
A NORMATIVE ANALYSIS OF HOUSING-RELATED TAX POLICY IN A GENERAL EQUILIBRIUM MODEL OF HOUSING QUALITY AND PRICES

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Abstract

We evaluate an array of housing-related tax policies in a general equilibrium model with endogenous housing quality and prices. The local government facilitates the provision of local public amenities, financed by an array of housing-related taxes, including a developer gross revenue tax, a property tax, a land tax, and a development license fee. In a competitive spatial equilibrium, all households optimize and reach the same utility, all monopolistically competitive developers optimize and receive zero profit, and both housing and land markets clear. We examine the effects of various tax policies on housing quality, housing prices, land rent, as well as the population and housing density. By evaluating quantitatively the welfare of the local economy, we establish a globally optimal tax scheme in the housing market: complete elimination of the property tax and imposition of a lower gross revenue tax (possibly fully eliminated) than both development and land taxes.

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1. Introduction

The study of housing markets and local public goods has long been a central issue in public economics and regional science. In addition to the quantity of the housing stock, housing quality has played an important role in influencing consumer demand, housing and land prices, and the associated urban structure. Despite the rapidly growing attention paid to housing research over the past two decades, the literature remains silent in a complete assessment of housing-related tax policies. Our paper attempts to fill this gap by developing a general equilibrium model of housing quality and prices with an active local government providing local public amenities financed by an array of housing-related taxes. The conclusions obtained herein may be useful for not only economists but also housing policymakers (wonks).¹

Specifically, we develop a general equilibrium model of housing where housing quality, housing density, and quality-augmented housing prices are all endogenously determined. The main features of the basic structure are summarized as follows.

- The model incorporates two crucial imperfections originated in housing markets: local housing congestions and imperfect housing markets with locally monopolistic developers.
- The provision and finance of local public amenities are important forces influencing housing quality and housing prices.
- The explicit spatial structure of the economy is modeled and the framework is particularly designed to quantitatively assess the effects of an array of housing-related taxes—a property tax, a developer gross revenue tax, a land tax, and a development tax—on the spatial distribution of housing-related variables, the urban structure, as well as the welfare of the local economy.

Concerning housing quality, we focus exclusively on the exterior and interior features of the dwelling as well as the congestion aspect of the local environment. While all households reach the same utility in locational equilibrium, all developers receive zero profit under free entry in a monopolistic competition framework. In competitive spatial equilibrium, the land rent, the population density, and the size of the city are also pinned down. The incorporation of a more comprehensive menu of local public finance in a general equilibrium setup of with endogenously determined housing quality and prices is the main contribution to the standard urban housing literature such as the models delineated in Fujita (1989, Chapters 2 and 6), Henderson (1985, Chapter 2), and those to be reviewed at the end of this section.

Since housing quality is an important ingredient of our model economy, it may be informative to provide a brief discussion on the measure of housing

¹For a study of housing markets and housing policy, the reader is referred to Green and Malpezzi (2000).

quality in the real world. Such measures include the quality of the lot and the existing dwelling (including the vintage and the exterior/interior features of the dwelling), the spending on maintenance, repairs and improvements, as well as the quality of public/private amenities and the local environment. The American Housing Survey (1997, 1999, 2001, 2003, and 2005) indicates that there exist upward trends in housing quality, housing values (measured by purchase prices), and monthly housing costs (including mortgages and maintenance costs). Moreover, owner occupied newer units (4 years or less) in Metropolitan Statistical Area (MSA) suburbs feature higher housing quality and values than other units.² We also find housing quality and housing values are positively related.

	All Units			Newer Units		
	All Areas	MSA		All Areas	MSA	
		Central	Suburb		Central	Suburb
Median housing value	\$165,344	\$161,096	\$205,218	\$238,365	\$241,366	\$256,556
Median housing costs	\$853	\$882	\$1,029	\$1,299	\$1,390	\$1,394
Proportions (%) of units featuring						
Porch	83.7	76.0	87.8	87.7	84.8	91.2
Fireplace	32.8	25.6	40.5	47.8	42.0	53.6
Garage	59.8	51.6	66.7	77.6	76.1	81.1
Separate dining rooms	46.3	44.0	50.9	53.9	50.8	57.2
Multiple living rooms	27.6	19.8	33.4	41.0	34.3	45.9

The main findings are summarized as follows. Concerning the urban structure, we find that the endogenous city size and the overall housing quality of the city are negatively related. While the population density, the land rent, and the number of floors (housing quantity) are all decreasing in the distance away from the city center, housing quality and quality-augmented housing prices (housing values) are both increasing in it. The latter result is consistent with the empirical observation in the United States. In response to a reduction in any of the housing-related taxes, urban fringe expands and both housing quality and housing values rise. While the effects of property and

²For example, based on the 2005 survey, we find newer units in MSA suburbs are most likely to have porch (or deck, patio, or balcony), fireplace, and garage, and to have separate dining room and multiple living or recreation rooms, while older units in MSA central cities are least likely to have such features.

gross revenue taxes on the population density and the (quality-augmented) housing stock are ambiguous, the effects of development and land taxes are unambiguously positive. In general, in response to any changes in housing-related taxes or in preferences and production, commuting and development technologies, housing quality, and housing prices/values are always positively related, corroborating with the empirical fact.

Most importantly, based on our quantitative welfare analysis, a globally optimal tax scheme in the housing market (without income or factor taxes) is to eliminate the property tax and to impose a lower gross revenue tax rate (possibly fully eliminated) than both development and land taxes. Under the assumptions that all tax rates are nonnegative (i.e., no subsidy), that the LPG is entirely financed by these four housing-related tax instruments and that local housing congestion externalities are nonnegligible, the welfare-maximizing tax mix in the benchmark economy is to eliminate both property and gross revenue taxes and to set the land tax rate at 45.4% and the development tax rate at 84%.

1.1. Literature Review

There is a conventional literature studying urban land use and durable housing pioneered by Brueckner (1981) and Fujita (1982), where housing quality is not fully characterized. In his pivotal work, Sweeney (1974) formalizes the notion of housing quality. Since then, there has been numerous empirical studies estimating housing demand and housing prices in which housing quality plays a crucial role. Because the majority of these papers are only remotely related, we will cite just a few for the sake of brevity. Theoretically, Arnott, Davidson, and Pines (1983) develop a dynamic model of housing quality supply based on profit maximization of the representative landlord. There is a growing number of followers continuing this stream of research emphasizing the durable good nature of housing quality. For example, Arnott et al. (1999) construct a general-equilibrium housing model with households choosing their demand for housing and developers determining the structure density (and time path) of housing. However, most of these studies do not model explicitly the internal urban structure. An exception is Lin, Mai, and Wang (2004), in which housing quality measured by housing capital in a monocentric city is regarded as an intermediate input to enhance household production and quality-augmented leisure time. None of these papers in this stream of research consider the provision or the finance of local public amenities.

The second stream of research studies housing-related taxes within the general equilibrium framework with local public good provision. For instance, Wilson (1985) compares a national capital tax and a local property tax that is used to finance local public expenditures. Turnovsky and Okuyama (1994) characterize housing demand and public goods in a two-sector growth model, whereas Leung (1999) examines the effects of property taxes in a small open setting. By considering two types of housing (with high and low quality) and

two types of households (the rich and the poor), Cremer and Gahvari (1998) study optimal taxation of housing that provides differential treatments to different types of occupants. More recently, Brueckner (2001) and Sieg et al. (2004) characterize housing demand in the presence of public goods and property taxes. None of these papers in this stream of research take into account the explicit spatial structure of the economy.

The third stream of research examines the effects of property taxes on the spatial structure without public good provision. In particular, Arnott and MacKinnon (1977) find that a higher property tax raises population density and shrinks the city size. This result has been reproduced by many follow-up studies, including those cited in Song and Zenou (2006), where an empirical test is also performed using the U.S. data. Recently, Brueckner and Kim (2003) show that this result may be upset when the dwelling size is fixed but the structural density per lot is flexible. There is also a long literature focusing on the dynamic effect of property taxes on housing demand (e.g., Kanemoto 1985 and the follow-up studies), which is too remote from our work and hence not discussed. In both the second and the third streams of research, the market of housing quality has not been incorporated.

In the present paper, we incorporate both the provision/finance of local public amenities and the explicit spatial structure of the local economy. This enables a more complete assessment of various housing-related taxes. Because it is too complex to incorporate both dynamics and internal urban structure (two-dimensional differential equation system in time and location), we restrict our analysis to static by leaving the durable goods nature of housing for future research.

2. The Model

There are three theatres of economic activities by households, housing developers, and the local government. We model housing quality demand and supply in a general equilibrium framework, allowing the provision and finance of local public amenities G to affect households' demand for housing quality and local monopoly power and local housing congestion to influence developers' housing supply. We consider four housing-related taxes: a property tax at rate τ_H , a developer gross revenue tax at rate τ_S , a land tax at rate τ_L , and a development tax at rate τ_D .

To characterize such a sophisticated structure, we further simplify the environment of the economy. Particularly, we are abstracting from the durable good aspects of housing. We fix the dwell density consumed by each household at one, but allow for variable housing supply at each location via the considerations of multiple floors per lot; thus, we measure housing quantity, h , by the number of floors (or the floor-area ratio).³ We do not consider differential taxes across locations.

³When a location-dependent variable (housing quality in our case) enters the utility function, it has been shown that with variable lot size (or floor-area ratios in our case), a

Consider a “long-narrow” featureless, uniformly distributed landscape over $[-\bar{x}, \bar{x}]$, within which a linear city is spread over $[-x_f, x_f]$, where the urban fringe x_f is endogenously determined. The area of $[-\bar{x}, -x_f] \cup [x_f, \bar{x}]$ is for agricultural uses, whose land rent is given exogenously at R_A . By construction, no household can reside in the agricultural area. There is an exogenously determined central business district (CBD) at location 0 in which a “travel-for” local public good (LPG) is provided.

2.1. Households

There is a continuum of households of measure N with identical income, y , and identical preferences. All households commute to the CBD to work (with an inelastic labor supply), to shop, and to enjoy local public amenities by incurring a commuting cost at a constant rate t per unit of distance. More precisely, households must travel to the CBD to consume the LPG service and the LPG is both nonrival and nonconclusive.

By residing in a house whose quantity is fixed at one unit, a representative household at location z values composite good consumption, c , housing quality, q , and the level of the LPG provided by the local government. Her utility function takes the following form:

$$U(c, q; G) = c + \beta q^\theta + \gamma \ln G, \quad (1)$$

where β measures the household’s preference toward housing quality, γ indicates her preference toward the LPG and $\theta \in (0, 1)$. This setup extends the quasi-linear utility function form employed by Bergstrom and Cornes (1983) and Berliant, Peng, and Wang (2006). When $\beta = 0$, the utility function reduces to that in Berliant, Peng, and Wang (2006). The utility function specification implies that both the unit bid price for housing quality and the marginal willingness to pay for the local public good are independent of the composite good, thus simplifying the analysis greatly.⁴

Given the unit price associated with quality q at location z (denoted by P) and the local property tax rate, the representative household allocates her net income (income net of the commuting cost incurred) to consumption of the composite good and housing (inclusive of the property tax payment). Her budget constraint can therefore be written as:

$$c + (1 + \tau_H)Pq = y - tz. \quad (2)$$

The representative household’s optimization problem is thus to choose composite good consumption, housing quality, and residential location, (c, q) , to maximize U given in (1) subject to the budget constraint (2). Due to

competitive spatial equilibrium may not exist and a social welfare maximum may not be well defined (see Berliant and ten Raa 1991 and Wang 1993, respectively).

⁴Notice that under our setup with homogeneous households, there is no need to consider voting behavior because all households would vote for the same level of local public good provision no matter where they live.

the quasi-linear property of the utility function, one can simply substitute (2) into (1) to eliminate c and manipulate the first-order condition with respect to q to yield the unit bid price schedule:

$$P(q) = \frac{\beta\theta}{1 + \tau_H} q^{\theta-1}. \tag{3}$$

The housing price, defined as the corresponding unit bid price augmented by quality at location z , can be derived below:

$$v(z) \equiv P(q(z))q(z) = \frac{\beta\theta}{1 + \tau_H} q(z)^\theta. \tag{4}$$

We then substitute (3) into (2) to derive $c(q, z)$ and then into (1) to obtain

$$U(c(q, z), q; G) = y - tz + \beta(1 - \theta)q^\theta + \gamma \ln G.$$

Straightforward differentiation of this utility expression with respect to z yields the Muth-like envelope condition:

$$\frac{dq}{dz} = \frac{t}{\beta\theta(1 - \theta)} q^{1-\theta}. \tag{5}$$

Integrating this first-order differential equation (5) gives the demand schedule for housing quality:

$$q^d(z) = \left[q_0 + \frac{t}{\beta(1 - \theta)} z \right]^{\frac{1}{\theta}}, \tag{6}$$

where the integration constant, q_0 , measures the housing quality at location 0 which will be endogenously determined in equilibrium. Thus, housing quality demand is independent of housing quantity measured by the number of floors.

2.2. Developers

There is a continuum of developers of measure M , each developing a lot at a particular location. The land available at each location is one unit, which is owned entirely by an absentee landlord and rented to a developer (or housing producer). Under free entry and no vacant land, the endogenous measure of developers is given by, $M = 2x_f$.

Each developer employs capital per house k , together with one unit of land, to produce houses, whose quantity measured by the number of floors (the floor-area ratio) is denoted by h . The developer at a given location has local monopoly power and compete with each other in a monopolistically competitive fashion.

We specify the housing quality production function as follows:

$$q(x) = Ak(x)^\alpha h(x)^{-\varepsilon}, \tag{7}$$

where $A > 0$, $0 < \alpha < 1$, and $\varepsilon > 0$. The production scaling parameter, A , measures the developer’s productivity. The parameter ε captures the degree of housing congestion: the more houses per unit of land (or, precisely, more floors) developed in a given location, the less the quality of a given house is. Three remarks are now in order. First, under our setup, it is isomorphic between a gross revenue tax rate and a profit tax—both taxes create distortion only via developers’ capital demand (and hence housing supply). The reason for selecting the former tax is because it has the same tax base as the property tax, which greatly simplifies the tax incidence analysis. Second, the development tax considered here is a percentage license fee imposed on the basis of the fixed development cost. Third, neither the land tax nor the development tax creates any direct distortion on developers’ capital demand or housing supply.

Denote $R(x)$ as the market land rent per unit of land at location x , r as the capital rental rate, and ϕ as the fixed lot-development fee (which is a sunk cost). The profit generated by the housing developer at location x is then given by:

$$\pi(x) = (1 - \tau_S) p(x)q(x)h(x) - rk(x)h(x) - (1 + \tau_L) R(x) - (1 + \tau_D)\phi, \tag{8}$$

which equals the after-tax revenue, net of capital costs, tax-included land rents, and tax-included development fees. Thus, a housing developer’s optimization is to maximize profit, given households’ inverse demand schedule $p(x) = P(q(x))$ derived from their optimization problem (3), to determine capital demand, housing quantity supply, unit housing price (or housing quality supply) and housing development location. Notably, due to the presence of the local monopoly power, the developer at x does not take $p(x)$ as given; rather he set both housing unit price and quality supply based on household demand.⁵ Moreover, capital is the only elastic factor input, which is perfectly mobile. Thus, there is no need for deriving the envelope condition for x because it is assured by the free entry condition. Furthermore, to avoid complexity from the joint production problem (of housing quantity and housing quality), our setup allows us to first pin down capital demand and then adjust housing quantity to meet household’s housing quality demand based on the production function.

By the above arguments, we substitute the inverse demand function (3) and the production function (7) into developer x ’s profit function (8) to reduce his optimization problem to:

$$\max_k \pi(x) = \left(\frac{1 - \tau_S}{1 + \tau_H} \beta \theta A^\theta k^{\alpha\theta} h^{-\varepsilon\theta} - rk \right) h - (1 + \tau_L) R(x) - (1 + \tau_D)\phi. \tag{9}$$

⁵Thus, housing units are developed until the marginal cost is equal to the marginal revenue, $p(x) + q(x) \frac{dp(q(x))}{dq(x)}$.

The first-order condition is:

$$k = \left[\left(\frac{1 - \tau_S}{1 + \tau_H} \right) \frac{\alpha \beta \theta^2 A^\theta}{r} \right]^{\frac{1}{1-\alpha\theta}} h^{-\frac{\epsilon\theta}{1-\alpha\theta}}. \tag{10}$$

Substituting (10) into (7), we have

$$q^s(x) = A^{\frac{1}{1-\alpha\theta}} \left[\left(\frac{1 - \tau_S}{1 + \tau_H} \right) \frac{\alpha \beta \theta^2}{r} \right]^{\frac{\alpha}{1-\alpha\theta}} h^{-\frac{\epsilon}{1-\alpha\theta}} \tag{11}$$

from which we can derive the quality-augmented housing stock $h^s(x) \equiv q(x)h(x)$. Thus, the quality supply schedule is decreasing in housing quantity measured by the floor-area ratio. As shown in Section 3 below, by equating this quality supply schedule with the quality demand schedule (6), one can obtain the housing quantity schedule.

2.3. Local Government

Consider a local government to provide public amenities at the CBD for all households residing in the city. Denote the level of the LPG provision as G . We suppose that this LPG is entirely financed by revenues collected from the four housing-related taxes: the gross revenue tax, the property tax, the land tax (imposed on both city and agricultural land rents), and the development tax. Then balancing the government budget requires

$$G = 2(\tau_H + \tau_S) \int_0^{x_f} p(z)h^s(z) dz + 2\tau_L \left[(\bar{x} - x_f) R_A + \int_0^{x_f} R(z) dz \right] + \tau_D \phi M. \tag{12}$$

It is easily seen that while the land tax and the development tax have different tax bases than the gross revenue tax and the property tax, the latter two have an identical tax base, $2 \int_0^{x_f} p(z)h^s(z) dz$. Thus, if the gross revenue tax and the property tax may affect the equilibrium outcomes differently, it must be due exclusively to differences in their marginal distortions to demands and supplies (and hence the associated prices).

3. Equilibrium

We begin by describing the equilibrium conditions in our local economy. Denoting households' indirect utility as $u(z)$, locational no-arbitrage implies $u(z) = u(0)$ for all z within the entire city. Under free entry, developers' zero profit conditions yield the bid rent at each location x as follows:

$$R(x) = \frac{1 - \alpha\theta}{1 + \tau_L} A^{\frac{1}{\epsilon}} \left(\frac{\alpha\theta}{r} \right)^{\frac{\alpha}{\epsilon}} \left[\left(\frac{1 - \tau_S}{1 + \tau_H} \right) \beta \theta \right]^{\frac{\alpha+\epsilon}{\epsilon}} \times \left[q_0 + \frac{t}{\beta(1-\theta)} x \right]^{-\frac{1-(\alpha+\epsilon)\theta}{\epsilon\theta}} - \left(\frac{1 + \tau_D}{1 + \tau_L} \right) \phi. \tag{13}$$

Since there are totally N households residing within the geographically symmetric linear city $[-x_f, x_f]$, the following population identity must be met:

$$2 \int_0^{x_f} h(x) dx = N. \tag{14}$$

Because there is only one developer at each location, the endogenous number of developers is pinned down by: $M = 2x_f$. Since each household owns exactly one unit of house (i.e., one floor of the dwelling at a particular lot), housing market clearance from the quantity perspective means that the population density at each location is equal to the floor-area ratio: $n(x) = h(x)$. From (6) and (11), housing market clearance from the quality perspective is given by

$$\left[q_0 + \frac{t}{\beta(1-\theta)} x \right]^{\frac{1}{\theta}} = A^{\frac{1}{1-\alpha\theta}} \left[\left(\frac{1-\tau_S}{1+\tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right]^{\frac{\alpha}{1-\alpha\theta}} h^{-\frac{\alpha}{1-\alpha\theta}}. \tag{15}$$

Furthermore, land market clearance implies that the linear city continues to be developed until the market land rent at the endogenously determined fringe x_f equals the agricultural land rent:⁶

$$R(x_f) = R_A. \tag{16}$$

A *competitive spatial equilibrium* is a list of nonnegative densities $\{q(z), k(x), h(x), n(x)\}$, a list residential and development locations $\{z, x\}$, and a list of aggregate quantities $\{G, M, x_f\}$, together with a list of positive prices $\{p(z), R(x)\}$, such that: (i) all households and developers optimize, (ii) all households reach the same indirect utility, (iii) all developers reach zero profit, (iv) the government budget is balanced, (v) the no-vacant land population identity is met, and (vi) the housing markets (quantity and quality) and the land market all clear.

Since solving the equilibrium is rather tedious, we relegate the mathematical details to the Appendix and only outline the solution method below.

- Step 1: The main task is to pin down a key endogenous variable, q_0 . We first use the housing quality market clearing condition at each location and the zero profit condition to express housing quantity and market land rent, respectively, as a function of q_0 . We then substitute housing quantity and land rent schedules into the land market clearing condition, the housing quantity clearing condition at each location, and the population identity to determine the equilibrium housing quantity schedule as well as the urban fringe and the value of q_0 .

⁶The tax-included gross land rents are $(1 + \tau_L)R(x)$ and $(1 + \tau_L)R_A$, respectively. Under a uniform land tax, these two land rents at the urban fringe must be equalized both before and after tax.

- Step 2: Substituting q_0 into the housing quality demand schedule, the unit bid price schedule, and the land rent schedule yield the corresponding equilibrium schedules as well as the equilibrium schedules of the (quality-augmented) housing stock and the (quality-augmented) housing price.
- Step 3: Substituting the above equilibrium outcomes into the government budget constraint as well as the household budget constraint and utility function, one obtains the equilibrium provision of the LPG (for given tax rates), the equilibrium consumption schedule and the constant value of indirect utility across all locations.

In the Appendix, we show that the following “normality” condition guarantees that a higher development fee will reduce the (quality-augmented) housing stock $h^s(x)$:⁷

CONDITION N: $\alpha\theta + \varepsilon < 1$.

We also show that this assumption is sufficient to guarantee $(\alpha + \varepsilon)\theta < 1$, which ensures that the market land rent is decreasing in x (a standard feature of monocentric city models).

From Step 1, the *overall quality* of housing in the city that is independent of z is derived as

$$q_0 = \left[\frac{A(\beta\theta)^{\alpha+\varepsilon} \left(\frac{\alpha\theta}{r} \frac{1-\tau_S}{1+\tau_H} \right)^\alpha}{\left\{ \frac{tN [1 - (\alpha + \varepsilon)\theta]}{2\varepsilon(1-\theta)} + \frac{(1+\tau_D)\phi + (1+\tau_L)R_A}{(1-\alpha\theta) \frac{1-\tau_S}{1+\tau_H}} \right\}^\varepsilon} \right]^{\frac{\theta}{1-(\alpha+\varepsilon)\theta}} \quad (17)$$

We can conclude:

PROPOSITION 1 (Overall Quality of Housing): *Under Condition N, the overall quality of housing is increasing in the household’s preference bias toward housing quality and the developer’s productivity of housing quality, but decreasing in the unit development fee, the unit commuting cost and each of the four tax rates.*

While other properties are not surprising, it is interesting that both development and land taxes are *distortionary* in our model. This is due to our consideration of the housing congestion externality. Should such an externality be removed (i.e., $\varepsilon = 0$), housing market clearance from the quality perspective, (15), immediately yields the solution of q_0 , which depends only on the property tax and the gross revenue tax (negatively). In the presence of the housing congestion externality, the distortionary effects of development and land taxes on the overall quality of housing imply that imposing a

⁷This normality condition is essentially Samuelson’s correspondence principle: it ensures stability and reasonable comparative statics.

100% tax on either land development fee or land rent need not be welfare-maximizing (to be discussed in Section 6 below).

By the recursive nature of the solution method, we can conveniently illustrate the determination of the competitive spatial equilibrium diagrammatically. To begin, we plot housing quality demand and supply schedules, (6) and (11), in (h, q) space (see the right panel of Figure 1). Given the solution of q_0 in (17) above, housing quality demand is independent of housing quantity, whereas housing quality supply is decreasing in housing quantity as a result of the housing congestion externality. The intersection (point E) determines equilibrium housing quality and quantity, as well as the (quality-augmented) housing stock. By housing market clearance from the quantity perspective, the equilibrium population density schedule is pinned down as well. Next, we can plot the unit bid price and the (quality-augmented) housing price against q (see the left panel of Figure 1). From (3), the unit bid price is downward-sloping, while, from (4), the housing price is upward-sloping. Given equilibrium housing quality determined in the right panel, the left panel then pins down the equilibrium unit price and the equilibrium housing price.

Thus, both panels of Figure 1 jointly determine three key housing variables: equilibrium housing quality, the equilibrium housing stock and the

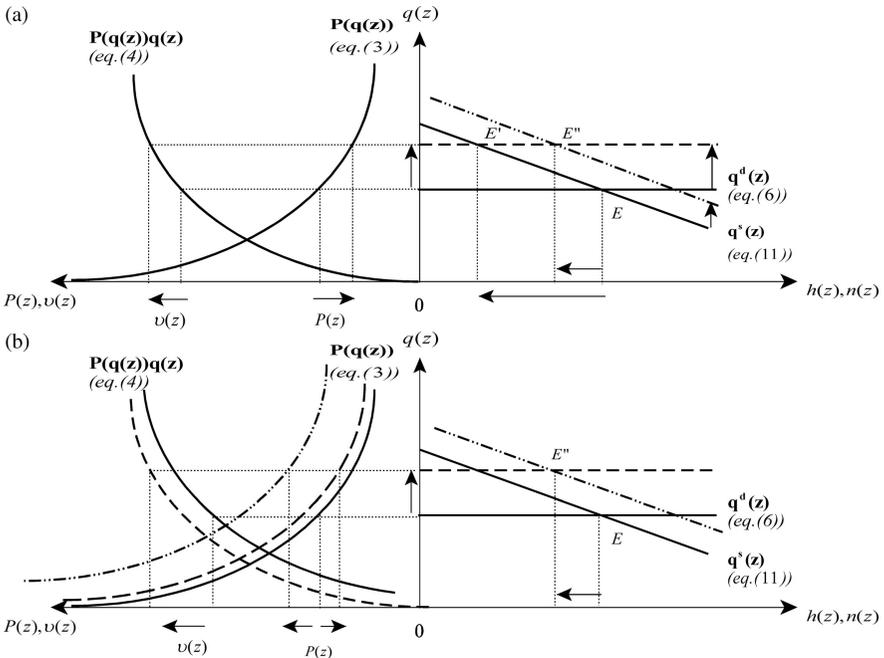


Figure 1: (a) Equilibrium determination and effects of a reduction in (τ_S, τ_L, τ_D) or ϕ , or an increase in A . (b) Effects of a reduction in τ_H .

equilibrium housing price (which also measures the equilibrium housing value):

$$q(z) = \left[q_0 + \frac{t}{\beta(1-\theta)} z \right]^{\frac{1}{\theta}} \tag{18}$$

$$h^s(z) = A^{\frac{1}{\varepsilon}} \left[\left(\frac{1-\tau_S}{1+\tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right]^{\frac{\alpha}{\varepsilon}} \left[q_0 + \frac{t}{\beta(1-\theta)} z \right]^{-\frac{1-(\alpha\theta+\varepsilon)}{\varepsilon\theta}} \tag{19}$$

$$v(z) = \frac{\beta\theta}{1+\tau_H} \left[q_0 + \frac{t}{\beta(1-\theta)} z \right]. \tag{20}$$

As it can be seen clearly from (18), equilibrium housing quality depends on two components: (i) the first component, q_0 , indicates the overall quality of houses in the city that is independent of z ; and (ii) the second component, $\frac{t}{\beta(1-\theta)}z$, measures the *quality gradient* that is increasing in z . However, since housing quality is not linear in the two components ($\theta < 1$), the overall quality of housing would not affect the housing quality schedule uniformly. Specifically, we have:

PROPOSITION 2 (Housing Quality Schedule): *Equilibrium housing quality is increasing in the distance away from the CBD. Under Condition N, an overall quality enhancement raises the equilibrium quality of houses in all locations, raising the quality of houses at the outskirts more than those in the inner city.*

An important consequence of this proposition is that an increase in any tax rate (which affects housing quality only via q_0) will create a larger distortion to the quality of houses at outskirts. Notably, while the housing quality schedule is upward-sloping in the distance away from the CBD, the housing quantity schedule (measured by the floor-area ratio) and the equilibrium population density are both downward-sloping as in the conventional monocentric city literature without the consideration of housing quality.

From (19) and (20), one can see that the housing stock and the equilibrium housing price are both closely related to equilibrium housing quality. In particular, we can show:

PROPOSITION 3 (Housing Stock and Price Schedules): *While the equilibrium housing price is always increasing in the distance away from the CBD, the equilibrium housing stock is decreasing in it if Condition N holds. An overall housing quality enhancement raises the equilibrium prices of houses in all locations uniformly, but raises housing stocks at the outskirts less than those in the inner city under Condition N.*

Thus, this proposition implies that an increase in any tax rate will create a larger distortion to housing stocks in the inner city. Concerning the equilibrium housing price, it is noted that the positive housing quality effect always dominates the negative unit bid price effect, leading to an upward-sloping

schedule in the distance away from the CBD. Indeed, the equilibrium housing price turns out to be linear in q_0 . Thus, an increase in q_0 raises housing prices uniformly across all locations. Other than the property tax that has a direct negative effect on housing prices, the other three tax instruments will all reduce housing prices uniformly across all locations (by affecting q_0).

Finally, we turn to the examination of the size of the city. As shown in the Appendix, the combination of zero profit, land and housing market clearance conditions, and the population identity yields the equilibrium urban fringe, satisfying the following properties:⁸

PROPOSITION 4 (City Size): *An overall housing quality enhancement reduces the urban fringe.*

Other things being equal, a larger city will feature houses in the inner city with lower quality than a smaller city. In equilibrium, higher overall quality of housing induces households to reside closer to the city center, thereby decreasing the city size.

4. Equilibrium Characterization

We are now ready to perform comparative-static exercises. We consider shifts in preference parameter (β), production parameter (A), development fee parameter (ϕ) and commuting cost parameter (t). We are interested in how such parameter changes affect an array of endogenous variables, including x_f (urban fringe), $q(z)$ (housing quality), $h^s(z)$ (housing stock), $v(z)$ (housing price/value), $n(z)$ (population density), $R(z)$ (land rent), and $u(0)$ (household welfare).

4.1. Analytical Results

The analytical results are best illustrated by utilizing Figure 1. First, we begin with the most straightforward case concerning a reduction in the unit development fee (ϕ), which only creates an overall quality enhancement effect via q_0 . Specifically, it causes the quality demand schedule to shift up without affecting the quality supply schedule. As a result, the equilibrium shifts from point E to point E' , which features higher housing quality, lower housing quantity (floors per location) and higher housing prices/values. Intuitively, a lower development fee enables firms in the outer city to maintain profitability more easily, thus leading to a larger city with lower density of houses (less floors) at each location. The reduction in housing congestion in turn improves the overall quality of houses in the city. Notice that these changes are simply along the housing quality supply curve. To restore housing quality equilibrium, housing quality demand must shift up, as indicated in the right panel of Figure 1(a). Since the positive quality effect is dominated

⁸For brevity, we do not display the solution of the urban fringe (which is rather complex) nor other endogenous variables in the main text.

by the negative quantity effect under Condition N, the equilibrium quality-augmented housing stock turns out to be lower. Moreover, as indicated in the left panel of Figure 1(a), while the unit bid prices decrease for every location in the city, quality-augmented housing prices (housing values) are always higher because the quality effect dominates.

Second, improved productivity of housing quality (A) creates similar effects via overall quality as a reduction in the development fee. Additionally, it shifts the housing quality supply schedule outward (see Figure 1(a)) and hence raises housing quantity. Due to the presence of the congestion effect, however, this positive effect is outweighed by the negative effect as a result of the shift in housing quality demand. Thus, the equilibrium shifts from point E to point E'' and housing quantity decreases. Due to the presence of a direct effect, however, the equilibrium housing stock turns out to be higher. Qualitatively, all the comparative static results, except that concerning the housing stock, are therefore identical to those with respect to a reduction in the development fee.

Third, an increase in the preference bias toward housing quality (β) encourages the development of a larger city size. While it creates similar effects via overall quality and the outward shift in the housing quality supply schedule as improved productivity, there is an additional negative quality gradient effect. Specifically, in response to this preference shift, the urban fringe expands and the resulting commuting cost for households residing in the outer city increases, so their willingness to pay for housing quality lessens. This induces a downward shift in the housing quality demand schedule, which may outweigh the overall quality effect and hence its net effects on housing quality and quantity become ambiguous. It is therefore possible that housing quality and quantity are positively related in response to a preference shift. Moreover, in response to this preference shift, the unit bid price locus shifts outward and the housing value locus rotates outward. Thus, while the effect on housing quality is generally ambiguous, it always raises equilibrium housing values as a result of this dominating direct effect via the shift in the housing value locus.

Finally, similar to the preference shift mentioned above, a reduction in the unit commuting cost (t) also has two opposing effects on housing quality: an overall quality enhancement effect and a negative quality-gradient effect. However, it generates two opposing effects on the urban fringe as well. The first is a *direct commuting* effect: cheaper commuting enables households to afford living farther from the city center, thereby causing the city to expand. The second is an indirect overall quality effect: cheaper commuting leads to housing quality enhancements even at locations near the center, thus discouraging households to move outward seeking houses of greater quality. As a consequence, an improvement in the commuting technology has an ambiguous effect on the urban fringe. This ambiguity leaves essentially all comparative static results indeterminate. In locations toward the city center, it is, however, expected that the overall quality enhancement effect dominates and hence housing quality improves unambiguously.

The analytical results are summarized below.

PROPOSITION 5 (Comparative Statics): *Under Condition N, a competitive spatial equilibrium possesses the following properties:*

- (i) *An increase in the household's preference bias toward housing quality raises overall housing quality and equilibrium housing prices, enlarges the city size, and enhances household welfare, but has ambiguous effects on the equilibrium quality of houses in the outer city and the equilibrium housing stock.*
- (ii) *An increase in the developer's productivity of housing quality or a decrease in the unit development fee raises overall and equilibrium housing quality as well as equilibrium housing prices, enlarges the city size, and enhances household welfare. While the former raises the equilibrium housing stock, the latter lowers it.*
- (iii) *While an increase in the unit commuting cost raises overall housing quality, its effects on equilibrium housing quality, prices, and stocks, the equilibrium city size, and household welfare are all ambiguous.*

4.2. Numerical Findings

We now turn to numerical analysis, which not only helps clarify the ambiguity in analytical comparative statics but also quantifies such responses. We set $\theta = 0.4$, $\varepsilon = 0.3$, $N = 1000$, and $y = 1$ (per capital income normalized as one). Since all housing-related taxes affect the urban fringe negatively, we set \bar{x} as one corresponding to the no-tax equilibrium, thereby ensuring $\bar{x} > x_f$ (in our benchmark case, $\bar{x} = 10.97$). We choose $\alpha = 2/3$ based on the share of structure capital in housing production and $r = 7.5\%$ based on the U.S. average capital rental rate. The preference parameter is set at $\beta = 0.2$ such that the share of flow housing expenditure to income ($\frac{TGR}{N_y}$, where $TGR = 2 \int_0^{x_f} p(z)q(z)h(z)z dz$ measures the total gross revenue from housing construction as well as the total flow housing expenditure) is around 10% (9.63% in our benchmark case). The unit commuting cost is set at $t = 0.02$ such that the share of commuting cost to household income ($\frac{ACC}{N_y}$, where $ACC = 2t \int_0^{x_f} h(z)z dz$ measures the aggregate commuting cost) takes a reasonable value in the range between 2% and 5% (3% in our benchmark case).

Further, we select $\phi = 0.2$ for the ratio of the total development fee ($TDF = 2\phi x_f$) to total gross revenue (TGR) to fall in the range between 3% and 5% (3.75% in our benchmark case) and $A = 20$ for the ratio of aggregate land rent to income ($\frac{ALR}{N_y}$) to take a reasonable value about 5% (5.46% in our benchmark case). In order for the ratio of agriculture land rent to the average city land rent ($\frac{R_A}{ALR/(2x_f)}$) to fall in the range between 1% and 5%, we set agricultural land rent as $R_A = 0.05$, from which we get $x_f = 9.02$ and $\frac{R_A}{ALR/(2x_f)} = 1.65\%$. Finally, we choose housing tax instruments mimicking a typical U.S. city, $(\tau_H, \tau_S, \tau_L, \tau_D) = (0.15, 0.075, 0.125, 0.08)$, which,

Table 1: Comparative statics with respect to preference and technology parameters

	x^f	$u(0)$	Ω	$\frac{ACC}{N_y}$	$\frac{ALR}{N_y}$	$\frac{TGR}{N_y}$	$\frac{TDF}{TGR}$	$\frac{G}{N_y}$
Benchmark	9.02	1.114	1167	0.0300	0.0546	0.0963	0.0375	0.0288
$\beta \cdot 0.8$	6.27 (-30.50)	1.086 (-2.49)	1123 (-3.77)	0.0209 (-30.50)	0.0379 (-30.50)	0.0669 (-30.50)	0.038 (0)	0.0201 (-30.35)
$\beta \cdot 0.9$	7.60 (-15.78)	1.100 (-1.26)	1145 (-1.93)	0.0253 (-15.78)	0.046 (-15.78)	0.0811 (-15.78)	0.038 (0)	0.0243 (-15.71)
$\beta \cdot 1.1$	10.54 (16.81)	1.128 (1.29)	1191 (2.01)	0.0351 (16.81)	0.0638 (16.81)	0.1125 (16.81)	0.038 (0)	0.0336 (16.73)
$\beta \cdot 1.2$	12.14 (34.62)	1.143 (2.62)	1215 (4.09)	0.0404 (34.62)	0.0735 (34.62)	0.1296 (34.62)	0.038 (0)	0.0387 (34.45)
$A \cdot 0.8$	7.80 (-13.54)	1.107 (-0.58)	1154 (-1.17)	0.0260 (-13.54)	0.0472 (-13.54)	0.0832 (-13.54)	0.038 (0)	0.0249 (-13.48)
$A \cdot 0.9$	8.42 (-6.64)	1.111 (-0.28)	1161 (-0.57)	0.0281 (-6.64)	0.0510 (-6.64)	0.0899 (-6.64)	0.038 (0)	0.0269 (-6.61)
$A \cdot 1.1$	9.60 (6.41)	1.116 (0.26)	1174 (0.54)	0.0320 (6.41)	0.0581 (6.41)	0.1024 (6.41)	0.038 (0)	0.0306 (6.38)
$A \cdot 1.2$	10.16 (12.63)	1.119 (0.50)	1180 (1.05)	0.0338 (12.63)	0.0615 (12.63)	0.1084 (12.63)	0.038 (0)	0.0324 (12.57)
$\phi \cdot 0.8$	9.56 (6.00)	1.114 (0.0+)	1168 (0.05)	0.0304 (1.14)	0.0552 (1.22)	0.0965 (0.24)	0.0317 (-15.40)	0.0289 (0.29)
$\phi \cdot 0.9$	9.28 (2.84)	1.114 (0.0+)	1168 (0.03)	0.0302 (0.56)	0.0549 (0.60)	0.0964 (0.12)	0.0346 (-7.55)	0.0288 (0.15)
$\phi \cdot 1.1$	8.79 (-2.59)	1.114 (0.0-)	1167 (-0.03)	0.0299 (-0.54)	0.0543 (-0.58)	0.0962 (-0.12)	0.0402 (7.28)	0.0288 (-0.14)
$\phi \cdot 1.2$	8.57 (-4.95)	1.114 (0.0-)	1167 (-0.05)	0.0297 (-1.07)	0.0540 (-1.13)	0.0961 (-0.23)	0.0428 (14.32)	0.0287 (-0.28)
$t \cdot 0.8$	10.90 (20.86)	1.116 (0.18)	1171 (0.29)	0.0309 (2.73)	0.0562 (3.03)	0.1002 (4.09)	0.0435 (16.11)	0.0299 (3.92)
$t \cdot 0.9$	9.87 (9.39)	1.115 (0.08)	1169 (0.14)	0.0304 (1.31)	0.0554 (1.44)	0.0981 (1.92)	0.0402 (7.33)	0.0293 (1.84)
$t \cdot 1.1$	8.31 (-7.84)	1.113 (-0.07)	1166 (-0.13)	0.0297 (-1.22)	0.0539 (-1.33)	0.0946 (-1.71)	0.0351 (-6.23)	0.0283 (-1.65)
$t \cdot 1.2$	7.71 (-14.49)	1.112 (-0.14)	1164 (-0.24)	0.0293 (-2.35)	0.0532 (-2.55)	0.0931 (-3.26)	0.0331 (-11.61)	0.0279 (-3.14)

(Continued)

together with $\gamma = 0.0001$, generate a reasonable LPG to aggregate income ratio in the range between 2% and 5% (2.88% in our benchmark case).⁹

We next present the numerical results with respect to preference (β) and technology (A , ϕ , and t) parameters in Table 1. The main findings are summarized as follows.

⁹More specifically, we must bear in mind that, in a static framework, housing values should be measured in flows. Consider a house with a market value of \$100,000. Given a 7.5% rental rate, its flow value is \$7,500. Under a typical 1.5% property tax rate imposed on the assessed value where the assessed value is assumed about 75% of the market value, the property tax payment becomes $1.5\% \cdot 75\% \cdot \$100,000 = \$1,125$. Dividing this payment by the flow housing value \$7,500, one obtains an imputed property tax rate of 15%. Next,

Table 1: Continued.

	$n(0)$	$n(x^f)$	$q(0)$	$q(x^f)$	$h^s(0)$	$h^s(x^f)$	$v(0)$	$v(x^f)$	$R(0)$
Benchmark	381	2.19	1.3679	11.29	520.96	24.70	0.0789	0.1834	17.92
$\beta \cdot 0.8$	547.98 (43.88)	3.15 (43.88)	0.9623 (-29.65)	7.94 (-29.65)	527.31 (1.22)	25.00 (1.22)	0.0548 (-30.50)	0.1275 (-30.50)	17.92 (0)
$\beta \cdot 0.9$	452.240 (18.74)	2.60 (18.74)	1.1586 (-15.30)	9.56 (-15.30)	523.95 (0.57)	24.84 (0.57)	0.0664 (-15.78)	0.1545 (-15.78)	17.92 (0)
$\beta \cdot 1.1$	326.04 (-14.39)	1.87 (-14.39)	1.5896 (16.21)	13.12 (16.21)	518.26 (-0.52)	24.58 (-0.52)	0.0921 (16.81)	0.2143 (16.81)	17.92 (0)
$\beta \cdot 1.2$	282.92 (-25.72)	1.63 (-25.72)	1.8232 (33.29)	15.05 (33.29)	515.82 (-0.99)	24.46 (-0.99)	0.1061 (34.62)	0.2469 (34.62)	17.92 (0)
$A \cdot 0.8$	440.52 (15.67)	2.53 (15.67)	0.9507 (-30.50)	7.85 (-30.50)	418.79 (-19.61)	19.86 (-19.61)	0.0682 (-13.54)	0.1586 (-13.54)	17.92 (0)
$A \cdot 0.9$	407.95 (7.11)	2.34 (7.11)	1.1520 (-15.78)	9.51 (-15.78)	469.94 (-9.79)	22.28 (-9.79)	0.0736 (-6.64)	0.1712 (-6.64)	17.92 (0)
$A \cdot 1.1$	357.90 (-6.03)	2.06 (-6.03)	1.5978 (16.81)	13.19 (16.81)	571.87 (9.77)	27.12 (9.77)	0.0839 (6.41)	0.1952 (6.41)	17.92 (0)
$A \cdot 1.2$	338.16 (-11.21)	1.94 (-11.21)	1.8414 (34.62)	15.20 (34.62)	622.67 (19.53)	29.53 (19.53)	0.0888 (12.63)	0.2066 (12.63)	17.92 (0)
$\phi \cdot 0.8$	379.89 (-0.25)	1.78 (-18.66)	1.3693 (0.10)	12.29 (8.82)	520.18 (-0.15)	21.86 (-11.49)	0.0789 (0.04)	0.1897 (3.44)	17.92 (0)
$\phi \cdot 0.9$	380.37 (-0.13)	1.98 (-9.41)	1.3686 (0.05)	11.76 (4.13)	520.57 (-0.07)	23.30 (-5.67)	0.0789 (0.02)	0.1864 (1.64)	17.92 (0)
$\phi \cdot 1.1$	381.34 (0.13)	2.40 (9.56)	1.3671 (-0.05)	10.88 (-3.67)	521.35 (0.07)	26.07 (5.54)	0.0788 (-0.02)	0.1807 (-1.47)	17.92 (0)
$\phi \cdot 1.2$	381.82 (0.25)	2.61 (19.26)	1.3664 (-0.10)	10.51 (-6.95)	521.74 (0.15)	27.41 (10.97)	0.0788 (-0.04)	0.1782 (-2.84)	17.92 (0)
$t \cdot 0.8$	292.83 (-23.11)	2.19 (0)	1.5231 (11.36)	11.29 (0)	446.02 (-14.38)	24.70 (0)	0.0823 (4.39)	0.1834 (0)	14.34 (-19.94)
$t \cdot 0.9$	336.37 (-11.68)	2.19 (0)	1.4391 (5.21)	11.29 (0)	484.09 (-7.08)	24.70 (0)	0.0805 (2.05)	0.1834 (0)	16.13 (-9.97)
$t \cdot 1.1$	426.20 (11.91)	2.19 (0)	1.3063 (-4.50)	11.29 (0)	556.77 (6.87)	24.70 (0)	0.0774 (-1.82)	0.1834 (0)	19.70 (9.97)
$t \cdot 1.2$	472.36 (24.03)	2.19 (0)	1.2525 (-8.43)	11.29 (0)	591.64 (13.57)	24.70 (0)	0.0761 (-3.46)	0.1834 (0)	21.49 (19.94)

Note: The percentage changes of the responses are reported in parentheses (in %).

First, we can evaluate theoretically ambiguous comparative-static effects numerically. We find that the direct effect of households' preferences on housing quality generally dominates the indirect effect via their willingness to pay.¹⁰ Thus, housing quality responds to changes in β positively. Nonetheless, the housing stock is slightly lower as β increases, due to its strong negative

concerning the gross revenue tax, a 30% profit tax rate together with a 25% profit rate implies an imputed gross revenue tax at 7.5%. Moreover, because the average land rent to gross revenue ratio is 1/3, a land tax rate of 12.5% is equivalent to a reasonable 4% lot surcharge attached to the housing value. Finally, since the average development fee to gross revenue ratio is about 12%, a development tax of 8% is equivalent to a plausible 1% licensing surcharge attached to the housing value.

¹⁰While most of these findings are robust to parameter changes with reasonable ranges, we would like to note that it is possible that housing quality in outer city may be lower in

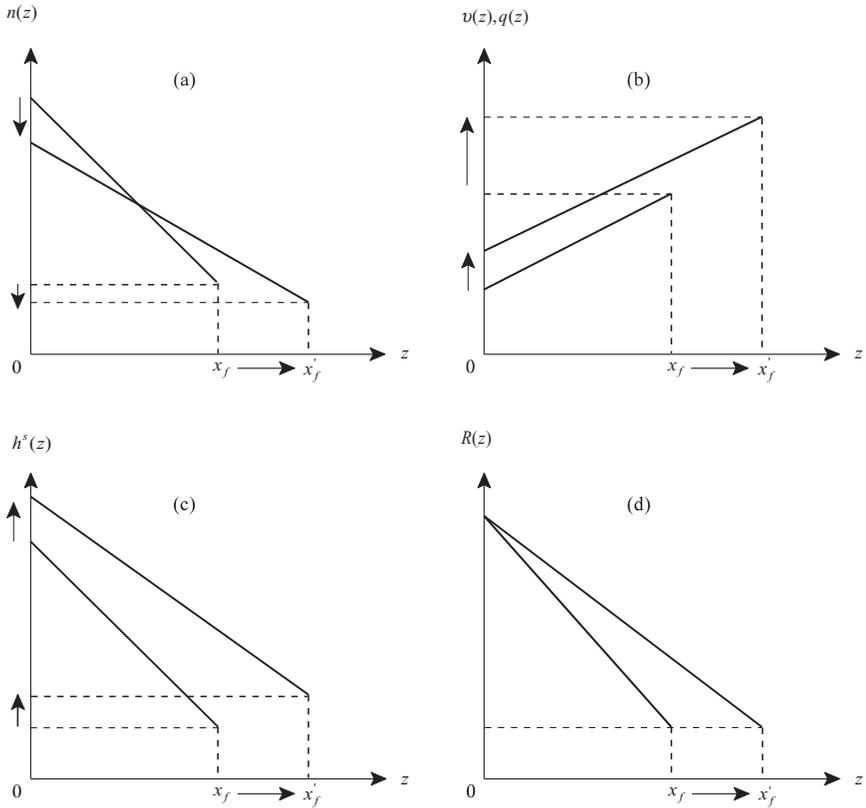


Figure 2: Effects of an increase in β or A , or a decrease in $\tau_H, \tau_S, \tau_L, \tau_D$ or ϕ .

effect on housing quantity. Moreover, we find that the direct commuting effect dominates the indirect overall quality effect. As a result, a lower unit commuting cost increases the city size.

Second, we can evaluate how the population density, housing quality, housing price, housing stock, and land rent schedules change in responses to shifts in preference and technology parameters. The results are conveniently plotted in Figure 2. Briefly, an increase in β or A , or a decrease in ϕ expands the urban fringe and encourages households to relocate from the inner to the outer city. As a result, both the population density and the land rent gradient are flatter, though the land rent at the CBD remains unchanged.

Third, it is interesting that the effects of a reduction in the unit commuting cost (t) on the housing stock, housing quality, and housing prices depend crucially on the location of the house (see Figure 3). In the inner city, lower

response to an increase in β or A , or a decrease in ϕ . This case may arise particularly when the urban fringe is very responsive to such parameter shifts.

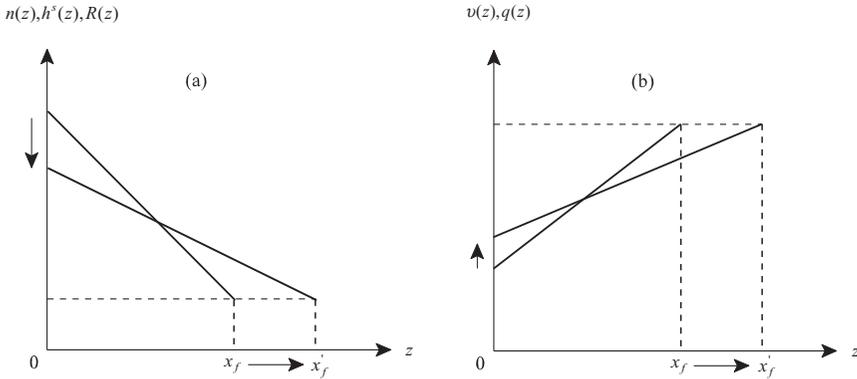


Figure 3: Effects of a reduction in t .

commuting cost reduces the number of floors and raises housing quality and prices; such results are reverse in the outer city. Accordingly, a rapid increase in commuting costs may result in a quick deterioration of the inner city from the aspect of housing quality and housing prices. Moreover, a decrease in the unit commuting cost not only flattens the population density and the land rent gradient but also reduces the land rent at the CBD.

Finally, the city size, the population density, and housing prices are most responsive to changes in the preference parameter (β), followed by the production and commuting technology parameters (A and t). Housing quality is mainly driven by the preference and the production technology parameters, whereas the housing stock is most responsive to the production technology parameter, followed by the commuting cost parameter. The land rent is only responsive to the commuting cost parameter. While the development cost parameter (ϕ) generates trivial effects on inner city allocations, its effects on the urban fringe and allocations in the outer city are nonnegligible.

4.3. Tax Policy Effects

We turn now to examining the effects of the four tax instruments ($\tau_H, \tau_S, \tau_L, \tau_D$) on $x_f, q(z), h^s(z), v(z), n(z), R(z)$, and $u(0)$.

First, land and development tax rates (τ_L and τ_D) generate qualitatively similar effects on the quality demand schedule as the development fee. From (17), a reduction in either tax rate shifts the housing quality demand schedule upward without affecting the quality supply schedule. It thus results in higher housing quality, lower housing quantity, and higher housing prices/values.

Second, in addition to their effects on the housing quality demand schedule, property, and gross revenue tax rates (τ_H and τ_S) create further distortions on housing quantity (see (15)) and capital demand (see (10)). A reduction in property or gross revenue tax rate lowers capital distortion, thus causing the housing quality supply schedule to shift outward. Although this seems to resemble an improvement in productivity, the presence of congestion no longer guarantees that the quality supply effect is outweighed by the quality

demand effect. Hence, equilibrium housing quantity at each location need not be lower (Figure 1 plots the case where the demand effect dominates).

Third, a reduction in the property tax rate (τ_H) has an additional effect shifting the unit bid price and housing value loci outward, as it raises the preference bias toward housing quality. This leads to further complication. From (3), (4), and the left panel of Figure 1(b), the housing value or the quality-augmented housing price (v) is unambiguously higher, but the equilibrium unit bid price (P) may rise or fall in response to a lower property tax rate.

Fourth, an increase in any housing-related tax rate always encourages relocation from the outer to the inner city, thereby shrinking the urban fringe. While the depressing effect of the property tax on the city size corroborates with findings in previous studies such as Arnott and MacKinnon (1977) and Song and Zenou (2006), our result suggests that other taxes can also be adopted to reduce the problem of urban sprawl.

Finally, under all tax policy changes, their negative effects on household welfare via housing quality (and the direct effect in the case of a property tax) may be offset by the positive effect via the provision of the local public good. As a consequence, the net effects are generally ambiguous.

These analytical findings can be summarized below.

PROPOSITION 6 (Tax Policy Effects): *Under Condition N, property, gross revenue, land, and development taxes all have negative effects on the overall quality of housing, equilibrium housing quality and prices and equilibrium city size. While land and development tax rates affect the housing stock positively, the effects of property and gross revenue tax rates on the housing stock are ambiguous. Their effects on household welfare are all ambiguous.*

Interestingly, the discussion above suggests that housing-related taxes may affect housing quality and quantity very differently within our general equilibrium framework.

We next report the numerical results with respect to tax policy parameters in Table 2. We find that a reduction in either property or gross revenue tax rate lowers housing quantity (and the population density) at each location, implying that the quality demand effect dominates the quality supply effect. Quantitatively, both property and gross revenue taxes have strong effects on the urban fringe, the population density, housing quality, the housing stock, and the aggregate commuting cost–income ratio. Housing prices/values and the total gross revenue–income ratio respond most significantly to changes in the property tax, followed by the gross revenue tax. While the property tax is most influential for the aggregate land rent–income ratio and local public good provision, gross revenue, and land taxes also have strong effects. Market land rents are most responsive to shifts in property and land taxes, followed by the gross revenue tax. The total development fee–total gross revenue ratio is decreasing in all but the property tax; that is, all but the property tax have stronger negative effects on the total development fee than on total gross revenue. Finally, all four taxes are harmful for household welfare.

Table 2: Comparative statics with respect to tax policy parameters

	x^f	$u(0)$	Ω	$\frac{ACC}{N_y}$	$\frac{ALR}{N_y}$	$\frac{TGR}{N_y}$	$\frac{TDF}{TGR}$	$\frac{G}{N_y}$
Benchmark	9.01	1.114	1167	0.0300	0.0546	0.0963	0.0375	0.0288
$\tau_H \cdot 0.8$	9.21 (2.07)	1.117 (0.13)	1173 (0.45)	0.0305 (1.34)	0.0568 (4.02)	0.1000 (3.91)	0.0368 (-1.77)	0.0269 (-6.52)
$\tau_H \cdot 0.9$	9.11 (1.02)	1.115 (0.40)	1170 (0.22)	0.0302 (0.66)	0.0557 (1.98)	0.0981 (1.92)	0.371 (-0.88)	0.0279 (-3.19)
$\tau_H \cdot 1.1$	8.93 (-1.00)	1.112 (-0.13)	1165 (-0.22)	0.0299 (-0.65)	0.0535 (-1.92)	0.0945 (-1.86)	0.0378 (0.88)	0.0297 (3.06)
$\tau_H \cdot 1.2$	8.84 (-1.98)	1.111 (-0.27)	1162 (-0.43)	0.0297 (-1.29)	0.0525 (-3.77)	0.0927 (-3.66)	0.0381 (1.75)	0.0305 (6.00)
$\tau_S \cdot 0.8$	9.13 (1.26)	1.114 (0.03)	1169 (0.14)	0.0303 (0.81)	0.0559 (2.43)	0.0970 (0.73)	0.0377 (0.53)	0.0277 (-3.92)
$\tau_S \cdot 0.9$	9.08 (0.63)	1.114 (0.01)	1168 (0.07)	0.0302 (0.41)	0.0552 (1.21)	0.0966 (0.36)	0.0376 (0.26)	0.0282 (-1.95)
$\tau_S \cdot 1.1$	8.96 (-0.63)	1.113 (-0.01)	1167 (-0.07)	0.0299 (-0.41)	0.0539 (-1.21)	0.0959 (-0.37)	0.0374 (-0.27)	0.0294 (1.93)
$\tau_S \cdot 1.2$	8.91 (-1.26)	1.113 (-0.03)	1166 (-0.14)	0.0298 (-0.82)	0.0533 (-2.41)	0.0956 (-0.73)	0.0373 (-0.53)	0.0299 (3.85)
$\tau_L \cdot 0.8$	9.03 (0.16)	1.114 (0+)	1169 (0.11)	0.0301 (0.03)	0.0558 (2.27)	0.096277 (0.007)	0.03753 (0.15)	0.0276 (-4.32)
$\tau_L \cdot 0.9$	9.03 (0.08)	1.114 (0+)	1168 (0.05)	0.0301 (0.02)	0.0552 (1.12)	0.096273 (0.003)	0.03751 (0.08)	0.0282 (-2.14)
$\tau_L \cdot 1.1$	9.01 (-0.08)	1.114 (0-)	1167 (-0.05)	0.0300 (-0.02)	0.0540 (-1.10)	0.096267 (-0.003)	0.03745 (-0.07)	0.0294 (2.09)
$\tau_L \cdot 1.2$	9.01 (-0.16)	1.114 (0-)	1166 (-0.10)	0.0300 (-0.03)	0.0534 (-2.17)	0.096264 (-0.007)	0.03742 (-0.15)	0.0300 (4.13)
$\tau_D \cdot 0.8$	9.06 (0.40)	1.114 (0-)	1167.31 (0.004)	0.0301 (0.08)	0.0546 (0.09)	0.09629 (0.017)	0.0376 (0.39)	0.0287 (-0.17)
$\tau_D \cdot 0.9$	9.04 (0.20)	1.114 (0-)	1167.29 (0.002)	0.0301 (0.04)	0.0546 (0.04)	0.09628 (0.009)	0.0375 (0.19)	0.0288 (-0.08)
$\tau_D \cdot 1.1$	9.00 (-0.20)	1.114 (0-)	1167.24 (-0.002)	0.0300 (-0.04)	0.0546 (-0.04)	0.09626 (-0.009)	0.0374 (-0.19)	0.0288 (0.08)
$\tau_D \cdot 1.2$	8.98 (-0.40)	1.114 (0-)	1167.22 (-0.004)	0.0300 (-0.08)	0.0545 (-0.09)	0.09625 (-0.017)	0.0373 (-0.38)	0.0288 (0.16)

(Continued)

5. Welfare Analysis

To conduct welfare analysis, we begin by computing the value of absentee landlords. The aggregate land rent (ALR) in the city is measured by $ALR \equiv 2 \int_0^{x_f} R(z) dz$, while the aggregate agricultural land rent is $2R_A(\bar{x} - x_f)$. Since the total outside option value facing the absentee landlords is $2R_A\bar{x}$, the aggregate value of absentee landlords is given by the net aggregate land rent: $NALR = ALR + 2R_A(\bar{x} - x_f) - 2R_A\bar{x} = ALR - 2R_Ax_f$.

Under our setup, all developers earn zero profit in equilibrium and hence their welfare measure is trivial. Moreover, because the household utility function is linear in consumption, the city-wide utilitarian welfare can simply

Table 2: Continued.

	$n(0)$	$n(x^f)$	$q(0)$	$q(x^f)$	$h^s(0)$	$h^s(x^f)$	$v(0)$	$v(x^f)$	$R(0)$
Benchmark	381	2.19	1.37	11.29	520.96	24.70	0.0789	0.1834	17.92
$\tau_H \cdot 0.8$	376.35 (-1.18)	2.10 (-4.22)	1.4080 (2.93)	11.7702 (4.25)	529.88 (1.71)	24.67 (-0.14)	0.0819 (3.87)	0.1915 (4.42)	18.39 (2.67)
$\tau_H \cdot 0.9$	378.61 (-0.59)	2.14 (-2.12)	1.3876 (1.45)	11.5260 (2.09)	525.37 (0.85)	24.69 (-0.07)	0.0804 (1.91)	0.1874 (2.18)	18.15 (1.32)
$\tau_H \cdot 1.1$	383.09 (0.59)	2.23 (2.14)	1.3486 (-1.41)	11.0617 (-2.02)	516.64 (-0.83)	24.72 (0.07)	0.0774 (-1.85)	0.1796 (-2.07)	17.69 (-1.28)
$\tau_H \cdot 1.2$	385.31 (1.17)	2.28 (4.29)	1.3299 (-2.78)	10.8409 (-3.98)	512.41 (-1.64)	24.74 (0.14)	0.0760 (-3.63)	0.1759 (-4.09)	17.46 (-2.54)
$\tau_S \cdot 0.8$	378.11 (-0.72)	2.13 (-2.59)	1.3921 (1.77)	11.5798 (2.57)	526.37 (1.04)	24.68 (-0.09)	0.0794 (0.71)	0.1853 (1.04)	18.21 (1.62)
$\tau_S \cdot 0.9$	379.47 (-0.36)	2.16 (-1.31)	1.3800 (0.89)	11.4346 (1.28)	523.67 (0.52)	24.69 (-0.04)	0.0791 (0.35)	0.1844 (0.55)	18.06 (0.81)
$\tau_S \cdot 1.1$	382.26 (0.37)	2.22 (1.34)	1.3557 (-0.89)	11.1460 (-1.27)	518.24 (-0.52)	24.71 (0.04)	0.0786 (-0.36)	0.1825 (-0.49)	17.77 (-0.81)
$\tau_S \cdot 1.2$	383.67 (0.74)	2.25 (2.70)	1.3436 (-1.77)	11.0028 (-2.54)	515.51 (-1.05)	24.72 (0.09)	0.0783 (-0.71)	0.1815 (-1.04)	17.63 (-1.62)
$\tau_L \cdot 0.8$	380.83 (0.0-)	2.18 (-0.55)	1.36790 (0.003)	11.3154 (0.23)	520.93 (-0.004)	24.62 (-0.32)	0.078852 (0.0012)	0.1836 (0.11)	18.32 (2.27)
$\tau_L \cdot 0.9$	380.84 (0.0-)	2.18 (-0.27)	1.36787 (0.002)	11.3026 (0.11)	520.95 (-0.002)	24.66 (-0.16)	0.078852 (0.0006)	0.1835 (0.05)	18.12 (1.12)
$\tau_L \cdot 1.1$	380.87 (0.0+)	2.19 (0.27)	1.36783 (-0.002)	11.2773 (-0.11)	520.97 (0.002)	24.74 (0.16)	0.078851 (-0.0006)	0.1833 (-0.05)	17.72 (-1.10)
$\tau_L \cdot 1.2$	380.88 (0.0+)	2.20 (0.55)	1.36781 (-0.003)	11.2647 (-0.22)	520.98 (0.004)	24.78 (0.32)	0.078850 (-0.0012)	0.1833 (-0.05)	17.53 (-2.17)
$\tau_D \cdot 0.8$	380.79 (-0.02)	2.16 (-1.40)	1.3680 (0.008)	11.36 (0.58)	520.90 (-0.011)	24.50 (-0.83)	0.078854 (0.003)	0.1839 (0.27)	17.92 (0)
$\tau_D \cdot 0.9$	380.82 (-0.01)	2.17 (-0.70)	1.3679 (0.004)	11.32 (0.29)	520.93 (-0.006)	24.60 (-0.42)	0.078852 (0.002)	0.1836 (0.11)	17.92 (0)
$\tau_D \cdot 1.1$	380.89 (0.01)	2.20 (0.70)	1.3678 (-0.004)	11.26 (-0.29)	520.99 (0.006)	24.81 (0.41)	0.078850 (-0.002)	0.1832 (-0.11)	17.92 (0)
$\tau_D \cdot 1.2$	380.93 (0.02)	2.22 (1.41)	1.3677 (-0.008)	11.23 (-0.57)	521.01 (0.011)	24.91 (0.83)	0.078849 (-0.003)	0.1830 (-0.22)	17.92 (0)

be measured by aggregating the values of households and absentee landlords obtained above with equal weights:

$$\Omega \equiv Nu(0) + NALR. \tag{21}$$

Should the government be more concerned with inequalities between households and absentee landlords, one may consider a more egalitarian social welfare measure using harmonic average of the welfare of households and the welfare of absentee landlords:

$$\Omega^{EGA} \equiv \left(\frac{\frac{1}{y}}{\frac{1}{y} + \frac{1}{NALR}} \right) Nu(0) + \left(1 - \frac{\frac{1}{y}}{\frac{1}{y} + \frac{1}{NALR}} \right) NALR.$$

One may also examine the welfare of households only (using $u(0)$).

We next simulate the model to conduct policy and welfare analysis. We are interested in conducting the following “tax incidence” exercises: (1) property tax (τ_H) versus gross revenue tax (τ_S); (2) property tax (τ_H) versus land tax (τ_L); (3) gross revenue tax (τ_S) versus land tax (τ_L); and (4) gross revenue tax (τ_S) versus development tax (τ_D). In each of the tax-incidence exercise for a pair of taxes, the local public amenity spending is fixed at the benchmark value with the other two tax rates also set at their benchmark values reported in Section 4.2. We have studied the three welfare measures as mentioned above, reaching qualitatively similar conclusions. For brevity, we thus focus on illustrating the best combination of various tax instruments to achieve highest welfare measured by Ω .

We begin by summarizing the main findings obtained from pairwise tax incidence exercises. First, we can identify the land tax as the most preferred tax instrument, followed by the development tax, then by the gross revenue tax and finally by the property tax. Second, while the land tax being the most preferred reaffirms the idea underlying the Henry George Theorem in urban economics, the land tax should not be set at 100%. As discussed in Proposition 1, the land tax is distortionary in our model due to housing congestion externalities through which overall housing quality and hence other endogenous variables are affected. Third, since the development tax generates less distortion (no distortion on capital demand or households’ willingness to pay), it is more preferred to property and gross revenue taxes. Fourth, it is somewhat surprising that the gross revenue tax is more preferred to the property tax—in the optimal taxation literature under perfect competition, a tax on the supply side usually generates a larger distortion. Our anti-conventional finding is due mainly to household’s demand for housing quality and firm’s local monopoly power over the housing market. Notably, all of these findings are robust to the three welfare measures and a reasonably wide range of parameters ($\pm 20\%$ from their benchmark values).

Next, we would like to inquire what the best combination of all tax instruments is to achieve highest welfare measured by Ω . Under the assumptions that all tax rates are nonnegative (i.e., no subsidy) and that the LPG is entirely financed by these four tax instruments, the welfare-maximizing tax mix is given by: $\tau_H = \tau_S = 0$, $\tau_L = 45.4\%$, and $\tau_D = 84\%$. That is, in the absence of income and factor taxes, a globally optimal tax scheme in the housing market is to eliminate property and gross revenue taxes while imposing a higher development tax rate than the land tax rate.¹¹ This optimal tax mix

¹¹That the development tax is chosen at a higher rate than the land tax under the globally optimal tax scheme gives rise to a different preference ordering from one under pairwise tax incidence exercises. This is because property and gross revenue tax rates are zero in the globally optimal tax mix, while they are set at their respective benchmark values in the pairwise tax incidence case.

result is robust to different welfare measures and to nondrastic parameter changes.

To the end, we must note that one of our tax incidence findings may be upset when the degree of the housing congestion externality is sufficiently low (say, ε down from its benchmark value of 0.3 to 0.05). In this case, pairwise tax incidence exercises suggest that the development tax is now more preferred than the land tax. The welfare-maximizing tax mix becomes: $\tau_H = 0$, $\tau_S = 2.9\%$, $\tau_L = 42.5\%$, and $\tau_D = 100\%$. Thus, while the gross revenue tax need not be fully eliminated, it is the development fee rather than the land rent that should be taxed 100%. Intuitively, in the absence of a significant housing congestion effect, both land and development taxes generate negligible distortions to the housing market. In this case, the welfare ordering of these two taxes is mainly driven by their effects on the local public good provision, which turns out to be in favor of the development tax.

6. Concluding Remarks

We have developed a competitive spatial equilibrium model with local public amenities to characterize the schedules of housing quality, housing prices, land rent, and population and housing density. We have also evaluated quantitatively the equilibrium effects and welfare implications of an array of housing-related tax policies, including a gross revenue tax, a property tax, a land tax, and a development tax. We find that housing quality and housing prices/values are higher in the outer city than in the inner city and are positively related in response to any tax, preference, and technology shifts, consistent with the observations from the American Housing Survey. Moreover, in response to a reduction in any of the housing-related taxes, the urban fringe expands, though both housing quality and housing values rises. Thus, the resulting problem of urban sprawl need not be harmful for individual households. While a reduction in development and land taxes raises quality-augmented housing stocks, the corresponding effects of property and gross revenue taxes are ambiguous. Numerically, we find that a globally optimal tax scheme in the housing market is always to eliminate the property tax and to impose a lower gross revenue tax rate (possibly fully eliminated) than either the development or the land tax.

Along these lines, there are at least four potentially interesting avenues for future research. One is to consider variable property tax rates across locations and derive the optimal property tax schedule within the city. Because housing quality increases in the distance away from the CBD, property taxes distort housing demand in the outer city more severely. Thus, it is expected that the property tax schedule ought to be downward sloping in the distance away from the CBD.

The next is to permit location-dependent local public good consumption. On the one hand, one may allow the service of public amenities to depend

negatively on the distance from the public amenity site (the city center).¹² We may now define households' valuation of housing quality to include public amenity services. When households' preferences for public amenities are sufficiently strong and the spatial discount of the public good service is sufficiently large, it is possible to have downward sloping housing price and housing quality schedules, capturing a common feature in Asian and European cities.¹³ On the other hand, one may consider public highway service that benefits outskirts residents more than inner city residents. This will yield exactly opposite results to that discussed above.

Finally, one may generalize the model to a dynamic setting such that the durability feature of housing can be incorporated. In this case, one may tie housing quality with the age of the structure. Moreover, a distortionary housing tax, particularly, property and gross revenue taxes, may now affect households' investment in housing quality (home improvements) and their intertemporal consumption-saving trade-offs, leading to greater welfare costs. Of course, in doing so, the equilibrium system will feature two-dimensional differential equations in both location and time. This complicates the analysis significantly and may require further simplification in order for the study to be carried out.

Appendix

Mathematical Details of Step 1: From (15), we obtain

$$h(x) = A^{\frac{1}{\varepsilon}} \left[\left(\frac{1 - \tau_S}{1 + \tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right]^{\frac{\alpha}{\varepsilon}} \left[q_0 + \frac{t}{\beta(1 - \theta)} x \right]^{-\frac{1-\alpha\theta}{\varepsilon\theta}} \tag{A1}$$

Substituting (A1) into (10) we have

$$k(x) = \left[\left(\frac{1 - \tau_S}{1 + \tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right] \left[q_0 + \frac{t}{\beta(1 - \theta)} x \right] \tag{A2}$$

Using (13) and (16), we can solve q_0 as a decreasing function of the urban fringe x_f :

$$q_0 = B - \frac{t}{\beta(1 - \theta)} x_f, \tag{A3}$$

¹²See the simple setup of a "travel-for" local public good and further discussion in Peng and Wang (2005).

¹³Denoting the service of public amenities as g , one may specify: $g(z) = e^{-\delta z} G$, where $\delta > 0$ measures the spatial discount of the public good service. Then households' valuation of housing quality inclusive of public amenity services is given by, $\beta q(z)^\theta + \gamma (\ln G - \delta z)$. Downward sloping housing price and quality gradient can be derived when $\gamma\delta$ is sufficiently high compared to β .

where $B(\beta, A, \phi, \tau_H, \tau_S, \tau_L, \tau_D) \equiv \left\{ \frac{(1-\alpha\theta)A^{\frac{1}{\varepsilon}} \left(\frac{\alpha\theta}{r}\right)^{\frac{\alpha}{\varepsilon}} \left[\left(\frac{1-\tau_S}{1+\tau_H}\right)\beta\theta\right]^{\frac{\alpha+\varepsilon}{\varepsilon}}}{(1+\tau_D)\phi + (1+\tau_L)R_A} \right\}^{1-\frac{\varepsilon\theta}{(\alpha+\varepsilon)\theta}}$. Under Condition N, $\frac{\partial B}{\partial \beta} > 0$, $\frac{\partial B}{\partial A} > 0$, $\frac{\partial B}{\partial \phi} < 0$, and $\frac{\partial B}{\partial \tau_i} < 0$ ($i = H, S, L, D$). Substituting (A3) into (A1) implies

$$n(x) = h(x) = A^{\frac{1}{\varepsilon}} \left[\left(\frac{1 - \tau_S}{1 + \tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right]^{\frac{\alpha}{\varepsilon}} \left[B - \frac{t}{\beta(1 - \theta)} (x_f - x) \right]^{-\frac{1-\alpha\theta}{\varepsilon\theta}}, \quad (A4)$$

which is decreasing in the distance from the CBD.

Next, we can plug (A4) into (14) and integrate it to obtain

$$x_f = \frac{\beta(1 - \theta)B}{t} \left[1 - \left(1 + D\frac{N}{2} \right)^{\frac{-\varepsilon\theta}{1-(\alpha+\varepsilon)\theta}} \right], \quad (A5)$$

where $D(\phi, t, \tau_H, \tau_S, \tau_L, \tau_D) \equiv \frac{\left(\frac{1-\tau_S}{1+\tau_H}\right)t[1-(\alpha+\varepsilon)\theta](1-\alpha\theta)}{\varepsilon(1-\theta)[(1+\tau_D)\phi + (1+\tau_L)R_A]}$. Under Condition N, $\frac{\partial D}{\partial \phi} < 0$, $\frac{\partial D}{\partial t} > 0$, and $\frac{\partial D}{\partial \tau_i} < 0$ ($i = H, S, L, D$). We can then substitute (A5) into (A4) and use (A3) (and change uniformly the location index in equilibrium to z) to derive

$$n(z) = h(z) = A^{\frac{1}{\varepsilon}} \left[\left(\frac{1 - \tau_S}{1 + \tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right]^{\frac{\alpha}{\varepsilon}} \left[q_0 + \frac{t}{\beta(1 - \theta)} z \right]^{-\frac{1-\alpha\theta}{\varepsilon\theta}} \quad (A6)$$

as well as to determine q_0 as in (17). Under Condition N, straightforward differentiation leads to $\frac{\partial q_0}{\partial \beta} > 0$, $\frac{\partial q_0}{\partial A} > 0$, $\frac{\partial q_0}{\partial \phi} < 0$, $\frac{\partial q_0}{\partial t} < 0$, and $\frac{\partial q_0}{\partial \tau_i} < 0$ ($i = H, S, L, D$).

Mathematical Details of Step 2: Substituting (17) into (6), we obtain the housing quality schedule in (18). We can further substitute (17) into (3) and (13) to derive

$$p(z) = \frac{\beta\theta}{1 + \tau_H} \left[q_0 + \frac{t}{\beta(1 - \theta)} z \right]^{-\frac{1-\theta}{\theta}} \quad (A7)$$

$$R(z) = \frac{(1 + \tau_D)\phi + (1 + \tau_L)R_A}{1 + \tau_L} \times \left[\left(1 + D\frac{N}{2} \right)^{\frac{-\varepsilon\theta}{1-(\alpha+\varepsilon)\theta}} + \frac{t}{\beta B(1 - \theta)} z \right]^{-\frac{1-(\alpha+\varepsilon)\theta}{\varepsilon\theta}} - \frac{1 + \tau_D}{1 + \tau_L} \phi, \quad (A8)$$

where $R(z)$ is decreasing in z under Condition N. Combining (A6) and (18), we obtain the equilibrium housing stock in (19). Finally, (4) and (17) together yield the equilibrium housing price in (20).

Mathematical Details of Step 3: From (2), (3), and (6), we have

$$c(z) = y - tz - (1 + \tau_H)\beta\theta \left[q_0 + \frac{t}{\beta(1 - \theta)}z \right]^\theta. \tag{A9}$$

Substituting (A9) into (1), we obtain the indirect utility

$$u(z) \equiv U(c(z), q(z), z) = y - tz + \beta [1 - (1 + \tau_H)\theta] \left[q_0 + \frac{t}{\beta(1 - \theta)}z \right]^\theta + \gamma \ln G,$$

where the equilibrium provision of LPG, G , is pinned down by plugging (A7), (A8), and (19) into (12) (which is not fully spelled out for the sake of brevity). Under locational no-arbitrage, the equilibrium utility level can thus be computed as

$$u(0) = y + \beta [1 - (1 + \tau_H)\theta] (q_0)^\theta + \gamma \ln G. \tag{A10}$$

Comparative Statics: Straightforward differentiation under condition N yields the comparative-static results as reported in Section 4.1. Specifically, from (A5), we derive

$$\frac{\partial x_f}{\partial \beta} > 0, \frac{\partial x_f}{\partial A} > 0, \frac{\partial x_f}{\partial \phi} < 0, \frac{\partial x_f}{\partial \tau_H} < 0, \frac{\partial x_f}{\partial \tau_S} < 0, \frac{\partial x_f}{\partial \tau_L} < 0, \frac{\partial x_f}{\partial \tau_D} < 0.$$

Because (A6) can be rewritten as

$$n(z) = A^{\frac{1}{\varepsilon}} \left[\left(\frac{1 - \tau_S}{1 + \tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right]^{\frac{\alpha}{\varepsilon}} \left[q_0 + \frac{t}{\beta(1 - \theta)}z \right]^{-\frac{1-\alpha\theta}{\varepsilon\theta}},$$

we obtain

$$\begin{aligned} \frac{\partial n(z)}{\partial \beta} &\geq 0, \frac{\partial n(z)}{\partial A} \geq 0, \frac{\partial n(z)}{\partial \phi} > 0, \frac{\partial n(z)}{\partial \tau_H} \geq 0, \\ \frac{\partial n(z)}{\partial \tau_S} &\geq 0, \frac{\partial n(z)}{\partial \tau_L} > 0, \frac{\partial n(z)}{\partial \tau_D} > 0 \end{aligned}$$

(the comparative statics of $h(z)$ are identical). Further from (A8) we derive

$$\begin{aligned} \frac{\partial R(z)}{\partial \beta} &> 0, \frac{\partial R(z)}{\partial A} > 0, \frac{\partial R(z)}{\partial \phi} \geq 0, \frac{\partial R(z)}{\partial \tau_H} < 0, \\ \frac{\partial R(z)}{\partial \tau_S} &< 0, \frac{\partial R(z)}{\partial \tau_L} \geq 0, \frac{\partial R(z)}{\partial \tau_D} \geq 0. \end{aligned}$$

Finally, one can easily show

$$\begin{aligned} \frac{\partial u(0)}{\partial \beta} > 0, \frac{\partial u(0)}{\partial A} > 0, \frac{\partial u(0)}{\partial \phi} < 0, \frac{\partial u(0)}{\partial \tau_H} \geq 0, \frac{\partial u(0)}{\partial \tau_S} \geq 0, \frac{\partial u(0)}{\partial \tau_L} \geq 0, \\ \frac{\partial u(0)}{\partial \tau_D} \geq 0; \frac{\partial ALR}{\partial \beta} > 0, \frac{\partial ALR}{\partial A} > 0, \frac{\partial ALR}{\partial \phi} \geq 0, \frac{\partial ALR}{\partial \tau_H} < 0, \frac{\partial ALR}{\partial \tau_S} < 0, \\ \frac{\partial ALR}{\partial \tau_L} \geq 0, \frac{\partial ALR}{\partial \tau_D} \geq 0; \frac{\partial \Omega}{\partial \beta} > 0, \frac{\partial \Omega}{\partial A} > 0, \frac{\partial \Omega}{\partial \phi} \geq 0, \frac{\partial \Omega}{\partial \tau_H} \geq 0, \frac{\partial \Omega}{\partial \tau_S} \geq 0, \\ \frac{\partial \Omega}{\partial \tau_L} \geq 0, \frac{\partial \Omega}{\partial \tau_D} \geq 0. \end{aligned}$$

References

- ARNOTT, R., and J. MACKINNON (1977) The effects of the property tax: A general equilibrium simulation, *Journal of Urban Economics* **4**, 389–407.
- ARNOTT, R., R. DAVIDSON, and D. PINES (1983) Housing quality, maintenance and rehabilitation, *Review of Economic Studies* **50**, 467–494.
- ARNOTT, R., R. BRAID, R. DAVIDSON, and D. PINES (1999) A general equilibrium spatial model of housing quality and quantity, *Regional Science and Urban Economics* **29**, 283–316.
- BERGSTROM, T., and R. CORNES (1983) Independence of allocative efficiency from distribution in the theory of public goods, *Econometrica* **51**, 1753–1765.
- BERLIANT, M., and T. TEN RAA (1991) On the continuum approach of spatial and some local public goods or product differentiation models: Some problems, *Journal of Economic Theory* **55**, 95–120.
- BERLIANT, M., S. PENG, and P. WANG (2006) Optimal number and locations of public facilities, *Regional Science and Urban Economics* **36**, 207–226.
- BRUECKNER, J. (1981) A dynamic model of housing production, *Journal of Urban Economics* **10**, 1–14.
- BRUECKNER, J. (2001) Do local governments engage in strategic property–tax competition?, *National Tax Journal* **54**, 203–230.
- BRUECKNER, J., and H. KIM (2003) Urban sprawl and the property tax, *International Tax and Public Finance* **10**, 5–23.
- CREMER, H., and F. GAHVARI (1998) On optimal taxation of housing, *Journal of Urban Economics* **43**, 315–335.
- FUJITA, M. (1982) Spatial pattern of residential development, *Journal of Urban Economics* **12**, 22–52.
- FUJITA, M. (1989) *Urban Economic Theory: Land Use and City Size*, Cambridge: Cambridge University Press.
- GREEN, R. K., and S. MALPEZZI (2000) A Primer on U.S. Housing Markets and Housing Policy, Center for Urban Land Economics Research Working Paper, University of Wisconsin, Madison.
- HENDERSON, J. V. (1985) *Economic Theory and the Cities*, New York: Academic Press.
- KANEMOTO, Y. (1985) Housing as an asset and the effects of property taxation on the residential development process, *Journal of Urban Economics* **17**, 145–166.

- LEUNG, C. K.-Y. (1999) Income tax, property tax, and tariff in a small open economy, *Review of International Economics* **7**, 541–554.
- LIN, C., C. MAI, and P. WANG (2004) Urban land use and housing policy in an endogenously growing monocentric city model, *Regional Science and Urban Economics* **34**, 241–261.
- PENG, S., and P. WANG (2005) Sorting by foot: Consumable travel-for local public goods and stratification, *Canadian Journal of Economics* **38**, 1224–1252.
- SIEG, H., V. K. SMITH, H. S. BANZHAF, and R. WALSH (2004) Estimating the general equilibrium benefits of large changes in spatially delineated public goods, *International Economic Review* **45**, 1047–1077.
- SONG, Y., and Y. ZENOU (2006) Property tax and urban sprawl: Theory and implications for U.S. cities, *Journal of Urban Economics* **60**, 519–534.
- SWEENEY, J. (1974) Quality, commodity hierarchies, and housing markets, *Econometrica* **42**, 147–168.
- TURNOVSKY, S., and T. OKUYAMA (1994) Taxes, housing, and capital accumulation in a two-sector growing economy, *Journal of Public Economics* **53**, 245–267.
- WANG, P. (1993) Agglomeration in a linear city with heterogeneous households, *Regional Science and Urban Economics* **23**, 291–306.
- WILSON, J. D. (1985) Optimal property taxation in the presence of interregional capital mobility, *Journal of Urban Economics* **17**, 73–89.