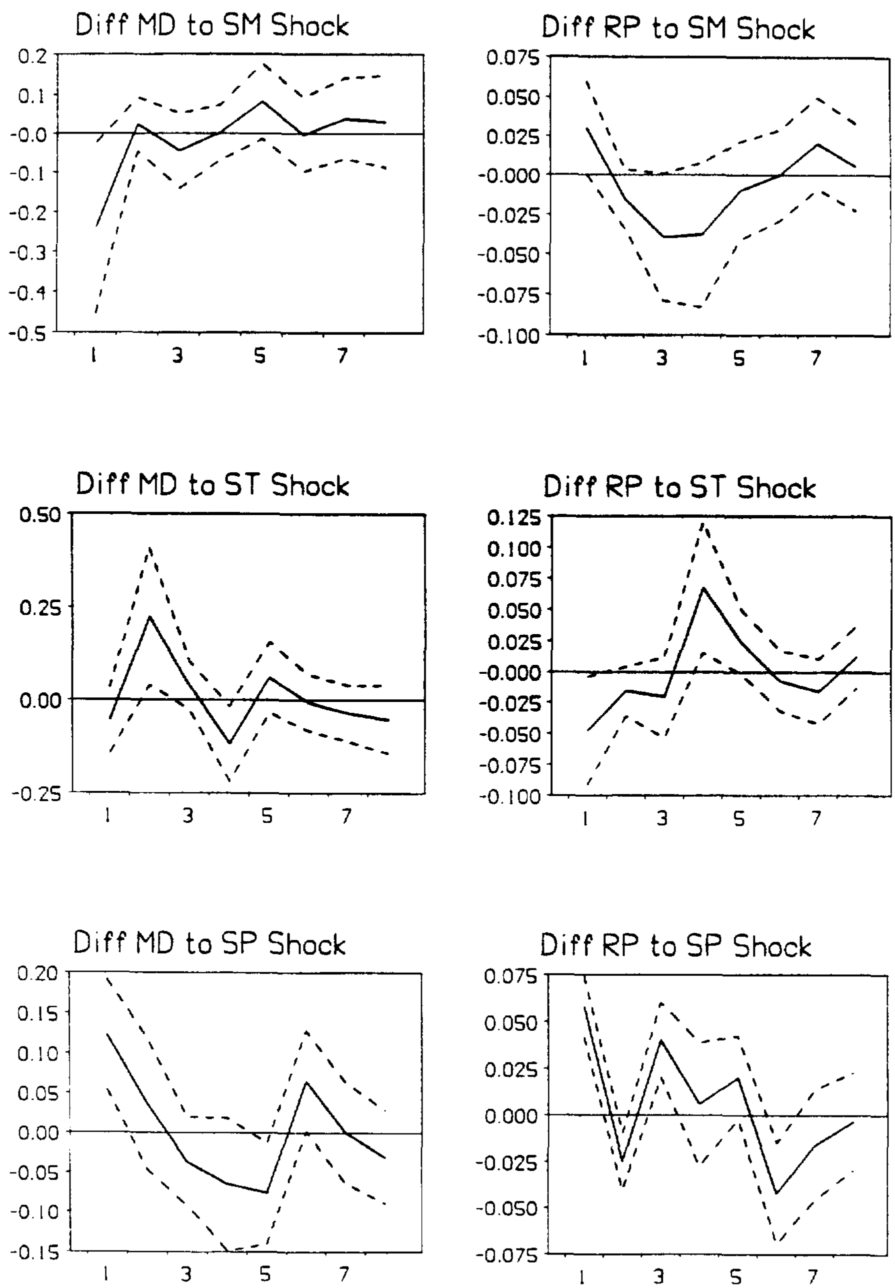

Fig. 2b. Impulse responses for China – *Differences* (SM = money supply growth rate, ST = transaction interval, SP = productivity (real), MD = money demand, RP = relative price ratio, *Diff* = first differences).

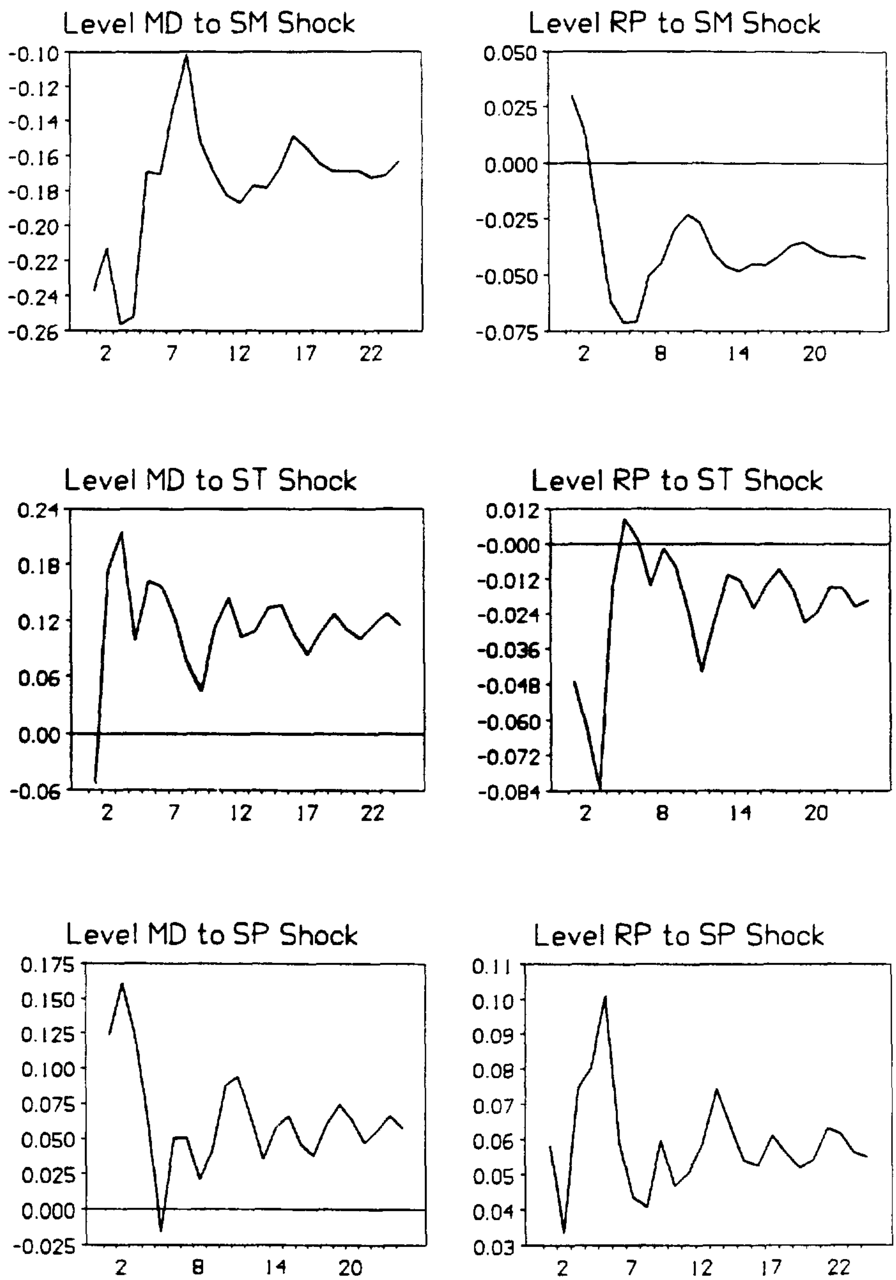

Fig. 2b. Impulse responses for China – *Levels* (SM = money supply growth rate, ST = transaction interval, SP = productivity (real), MD = money demand, RP = relative price ratio, *Diff* = first differences).

between March 1946 and March 1949. The inflation rate averaged 41 percent per month for the entire sample period, accelerating from a 26 percent monthly rate, before the failed reform of August 1948 to a rate of 106 percent per month afterward.

The political conditions within war-beleaguered Germany were in transition toward reconstruction following World War I. In contrast, China, though recovering from the Sino–Japanese War, faced widespread civil war with increasing political instabilities. The depreciations in the foreign exchange market reflected the relative risks of these two countries; the domestic real value of the Chinese yuan was 10 times greater than the international real value, whereas the real domestic value of the German mark was only 1.67 times the real international value.

In each country domestic money no longer served as a unit of account or store of value. Nonetheless, the Chinese monies still maintained the transactions role as media of exchange even in the most severe hyperinflationary periods. Money retained its role because there were strictly enforced regulations on the use of official currencies and Chinese are culturally law-abiding.[26] The lack of effective price controls in China also enhanced the use of money in transactions.[27] In contrast, Germany enforced extensive price controls that made barter more effective.[28] The enforced use of official money for transactions in China together with pessimistic expectations of any positive solution to the civil war made the velocity of money increase sharply. It is notable that the German fiscal/monetary reform was effective for stopping the hyperinflation. On the other hand, the Chinese central bank collapsed, and the data following the Communist takeover is relatively unavailable, so we cannot determine its end.

Comparing the empirical results from the two countries, we find that in both countries the transactions interval shock affects negatively the relative price ratio and accounts for about 40 percent of its forecast error variance. As mentioned above, this supports the importance of the store-of-value role of the capital good. However, there are noticeable differences between the countries in the responses of the relative price ratio to the money growth (and therefore, the productivity shock). First, the money growth shock has a larger impact on the relative price in Germany than in China. We interpret this finding as resulting from the effectiveness of price controls on certain German final goods prices

[26] Campbell and Tullock (1954, p. 244).

[27] Ibid., p. 244.

[28] Webb (1989) notes that in Germany certain public utility prices and rents were subject to price controls, so that the cost-of-living index did not adjust fully to inflationary increases. The wholesale price index averaged prices of imports and domestic production of mostly intermediate products and were freer to move with market forces.

versus the lack of enforced price ceilings in China. German price controls made consumption goods prices adjust only partially to money supply shocks relative to wholesale prices (our measure of capital goods prices) that move more freely to market forces. Money growth disturbances thus increase the relative price of capital goods to consumption goods to a greater extent than when consumption prices are unconstrained. In contrast, China lacked effective consumption price controls, and the productivity shock appears the main driving force for the behavior of the relative price. Nevertheless, in either case the effects of the money growth shock on the relative price ratio indicate a significant nonsuperneutrality during these hyperinflationary episodes.

As mentioned above, the Chinese official monies retained their dominant role as media of exchange despite the hyperinflation. The transactions interval shock is more important in the explaining money demand behavior in China than in Germany. In both cases, the transactions interval shock has a positive effect on money demand. However, the shock explains over 40 percent of the variance in the Chinese case, but only 6 percent of the variance in the German case. Continued pessimistic expectations and increased economic uncertainties due to the Chinese Civil War raised money velocity and shortened the transactions interval. Thus, we can anticipate the results that the transactions interval shock has greater explanatory power in the Chinese case. In both countries, productivity shocks are important for explaining money demand behavior. Thus, even in hyperinflationary episodes money demand behavior still responds to real economic activities.

The results discussed above correspond to the identification restrictions imposed by the long-run orderings of the variables. As mentioned before, the theoretical model implications suggest placing the money demand–relative price ratio ahead of the relative price variable in the structural VAR. In alternative orderings, we can maintain the relative placement of the money demand–relative price ratio before the relative price variable, but put the money growth variable either second or third in the ordering in contrast to the benchmark model. The money growth shock, in these alternative orderings, explains a smaller proportion of the movements in the relative price and money demand variables. A shock to money growth can account for from 11 to 48 percent of the variation in the rate of change of money demand in both countries at the 24-month forecast horizon; however, this nominal shock only accounts for 0 to 22 percent of the variation in the rate of change in the relative price.[29] On the other hand, the results for the transactions frequency and productivity shocks remain qualitatively unchanged yet quantitatively stronger when compared to evidence from the benchmark specification. According to our theory, the money

[29] Results are available upon request.

demand–relative price ratio should respond to innovations in the money growth rate. Placing money growth after the money demand–relative price ratio would imply that the innovations associated with the money demand–relative price ratio likely includes some components of the monetary shocks.[30] Thus, it is not surprising that monetary shocks account for less of the variation in the real variables in the alternative orderings.

5. Concluding remarks

Our study of two hyperinflationary instances emphasizes that shocks to nominal variables (e.g., money growth) can have important effects on real measures (the relative price) in such episodes. Also, we show that real (productivity) shocks may affect the dynamic behavior of money demand variables in hyperinflations. Both issues have not been addressed in prior research, mainly due to the lack of real aggregate measures at a monthly frequency. However, we are able to employ data, suggested by a theoretical model, that allow us to investigate issues, like, for example, whether hyperinflationary money demand shifts as a result of real shocks.

Our general equilibrium theoretical model generates results that provide an explicit framework for the empirical work. The model implications lead us to a structural empirical model using long-run restrictions to identify the sources of shocks to the system. Thus, we can give direct interpretations to the impulse responses and variance decompositions from the estimated structural VAR. We find empirical evidence suggesting that real (productivity) shocks can affect money demand significantly, as well as that nominal shocks affect real variables. Contrasting results from the two countries emphasize that there can be important differences in the behavior of the relative price and money demand in hyperinflationary episodes. We note that these differences in results are consistent with institutional differences found in descriptions of each hyperinflationary period.

In summary, both theory and estimation imply that dynamic interactions between nominal and real variables are significant in hyperinflationary periods. We believe that both nominal and real shocks are relevant for understanding the fluctuations of macroeconomic aggregates in these episodes. This finding generates relevant implications for monetary economics. For instance, the results suggest that real activity influences money demand behavior even during hyperinflationary episodes and that expected inflation cannot fully account for movements in real money balances, in contrast to Cagan's claim. Also, the

[30] Implicitly, this ordering assumes that any long-run variations in the money demand–relative price ratio must be due to nonmonetary factors, which is inconsistent with standard monetary models.

results indicate that monetary policy may have nonnegligible real effects on macroeconomic aggregates through relative price changes. In particular, shifts in relative prices can lead to production and consumption reallocations that are distortionary. In order to assess the severity of a hyperinflation accurately, one must take these nonneutral effects into account. Moreover, our results also relate to the typical welfare analysis of inflation in contemporary macroeconomics. Conventional studies measure the welfare loss from inflation in terms of the Harberger triangle of money demand specified as a stable function of expected inflation. The application of this partial equilibrium method overlooks the welfare loss from real distortions from relative price fluctuations arising from nominal disturbances, thus underestimating the true cost of hyperinflation.

Appendix

This appendix displays the algebraic manipulations that we perform to derive the results discussed in the text. Recall the government's money supply process ($\tau_t = \mu_{t+1} m_t$), money market equilibrium ($m_{t+1}/m_t = (1+\mu_{t+1})/(1+\pi_{t+1})$), and the goods market clearing condition ($c_t = y_t$). Using these relations, the generalized CIA constraint ($c_t = v_t m_t$), and the first-order conditions for both the firm (2) and the consumer (6)–(8), we find

$$\beta^{t+1} = \mathrm{E}_{t-1}\,\lambda_{1,t}\left[\frac{m_t}{q_t}(1+\mu_{t+1})\gamma + (v_{t+1}-1)\left(\frac{m_{t+1}}{q_{t+1}}\right)\right]. \tag{A.1}$$

By taking ratios, we obtain (9c). Also, we have (9a), (9b), and $x_t = (\alpha v_t m_t)/q_t$.

To simplify the analysis, we make the following transformations of the variables. First, we define five growth factors: θ^μ, θ^v, θ^a, θ^m, and θ^q (the money supply growth, the velocity growth, the technology growth, the money demand–relative price ratio growth, and the relative price ratio growth, respectively).

$$\theta_t^\mu = \frac{1+\mu_t}{1+\mu_{t-1}}, \quad \theta_t^v = \frac{v_t}{v_{t-1}} \approx \frac{v_t - 1}{v_{t-1}-1}, \quad \theta_t^a = \frac{a_t}{a_{t-1}}, \quad \theta_t^m = \frac{m_t/q_t}{m_{t-1}/q_{t-1}},$$

$$\theta_t^q = \frac{q_t}{q_{t-1}} = \theta_t^a\left[\frac{v_t}{v_{t-1}}\,\frac{m_t/q_t}{m_{t-1}/q_{t-1}}\right]^{\alpha-1} = \theta_t^a(\theta_t^v\theta_t^m)^{\alpha-1}. \tag{A.2}$$

Let $\psi_t = (v_t - 1)/(1+\mu_t)$. We can then rewrite the no-arbitrage equation (A.2) as

$$\beta\gamma = \mathrm{E}_{t-1}\left[\frac{\dfrac{m_t}{q_t}}{\dfrac{m_{t-1}}{q_{t-1}}}\right]\left[\frac{1+\mu_{t+1}}{1+\mu_t}\right]\left[\frac{\gamma + \left(\dfrac{v_{t+1}-1}{1+\mu_{t+1}}\right)\left(\dfrac{m_{t+1}/q_{t+1}}{m_t/q_t}\right)}{\gamma + \left(\dfrac{v_t - 1}{1+\mu_t}\right)\left(\dfrac{m_t/q_t}{m_{t-1}/q_{t-1}}\right)}\right] \tag{A.3}$$

$$= \mathrm{E}_{t-1}\, \theta_t^m\, \theta_{t+1}^\mu \left[\frac{\gamma + \psi_t \left(\dfrac{\theta_{t+1}^v}{\theta_{t+1}^\mu} \right) \theta_{t+1}^m}{\gamma + \psi_t\, \theta_t^m} \right]$$

$$\equiv \mathrm{E}_{t-1}\, \Delta_t .$$

Evaluate $\partial \Delta_t / \partial \cdot$ at $\theta_t^m = \theta^m$, $\theta_t^v = \theta^v$, $\theta_t^\mu = \theta^\mu$ (i.e., permanent effect), $\psi_t = \bar{\psi}$:

$$\frac{\partial \Delta_t}{\partial \theta^m} = [\gamma^2 \theta^\mu + \bar{\psi} \theta^m \theta^v (2 + \bar{\psi}\, \theta^m)]/(\gamma + \bar{\psi}\, \theta^m)^2 > 0, \tag{A.4}$$

$$\frac{\partial \Delta_t}{\partial \theta^\mu} = \frac{\gamma\, \theta^m}{\gamma + \bar{\psi}\, \theta^m} > 0, \tag{A.5}$$

$$\frac{\partial \Delta}{\partial \theta^v} = \frac{\bar{\psi} (\theta^m)^2}{\gamma + \bar{\psi} \theta^m} > 0. \tag{A.6}$$

Straightforward comparative-static analysis using (A.4) and (A.5) around the steady state generates the following implications:

$$\frac{\mathrm{d}\theta^m}{\mathrm{d}\theta^\mu} = - \frac{\partial \Delta / \partial \theta^\mu}{\partial \Delta / \partial \theta^m} = \frac{- \gamma\, \theta^m (\gamma + \bar{\psi}\, \theta^m)}{\gamma^2 \theta^\mu + \bar{\psi}\, \theta^m \theta^v (2 + \bar{\psi}\, \theta^m)} < 0, \tag{A.7}$$

suggesting that higher money growth implies decline in the money demand–relative price ratio.

Similarly, using (A.4) and (A.6), we have

$$\frac{\mathrm{d}\theta^m}{\mathrm{d}\theta^v} = - \frac{\partial \Delta / \partial \theta^v}{\partial \Delta / \partial \theta^m} = \frac{- \bar{\psi} (\theta^m)^2 (\gamma + \bar{\psi}\, \theta^m)}{\gamma^2 \theta^\mu + \bar{\psi}\, \theta^m \theta^v (2 + \bar{\psi}\, \theta^m)} < 0, \tag{A.8}$$

which implies that increased velocity lowers the money demand–relative price ratio.

We then derive the following relationships for the endogenous relative price:

$$\frac{\mathrm{d}\theta^q}{\mathrm{d}\theta^\mu} = - \frac{\theta^a (1 - \alpha)(\theta^m)^{\alpha - 2}}{(\theta^v)^{1-\alpha}} \frac{\mathrm{d}\theta^m}{\mathrm{d}\theta^\mu} > 0, \tag{A.9}$$

$$\frac{\mathrm{d}\theta^q}{\mathrm{d}\theta^v} = - (1 - \alpha) \frac{\theta^q}{\theta^v} \left[\frac{\gamma^2 \theta^\mu + \bar{\psi} \theta^m \theta^v (2 - \gamma)}{\gamma^2 \theta^\mu + \bar{\psi} \theta^m \theta^v (2 + \bar{\psi} \theta_t^m)} \right] \gtrless 0. \tag{A.10}$$

Thus, relative price increases with the money supply, but velocity shocks have ambiguous effects on the relative price.

From these relationships we get support for the implications discussed in the text.

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