Firm Heterogeneity and Wealth Distribution

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Abstract

In this paper I study how firm heterogeneity affects wealth distribution through entrepreneu-rial income and capital gains. As shown in Hottman, Redding and Weinstein 2016, size distribution of firms is highly skewed. Top 1% of firms on average have market shares of 50%. I develop a dynamic heterogeneous-agent general equilibrium model, where firms produce multiple products and households invest by bargaining with firm owners. Every product in this model has two dimensions: its rate of return and its optimal output. Every firm thus has two dimensions: its rate of return and size. Investors sort into different firms based on their wealth, which increases due to either their share of firm's profit or their capital gains. The distribution of product space is calibrated with Nielsen barcode dataset[1][2]. The simulation shows that the firm heterogeneity in its two dimensions implied by the dataset results in a severe wealth inequality. The model can explain 90% of the rise in top wealth concentration between 2003 and 2012 in the United States and shows that the upswing in the top 1% wealth share is due to the rise in the top 0.1%, which is consistent with Saez and Zucman 2016. Counterfactual exercises highlight entrepreneurship as the main driver in wealth equality and show that fluctuations in risk free rate of return and access to new investment opportunities reduce wealth inequality, of which each may offset up to half of the effect of entrepreneurship.

Key Words:Wealth Distribution, Entrepreneur Income, Firm Heterogeneity, Multi-product firms

JEL: D31, L11, M13

[1]Researcher(s) own analyses calculated (or derived) based in part on data from The Nielsen Company (US), LLC and marketing databases provided through the Nielsen Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business.

[2]The conclusions drawn from the Nielsen data are those of the researcher(s) and do not reflect the views of Nielsen. Nielsen is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.

1 Introduction

Wealth in the United States is heavily concentrated on the top of the distribution and the share of the wealthy keeps increasing over time. Wolff (2016) estimates a Gini coefficient of 0.871 in 2013. Saez and Zucman (2016) document a steady rising trend of wealth inequality from 2003 to 2012 as shown in Figure1.

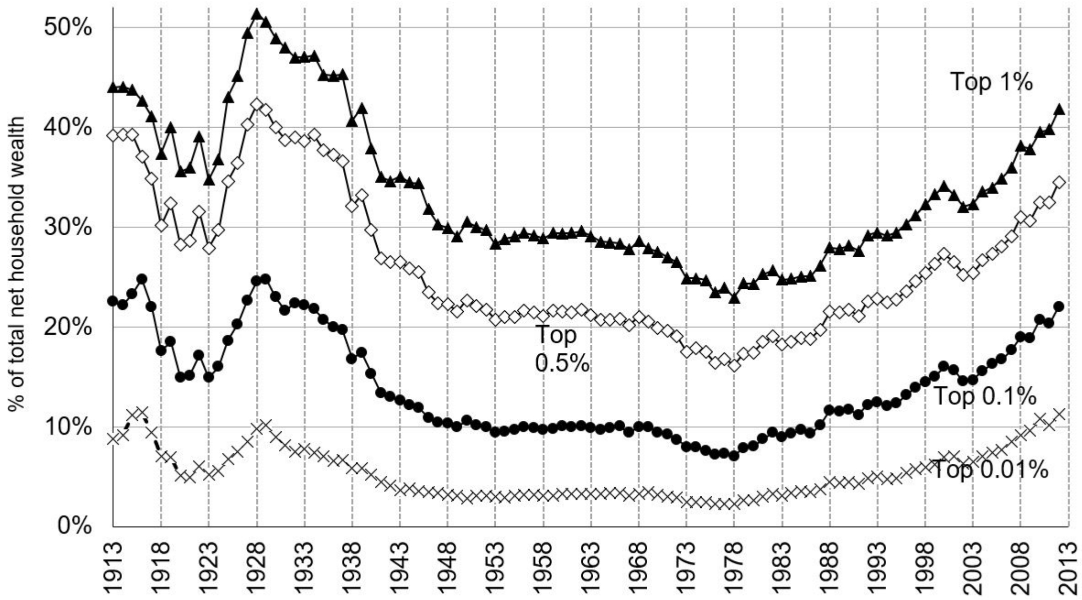

Figure 1: Top US household wealth shares obtained by capitalizing income. Taken from Saez and Zucman (2016).

Meanwhile, Hottman, Redding and Weinstein (2016) show that the size distribution of firms is also highly left-skewed. The largest firm in an industry typically sells 2,500 times more than the median firm. Ranked by revenue, on average the top 1% of firms in each product group have market shares of about 50%. Based on Nielsen barcode data, the size (measured by revenue) distribution of firms covered in the dataset in 2013 is 0.965 as shown in Figure2, which is even higher than that of wealth distribution. Considering the fact that the majority of richest Americans are entrepreneurs in large firms, one may wonder if it is possible that the wealthy people get rich simply by receiving huge profits from their dominating corporations.

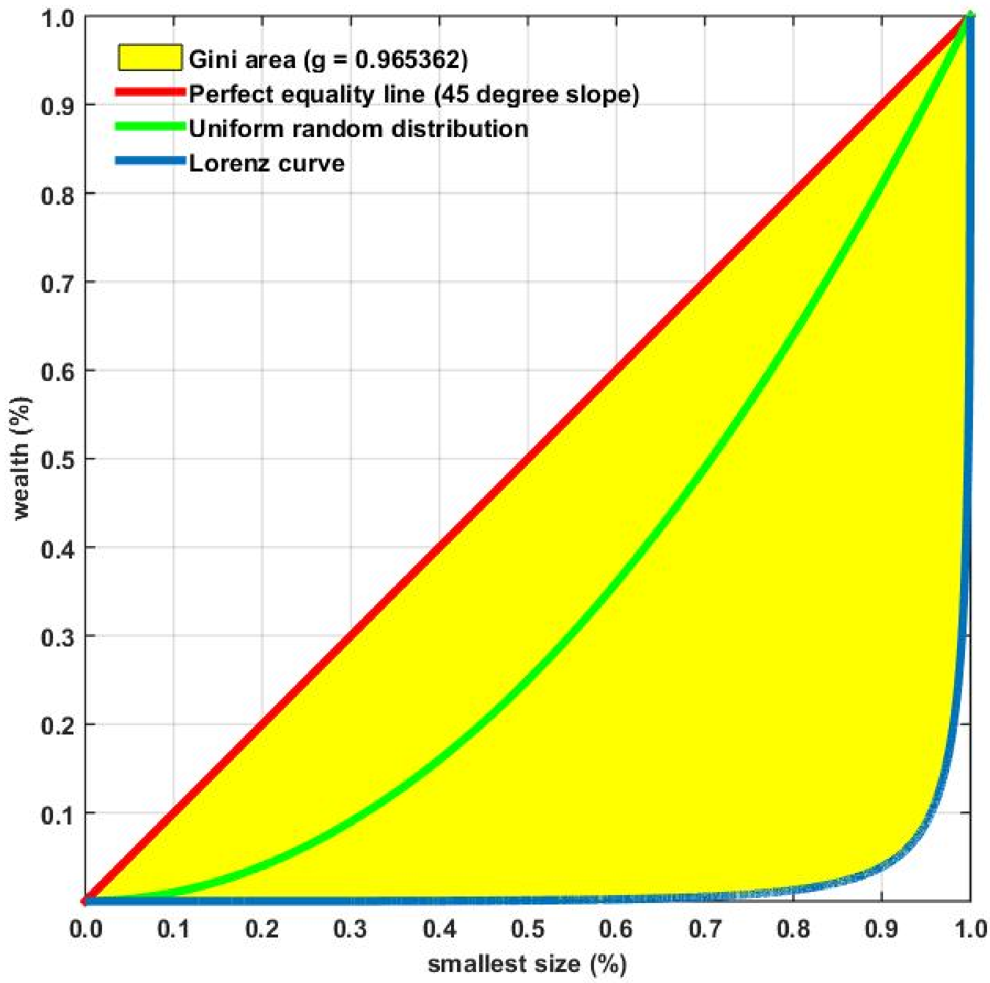

Figure 2: Cumulative distribution of firm size, measured with firm revenues, for firms covered in Nielsen barcode dataset in 2013.

This paper studies how firm heterogeneity in size and rate of return affects household wealth distribution through entrepreneurial income and capital gain. To that end I develop a dynamic general equilibrium model with heterogeneous households and multiproduct firms. In this model firm heterogeneity originates from entrepreneurs' inherent ability. Each entrepreneur has her own varieties of products which are produced by a multiproduct firm established by the entrepreneur and engaged in monopolistic competition. Each variety has its own rate of return and scale, which results in heterogeneity in the rate of return of firms and firm size. Firms with larger scope of products grow faster by benefiting more from stricter control in product quality. In other words, this generates a concentration on the top of distribution of firm size.

This model features a novel mechanism of investment between households with different wealth levels and firms with different sizes and rates of return. Investment in a firm is modeled as a bargaining between new incumbent investors over their own claims on firm profits, or equivalently, claims on firm assets. The difference between investment and new investors' claim on firm assets can be interpreted as, on one hand, an investment fee paid by new investors and, on the other hand, the capital gain to the incumbents. This bargaining process can be viewed as venture capital investment, IPO, or large trades on secondary markets in reality. Firms in the model have two dimensions: its rate of return and the firm size. This investment fee increases in both, which sorts households into different firms based on their wealth. Were it not for the investment fee, given the same rate of return, an investor would like to invest in relatively large firms because small firms may not be able to take in all her investment and thus a proportion of her wealth may be forced to be invested in risk free assets, which usually offers lower rate of return. Wealth of households, entrepreneurs in particular, accumulates partly because of the entrepreneurial income, which is their claim on their firm's profits, and partly because they receive capital gains.

I calibrate the model with the Nielsen barcode data and estimate the joint distribution of the two parameters governing the rate of return and the optimal output of products. The model is simulated with this distribution. The result shows that, even if beginning with an initial wealth distribution of relatively mild inequality, severe wealth concentration at the top appears and becomes increasingly heavier. The dynamics of wealth share of top 0.1% households match the rising trend in the United State between 2003 and 2012[3]. The simulation can predict 90% of the increase in wealth share of the top 0.1% in that period. However, the model tends to underestimate wealth shares of top 0.01% (because entrepreneurs in the model can only invest in their own firm), underestimate top 1% to 0.5% (because there is no human capital inequality in the model) and overestimate the bottom 90% (because the model uses linear preference which produces high marginal propensity of investment).

The model also reveals effects of several important factors on wealth inequality, including entrepreneurship, mobility in wealth distribution, saving rate, subsistence consumption, fluctuations in risk free rate and access to new investment opportunities.

Entrepreneurship is the most important driving forces behind wealth inequality. Entrepreneurs are defined as the initial owners of firms in the model. Being an

[3]I focus on the ten year interval of 2003-2012 to reduce the effects of other factors on wealth distribution, such as tax policy (the Bush tax cuts began at 2003), globalization and technological change etc. Moreover, since the distribution of products does not evolve over time it is reasonable to focus on a relatively short interval.

entrepreneur has two major benefits on her portfolio. Firstly, it offers a possibility of receiving large capital gains from new investors which sends herself up to the top of the distribution quickly. Secondly an entrepreneur usually claims more shares of firms than the rest of investors do. If her firm is invested by new investors, she receives capital gains in the form of shares in the firm. If her firm is not invested, she is the sole owner of the firm and, thus can invest as much as she like as long as the production cost of the firm is not met. This increases an entrepreneur's effective rate of return from her wealth and help her accumulate wealth faster. The effect of mobility in wealth distribution is highly correlated with that of entrepreneurship because most of the major mobilities are caused by entrepreneurs.

Fluctuations in risk free rate have complicated general equilibrium effects on wealth distribution. Overall, it mitigates wealth inequality and may offset up to a half of the impact from entrepreneurship, compared to fixing the risk free rate at a low level. Firstly, a lower risk free rate widens the gap from rates of return of firms and facilitate the wealth accumulation of the wealthy, whose portfolio have more assets in firms. Secondly, a lower risk free rate also stimulate the poor to consume more due to a lower rate of intertemporal substitution. As a result, the saving rate of the poor is lower than that of the wealthy.

Another important effect of fluctuations in risk free rate is that it facilitates incumbent investors in small firms switching to larger firms because fixing risk free rate at a low level reduces the opportunity cost of investing in a large firm. This results in a diversification among investors in choices of firms to invest. If access to new firms is allowed for investors, it reduces wealth inequality and induces the emergence of the middle class. This effect may also offset about a half of the effect of entrepreneurship.

Subsistence consumption has a significant impact on the left tail of wealth distribution. It depletes the wealth of the poor whose income cannot cover subsistence expense and renders them with no assets. After that its effect on inequality diminishes.

The result also matches some stylized facts in Saez and Zucman 2016. With tax return data, they establish that the upswing in the top 1% wealth share is due to the rise in the top 0.1% and that the increase in wealth inequality is not due to rate of return differential on corporate stocks but rather saving rate inequality.

2 Related Literature

A popular framework for studying wealth inequality is that of general lifecycle models. Cagetti 2003 estimates time preference and risk aversion in a life-cycle model and highlights the importance of precautionary savings in early stages of the life-cycle in explaining wealth inequality but preference heterogeneity alone is insufficient to account for the heavy concentration in wealth observed in the data. Cagetti and De Nardi 2008 present a thorough review of life-cycle models in wealth inequality literature. They shows that baseline versions of these models are unable to replicate the observed wealth inequality.

For the relatively difficult task of explaining the thickness of the right tail of wealth distribution, some authors introduce features such as bequest, capital income risks and wage inequality. The impact of bequest on wealth distribution could be either equalizing or dis-equalizing. Cagetti and De Nardi 2006 and Benhabib, Bisin and Luo 2017 show that non-homogeneous bequest, bequests as a fraction of wealth that are increasing in wealth, largely contributes to generating the thick tail of wealth distribution by inducing differential saving rates. On the other hand, Boserup, Kopczuk and Kreiner 2017 and Elinder, Erixson and Waldenstrom 2018 show that bequest may reduce wealth inequality because the less wealthy heirs inherit more relative to their pre-inheritance wealth. This is driven by the redistributional effect of inheritance taxation and the fact that heirs do not inherit debts, which makes the distribution of inheritances more equal than the distribution of wealth among the heirs. Benhabib, Bisin and Zhu 2015 analytically show that wealth distribution in Bewley economies with stochastic idiosyncratic returns to wealth displays a fat tail. Wealth inequality rises with the capital income risk. Quadrini 2000 and Benhabib, Bisin and Luo 2017 show that capital income risk induces a skewed distribution of wealth.

Piketty and Saez 2003 show that wage dispersion is the driving force of income inequality. This suggests that it may be behind wealth inequality as well. The literature has largely explored the role of wage inequality on wealth distribution in the Bewley-Huggett-Aiyagari model. Some literature have verified that labor income plays an important role on wealth inequality.[4] However, one issue about this is that earnings inequality is generally hard to match the top wealth concentration in data. Indeed, Cagetti and De Nardi 2008 show that there is no mechanism that induces wealthier households to keep saving at high rates once they accumulate a certain level of buffer stock to insure against earning risks. Benhabib, Bisin and Luo 2017

[4]See Benhabib, Bisin and Luo 2017 and Kaymak and Poschke 2015.

also show that one cannot explain wealth inequality with wage dispersion unless extraordinarily high earnings states are used.

Another strand of related literature is researches on span of control and entrepreneur's compensation. Its basic idea is similar to that of this paper. Modeling firms with span of control generates positive profits which are viewed as compensation to managers. The higher the hierarchy is the more the compensation. However, there are only a few papers in this area probably because data on firm hierarchy is difficult to obtain. Smeet and Warzynski 2007 document that span of control increases with wage inequality between job levels. Kemp-Benedict 2015 shows that when span of control increases, within-firm wage inequality also increases.

3 Model

Time, products, firms and households are all discrete in this model.[5] Households have linear utility and consume a set of products Ω_t^H at period t. In period $t-1$, entrepreneur i have a set of products available to produce at period t, denoted by $\Omega_{i,t}^G$, from which she picks $\Omega_{i,t}^{FG}$ to set up firm i to produce. In other words,

$$\Omega_t^H = \bigcup_i \Omega_{i,t}^{FG} \quad \text{and} \quad \Omega_{i_1,t}^{FG} \bigcap \Omega_{i_2,t}^{FG} = \emptyset \quad \text{for} \quad i_1 \neq i_2.$$

Firms are engaged in monopolistic competition. Each product in a firm is either successful or a failure. The probability of being successful is determined by the firm's investment in quality control. Before producing, the firm needs to finance the production for optimal outputs, fixed entry costs of products and investment in quality control.

Figure3 illustrates the timing of various events in the model.

[5] In the investment mechanism that sorts heterogeneous households into heterogeneous firms, firm size becomes a relevant factor. This implies that firms in the model have to be discrete because to generate a continuum of firms one has to manipulate the density and, thus, firm size.

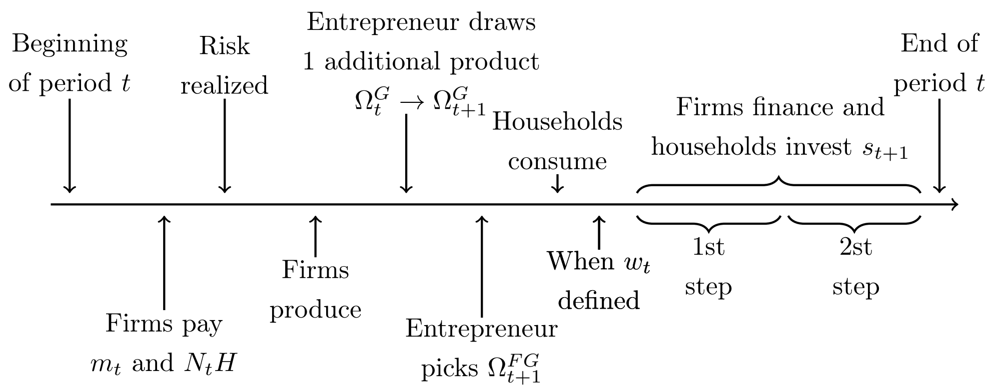

Figure 3: Timeline.

For the production cycle from period t to $t+1$, each entrepreneur first picks her own Ω_{t+1}^{FG} and set up the firm. Then firms finance and at the same time households consume and invest. Without loss of generality, I assume that, after households make their own consumption and investment decision, they first consume and then invest. In period $t+1$ firms first pay the investment in quality control and fixed entry costs for each product, then risks in products are realized. Given the realization of risks, firm then produce the survived products. In this paper, wealth of each household is measured immediately after she consume and this is the wealth I use to calculate wealth distribution. For the rest of the section I will discuss households' problem, entrepreneur and firm's problem, investment mechanism and general equilibrium of the model in detail.

3.1 Households

I assume that the expected utility for households at period $t = t_0$ is:

$$\mathcal{U}_{t_0} = \mathbb{E} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U_t,$$

where β is the discount factor and U_t is the stage utility. The stage utility is linear in consumption C_{gt} adjusted with the appeal of each product[6] and the subsistence

[6]The definition of consumer appeal of product is utility per common physical unit of product, as in Hottman, Redding and Weinstein 2016.

level of consumption[7]:

$$U_t = \begin{cases} \sum_{g \in \Omega_t^H} \varphi_g C_{gt} - \underline{C}_t, & \sum_{g \in \Omega_t^H} \varphi_g C_{gt} \geq \underline{C}_t, \\ -\infty, & \sum_{g \in \Omega_t^H} \varphi_g C_{gt} \leq \underline{C}_t, \end{cases}$$

where Ω_t^H is the set of all products available for the household, $\varphi_g > 0$ is the consumer appeal of product g and measures utility per unit of product g, and $\underline{C}_t$ is the subsistence level of consumption.[8]

Normalize the price of one unit of utility at $t = t_0$ to one, in equilibrium we have

$$p_{gt}^{t_0} = \beta^{t-t_0} \varphi_g \tag{1}$$

where $p_{gt}^{t_0}$ is the price of product g at time t with numeraire set at t_0. In other words, the price of the product is proportional to its appeal.

Households can invest in firms and/or a pooled fund which offers a common and endogenous rate of return r. Household's budget constraint is related to investment mechanism and is presented in the subsection on investment.

3.2 Entrepreneurs and Firms

I assume that firms engage in monopolistic competition.

A fraction, μ, of households are entrepreneurs. Suppressing the subscript index for entrepreneurs, at time $t-1$ each entrepreneur is given a set of products (in this paper a product means a variety of goods, rather than a unit of goods) available to produce at time t, Ω_t^G. She can produce an arbitrary nonempty subset, Ω_t^{FG}, from Ω_t^G.[9] Should she decide to produce anything, she set up a firm to raise funds from

[7] I incorporate subsistence consumption in the preference to reduce household savings. One drawback of linear utility is that household's motive to save is exaggerated. As long as the rate of return of investment is larger than the inverse of the discount factor, the household invests without consuming, which leads to overestimation of wealth shares of relative poor households. Although this may result in a common consumption level for households of all wealth levels, recent literature shows that consumption inequality is much milder and stabler than income inequality (see Krueger and Perri 2006, Heathcote, Perri and Violante 2010, Meyer and Sullivan 2018)

[8] Linear preference has been used commonly in the literation. Its major benefit is that households are thus risk-neutral which allows me to focus on the effects of entrepreneurship and firms. An alternative choice of demand system is the nested CES model used in Hottman, Redding and Weinstein 2016, where it is adopted to study the cannibalization effect. Linear preference is its special case with a cannibalization rate of 1.

[9] Generally firms in the literature grow either by increasing their scale or by expanding their product scope. I choose the latter because it generates a clean solution on the firm side.

households or the pooled fund to finance the production for optimal outputs and enters time t. After paying the fixed costs for products and the quality control cost, the risk in each variety is realized. If a product is successful it goes to the market and the firm pays the variable cost to produce it. If it is a failure instead, it does not go to market and the firm does not produce it. The objective of the firm is to maximize its expected profit:

$$\max_{\{y_{gt}^G\},m_t} \mathbb{E}\,\Pi_{ft}^F = \mathbb{E} \sum_{g\in\Omega_t^{FG}} I_g[p_{gt}^t y_{gt} - a(y_{gt})^{1+\delta_g}] - m_t - N_t H \tag{2}$$

where I_g is an indicator function, $I_g = 1$ means that product g is successful and goes to the market while $I_g = 0$ means that product g is a failure and does not go to the market, y_{gt} is the output of product g whose variable cost is $a(y_{gt})^{1+\delta_g}$ with $\delta_g \leq \bar{\delta}$, m_t is the firm's investment in quality control, H is the fixed cost for each product and N_t is the number of elements in Ω_t^{FG}.

The expectation of the firm's maximization problem is taken over risks in products it produces, which are captured by I_g. To specify the probability function of I_g, I begin with the assumption that every product has one single quality issue with a discover rate λ per unit of quality control investment m_t. The discovery of the quality issue then follows a Poisson process and the probability of discovering the issue is $1 - e^{-\lambda m_t}$, which is firm-specific and independent across all the products in the firm. If the quality issue for product g is discovered $I_g = 1$, otherwise $I_g = 0$. In reality a failure may represent a quality issue or a wrong evaluation in consumers' appeal. Reducing such a risk corresponds to firm investing in quality control or consumer survey. The assumption that the probability of success is independent across products is similar to the unrelated diversification of multi-product firms in industrial organization, which is a popular corporate strategy adopted by large corporations across the globe.[10]

It follows that the firm's expected profit becomes

$$\sum_{g\in\Omega_t^{FG}} (1 - e^{-\lambda m_t})[p_{gt}^t y_{gt} - a(y_{gt})^{1+\delta_g}] - m_t - N_t H.$$

The firm maximizes its expected profit subject to the constraint that in equilibrium price of each product equals to its product appeal as equation 1.

[10]Briglauer 2000 and Picone 2012 explore various reasons for unrelated diversification in the views of strategic management, synergy, agency and market power theoretically and empirically.

The optimal conditions for the firm are

$$m_t = \frac{1}{\lambda} ln(\lambda \sum_{g \in \Omega_t^{FG}} [p_{gt}^t y_{gt} - a(y_{gt})^{1+\delta_g}]),$$

or

$$1 - e^{-\lambda m_t} = 1 - \frac{1}{\lambda \sum_{g \in \Omega_t^{FG}} [p_{gt}^t y_{gt} - a(y_{gt})^{1+\delta_g}]}$$

and

$$\varphi_{gt}^G = a(1+\delta_g) y_{gt}^{\delta_g}$$

or

$$sales = a(1+\delta_g) y_{gt}^{1+\delta_g}. \tag{3}$$

In other words, the probability of being successful increases the sum of product profits and the optimal product output increases with its product appeal. This also leads to the following proposition.

Proposition 1. The larger the product scope or the profit of the firm are, the larger the increase in its size, measured with overall costs, over time tends to be.

The proof of proposition 1 is provided in the Appendix. Proposition 1 implies that large firms tend to remain large while small firms are relatively hard to grow. This may lead to a self-reinforcing left-skewed distribution of firm size.

Define the rate of return of product g, R_{gt}^G, as the ratio of its revenue of the variable cost, then

$$R_{gt}^G = \frac{p_{gt}^t y_{gt}}{a(y_{gt})^{1+\delta_g}} = 1 + \delta_g.$$

Since the output and rate of return of each product are characterized by two product-specific parameters φ_g and δ_g, I assume that elements in Ω_t^G are independently drawn from a bivariate distribution of φ_g and δ_g, $F(\varphi, \delta)$. These draws are also independent over all the entrepreneurs. At the beginning of every period, every active entrepreneur, that is, an entrepreneur who set up a firm last period, draws an additional product from $F(\varphi, \delta)$.

The expected rate of return of firm at t is then

$$R_t^e = \frac{\sum_{g\in\Omega_t^{FG}} [p_{gt}^t y_{gt} - a(y_{gt})^{1+\delta_g}] - \frac{1}{\lambda} - m_t - N_t H}{\sum_{g\in\Omega_t^{FG}} a(y_{gt})^{1+\delta_g} + m_t + N_t H} < 1 + \bar{\delta}.$$

Lastly, the entrepreneur's problem is to maximize the expected profit of the firm by choosing all the products in Ω_t^G that can bring profits. Note that $1 + r$ is an opportunity cost of setting up a firm since households can always invest in the pooled fund which offers a rate of return of r. As a result, given r, Ω_t^{FG} is determined in the following problem:

$$\max_{\Omega_t^{FG}\subseteq\Omega_t^G} \sum_{g\in\Omega_t^{FG}} (1 - e^{-\lambda m_t})[p_{gt}^t y_{gt} - a(y_{gt})^{1+\delta_g}] - ra(y_{gt})^{1+\delta_g} - (1+r)(m_t + N_t H). \quad (4)$$

3.3 Investment

All Households, except entrepreneurs, can invest in either firms directly or a pooled fund or both of them. An entrepreneur can only invest in the firm she set up. For each firm there are two types of investors in every period: the incumbent investors who have invested in the firm previously and hold equity of the firm; the new investors who invest in the current period. Note that if an incumbent investor wants to invest in the firm in the current period she also becomes a new investor in the firm by definition. If a firm has not been invested by any investor other than the entrepreneur, the only incumbent investor is the entrepreneur herself. To simplify the problem, I assume that an investor cannot invest in multiple firms at the same time.[11] That said, an incumbent can quit from the current firm and become a new investor in an alternative firm. In each firm, new investors invest through a joint Nash bargaining between new investors and incumbents. As shown later in this subsection, the bargaining generates investment fees for new investors and capital gains to incumbents. A firm can also finance from the pooled fund at a cost equaling to the rate of return of this firm, which means the pooled fund does not need to pay the fee to invest.[12] Hence, the firm prioritizes financing from investors directly.

A household's investment decision consists of which firm to invest, how much to invest in the firm and how much to invest in the pooled fund. For simplicity, I

[11]This assumption is not as strong as it may appear. The benefit of risk diversification from investing in multiple firms is limited because households are risk neutral due to linear preference. The investor can switch to a larger firm instead of invest in multiple firms, if the current one is too small for her wealth.

[12]Another way to interpret this is that, since the pooled fund acts as the last resort to the firm, the firm loses its bargaining power when financing from the pooled fund and thus requests no fee.

assume that households make one-period-ahead investment decision,[13] which means that the household's problem at period $t = t_0$ is to maximize her utility over only two periods:

$$\max \mathbb{E} \sum_{t=t_0}^{t_0+1} \beta^{t-t_0} U_t, \tag{5}$$

It is easily shown that he household invests only when the expected rate of return of the investment, R^e, is larger than the inverse of discount factor:

$$\beta R^e \geq 1. \tag{6}$$

The joint Nash bargaining is between two groups of investors. On one hand it is the group of new investors, on the other hand it is the group of incumbent investors. It determines how the incumbents and new investors split the revenue of the firm. Since alternatively the incumbents can always acquire sufficient investment from the pooled fund at a cost equaling to its own rate of return, R^e, and new investor can always invest in the pooled fund for a rate of return of r, the negotiated outcome maximizes

$$(X - R^e D)^\alpha [Y - (1+r)T]^{1-\alpha}, \quad \text{with} \quad X + Y = (D+T)R^e \quad \text{and} \quad D + T \leq OC$$

where X and Y are the incumbents and new investors' claims on the firm's revenue respectively, α is the bargaining power of the incumbents, T is the total investment from new investors and R^e is the expected rate of return of the firm, D is the existing investment from the incumbents and OC is the overall cost of the firm which is also the maximum investment it may take in,

$$OC_t = \sum_{g \in \Omega_t^{FG}} a(y_{gt})^{1+\delta_g} + m_t + N_t H_t.$$

The solution to the joint Nash bargaining is

$$X = \alpha(R^e - 1 - r)T + DR^e.$$

[13] To justify the assumption on one-period-ahead investment decision, note that in traditional framework of linear demand system with no investment fee, one-period-ahead investment decision is equivalent to perfect foresight. This is because there is no consumption smoothing in linear utility. Imposing the assumption of one-period-ahead investment decision does not discard any element in the traditional framework. The only thing it assumes away is the effect that new investors may want to invest in firms that can take in larger investments in the future. This assumption can be extended to multiple-period-ahead. A robust test with the assumption extended to two-period is performed in the section of robustness.

Define B as the assets transferred to the incumbents from new investors so that the incumbents get X in the next period:

$$R^e(B+D)=X.$$

It can then be shown that

$$B=\alpha(1-\frac{1+r}{R^e})T,\quad T\leq OC-D.$$

This joint bargaining process can be visualized as venture capital investment, private equity investment or IPO (with the investment bank as the intermediary between firm owners and new investors) etc. Take IPO as an example. A classic theory views IPO as the underwriter discovering the value of the firm with the help of owners, and then the owners monetize a proportion of the firm value to investors. The value of the firm, or say, equity is determined as the present value of future revenues. One may thus conclude that no bargaining is involved in IPO. However note that, firstly, during an IPO the underwriter usually produces a filing range of the offer price to the public before the roadshow, which can be seen as a negotiation between owners and investors. Secondly, it is a well known fact that IPOs are usually underpriced as shown by Hanley 1993 and many others, which indicates that IPO offer prices do not fully reflect the values of firms. Hence it may be reasonable to assume that there is a bargaining over the offer price, and thus the firm value, in IPO.

Indeed, it can be shown that the classic view is a special case of the investment mechanism in my model. Consider a twp-period case of IPO with a firm with overall cost OC, rate of return R, existing investment D, new investment from IPO T, risk free rate $1+r$ and $D+T=OC$.
In the classic view, the value of incumbents' equity in the firm increases from $\frac{R}{1+r}D$ to

$$\frac{R}{1+r}OC-T\quad\text{or}\quad\frac{R}{1+r}(D+T-\frac{1+r}{R}T)$$

after IPO due to capital gains. While under the investment mechanism of my model, the value of incumbents' equity increases from $\frac{R}{1+r}D$ to

$$\frac{R}{1+r}(D+\alpha(1-\frac{1+r}{R})T)\quad\text{or}\quad\frac{R}{1+r}(D+B).$$

Therefore, the classic view is a special case of the investment mechanism in my model with $\alpha=1$. Note that the value in the classic view is less than the one in my model, which is consistent with the well documented fact that IPOs are usually underpriced. Moreover, this example also shows that capital gains can be decomposed into two terms: $\frac{R}{1+r}$ which measures the market price of the firm's equity and $D+B$ which measures the incumbents' claim on assets of the firm. In this model all the capital gains are due to the increase in the second term, B. During the IPO, the value for new investors also changes from $\frac{R}{1+r}$ to

$$\frac{R}{1+r}(T-\alpha(1-\frac{1+r}{R})T) \quad \text{or} \quad \frac{R}{1+r}(T-B)$$

This shows that new investors' claim on the firm's assets is the new investment T subtracted by B. As a result, B can be interpreted as the difference between new investment and the new investors' claim on assets of the firm, investment fees for new investors and capital gains for incumbents.

Given B, new investors equally split the premium B and each pays

$$b^N = \frac{B}{n^N}$$

where n^N is the number of new investors. And incumbent investors equally split the premium as well and each receives

$$b^I = \frac{B}{n^I}$$

where n^I is the number of incumbent investors.

Denote the household's new investment in a firm at period t (for the production at period $t+1$) by s_{t+1}, and the stock of asset in the firm at the end of $t+1$ by s^{t+1}. The evolution of s^{t+1} is

$$s^{t+1} = \begin{cases} s_{t+1} - b^N_{t+1}, & \text{if investing as a new investor only,} \\ s^t + s_{t+1} - b^N_{t+1} + b^I_{t+1}, & \text{if investing as an incumbent.} \end{cases}$$

Denote the household wealth after consumption at t by w_t as indicated in Figure3. Given the realized rate of return of the firm, R_{t+1}, the evolution of w_t, which is also the budget constraint of the household, is

$$C_{t+1} + w_{t+1} = s^{t+1}R_{t+1} + (w_t - s^{t+1} - I^I_{t+1}b^I_{t+1} + I^N_{t+1}b^N_{t+1})(1+r_{t+1}), \tag{7}$$

where $I^I_{t+1} = 1$ if the household is an incumbent, otherwise $I^I_{t+1} = 0$, $I^N_{t+1} = 1$ if the household makes any new investment at t, otherwise $I^N_{t+1} = 0$.

3.4 Equilibrium

To ensure an equilibrium exists in the model, I assume that the household makes her investment decisions in two steps. Given the rate of return of the pooled fund r, she first decides which firm she would like to invest. The choice is fixed for the rest of the period. It can be the firm she has invested in or a new firm as a new investor. She cannot invest in firms other than the one she chose until the next period. She then decides if she invests in the chosen firm or the pooled fund only, and the amount of investment.

•the first step

Suppressing the subscript index of time, in the first step the household only observes expected rates of return and overall costs for each firm, that is R^e_f and OC_f for all $f \in \Omega^F$, where Ω^F is the set of firms. Her expectations over investment fees she may pay and receive in firm f are

$$\mathbb{E}\,\frac{B_f}{n^N_f} = \alpha(1 - \frac{1+r}{R^e_f})\gamma_1 OC_f$$

$$\mathbb{E}\,\frac{B_f}{n^I_f} = \alpha(1 - \frac{1+r}{R^e_f})\gamma_2 OC_f$$

respectively where γ_1 and γ_2 are rational expectations of $\frac{T_f}{n^N_f}$ and $\frac{T_f}{n^I_f}$ over $f \in \Omega^F$

$$\gamma_1 = \mathbb{E}\,\frac{T_f}{n^N_f} \quad \text{and} \quad \gamma_2 = \mathbb{E}\,\frac{T_f}{n^I_f}.$$

In equilibrium, they satisfy expectation consistency, which means that they are equal to the averages of their realizations:

$$\gamma_1 = \frac{1}{card(\Omega^F)} \sum_{f \in \Omega^F} \frac{T_f}{n^N_f}, \tag{8}$$

$$\gamma_2 = \frac{1}{card(\Omega^F)} \sum_{f \in \Omega^F} \frac{T_f}{n^I_f}, \tag{9}$$

In particular, in the first step the household first decides her best outside option of firms, that is, the best firm to invest other than the one she invested last period, if any. And then she compares her last firm and her best outside option to determine the best firm to invest in the current period. Under the assumption of one-period-ahead investment decision, the best outside option is the solution to the following problem:

$$\max_{f \in \Omega^F} K \triangleq \mathbb{E}(s_f - \frac{B_f}{n^N_f})R^e_f + (w - s_f)(1+r) \tag{10}$$

where w is the wealth of the investor, Ω^F is the set of firms and s_f is her investment in firm f. Denote the value of the problem by K. It can be shown that
if $\frac{w}{\alpha} > \gamma_1 OC_f \geq w$,

$$K = wR^e_f - \alpha\left[R^e_f - (1+r)\right]\gamma_1 OC_f;$$

if $\gamma_1 OC_f \leq w$,

$$K = (1-\alpha)\left[R^e_f - (1+r)\right]\gamma_1 OC_f + (1+r)w.$$

This result can be illustrated by Figure 4. Given the wealth of the household, the best outside option is the the one attained by the highest indifference boundary. Since R^e is bounded above by $1+\bar{\delta}$ and indifference curves are right bounded by $\frac{w}{\alpha}$, the solution for the best outside option of firms is well defined. Were it not for the investment fee, higher values in both two dimensions of firms, rate of return, R^e, and size, measured by OC, are all favored by households. But since the fee increases with OC, the household may prefer the firm with smaller size. If $\gamma_1 OC > w > \alpha\gamma_1 OC$, increasing OC does not increase the investment in the firm but raises the fee. To keep the household indifferent with the firm, an increase in R^e is needed. If $\gamma_1 OC < w$, the increase in investment dominates the increase in fee so the value K goes up. A reduction in R^e is required to drive down the value. For instance, consider the four firms shown on the diagram. Firm 1 is preferred over firm 2 while firm 2 is preferred over firm 3 according to the indifference boundaries. Firm 4 is to the right of line $\frac{w}{\alpha}$ and is not feasible for the household because its investment fee is even higher than her wealth. However, a household with higher wealth may prefer firm 4 over firm 1. This leads to the following proposition.

Proposition 2. Wealthier households tend to invest in firms with larger size, measured with overall costs.

The proof of proposition 2 is provided in the Appendix. Proposition implies that investors sort themselves into firms based on their wealth. This facilitates the accumulation of wealth for the rich because larger firms tend to take in larger investments.

Proposition 1 and 2 state that wealthier households tend to invest in larger firms while the size of larger firms tend to increase more over time. This implies that the rich may be able to invest a larger fraction of wealth in their large firms. This again reinforces the portfolio effect.

Given the maximized value of the best outside option f^*, K^*, the household leaves the current firm f_0 for f^* if and only if its value is larger than the value from

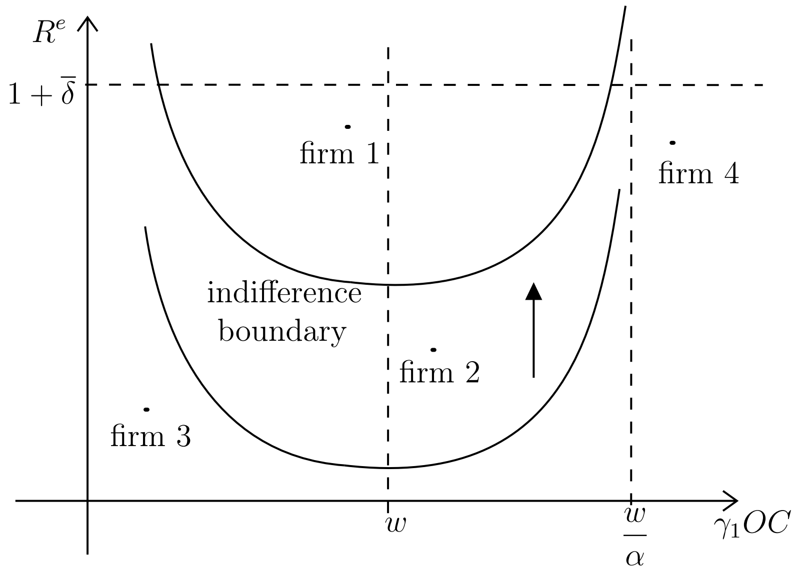

Figure 4: Indifference Boundary with an example of 4 firms.

staying in f_0

$$K^* > (s^T_{f_0} + s_{f_0} I^N - \gamma_1 OC_{f_0} I^N + \gamma_2 OC_{f_0}) R^e + (w - s^T_{f_0} - s_{f_0} I^N)(1+r), \text{ for } I^N = 0, 1$$

where $s^T_{f_0}$ is the household's claim of assets in the current firm in the previous period, s_{f_0} is the new investment in the current firm and I^N indicates whether the household makes new investment or not.

•the second step

In the beginning of the second step, households are locked with the firms they chose in the first step. Given r, each investor's choice on firm, and T_f and n^N_f for each firm, the household decides whether she make investment in the firm or not. Note that an incumbent never leaves the firm for the pooled fund because the solution of entrepreneur's problem guarantees that the expected rate of return of the firm is larger than $1 + r$. An investor makes new investment in firm f if and only if investing in the firm generates higher value than investing in the pooled fund only does

$$(s_f - b^N_f) R^e_f + (w - s_f)(1 + r) > w(1 + r).$$

or

$$s_f > \alpha \frac{T_f}{n^N_f}. \tag{11}$$

The total new investment for firm f, T_f, is the sum of all the new investors' investments or $OC_f - D_f$, whichever is smaller, because the firm does not take in investment more than it needs

$$T_f = \min\{\sum s_f, OC_f - D_f\}. \tag{12}$$

Denote the wealth for new investment, $w - s_f^T$, by w^N. If $T_f = \sum s_f$ then

$$s_f = w^N. \tag{13}$$

If $T_f = OC_f - D_f$, it can be shown that there exists a unique $\bar{s}$ such that $T_f = \sum \min\{\bar{s}, w^N\}$. We have

$$s_f = \min\{\bar{s}, w^N\}. \tag{14}$$

Note that other investors' decisions in the second step affect T_f and n_f^N and thus affect the household's decision, which in turn affect other investors' decisions. It can be shown that there exists a Nash equilibrium for each firm where every investor makes her best decision given all the other investors' decisions.

Proposition 3. Given r and each investor's choice on firm, for each firm there exists a threshold w^{N*} such that all investors with $w^N > w^{N*}$ investing in the firm and all the investors with $w^N < w^{N*}$ investing w^N in the pooled fund is a Nash equilibrium.

The proof of proposition 3 is provided in the Appendix. Proposition 3 implies that households with higher wealth are more likely to invest in firms while those with lower wealth are more likely to invest in the pooled fund only. In the Nash equilibrium, every household's portfolio is finalized. T_f, n_f^N and n_f^I for each firm are all determined.

To better illustrate the process of finance and investment, I provide an example in Table 1. There are two firms and six households in the economy. 1 and 2 are the entrepreneurs behind firm 1 and 2 respectively. Note that column "N" shows new investors and column "I" shows incumbents. At the beginning of the period, 1 and 3 are the incumbents in firm 1 while 2 and 4 are the incumbents in firm 2. 5 and 6 invest in the pooled fund only. In the first step of investment, every investor is required to choose the best firm for herself. 1 and 2 can only stay their own firm because they are entrepreneurs. 3 chooses to stay in the current firm while 4 decides to leave firm 2 for firm 1 as a new investor. As a result, 1, 3, 4 and 5 choose firm 1 while 2 and 6 choose firm 2. In step 2, 1, 4 and 6 decide not to make any new investment. Since 4 and 6 are not incumbents in any firm they have to invest all of their wealth in the pooled fund. In firm 1, 3 and 5 as a group bargains with the other group of 1 and 3. In firm 2, the trivial bargaining is between 2 and 2, which means that 2 pays the investment fee to herself and there is no fee effectively.

Before Step 1:

Firm 1	
N	I
	1
	3

Firm 2	
N	I
	2
	4

Fund Only
5
6

After Step 1:

Firm 1	
N	I
1,3	1
4,5	3

Firm 2	
N	I
2	2
6	

After Step 2:

Firm 1	
N	I
3	1
5	3

Firm 2	
N	I
2	2

Fund Only
4
6

Table 1: An example of 2 firms and 6 investors. "N" means new investors and "I" means incumbents.

Now we can define the equilibrium of the model. At period t, given the set of entrepreneurs Ω^E, the set of households Ω^A, the set of products available to produce for each entrepreneur $\{\Omega^G_{i,t+1}\}_{i\in\Omega^E}$, each household's wealth $\{w_{i,t}\}_{i\in\Omega^A}$, the existing investment at the beginning of t $\{s^{t-1}_{i,f}\}_{i\in\Omega^A, f\in\Omega^F}$, and the realization of risks in products at $t+1$, the general equilibrium in this model economy consists of $\{\Omega^{FG}_{f,t+1}, m_{f,t+1}, \{y_{f,g,t+1}\}_{g\in\Omega^{FG}_{t+1}}, \{p^{t+1}_{f,g,t+1}\}_{g\in\Omega^{FG}_{t+1}}, n^I_{f,t}, n^N_{f,t}, T_{f,t+1}, D_{f,t+1}, \{s_{f,i,t+1}\}_{i\in\Omega^A}\}_{f\in\Omega^F}$, $\{C_{g,i,t+1}\}_{g\in\Omega^H_{t+1}}$, $\gamma_{1,t+1}$, $\gamma_{2,t+1}$ and r_{t+1} such that

1. Given r_{t+1}, $\{\{p^{t+1}_{f,g,t+1}\}_{g\in\Omega^{FG}_{t+1}}, n^I_{f,t}, n^N_{f,t}, T_{f,t+1}, D_{f,t+1}, \{s_{f,i,t+1}\}_{i\in\Omega^A}\}_{f\in\Omega^F}$, $\gamma_{1,t+1}$ and $\gamma_{2,t+1}$, $\{C_{g,i,t+1}\}_{g\in\Omega^H_{t+1}}$ and $\{s_{f,i,t+1}\}_{i\in\Omega^A, f\in\Omega^F}$ solve the household's problem defined in 5, subject to the budget constraint 7.

2. Given r_{t+1}, $\{\Omega^{FG}_{f,t+1}\}_{f\in\Omega^F}$ solves entrepreneur's problem defined in 4.

3. Given $\{\Omega^{FG}_{f,t+1}\}_{f\in\Omega^F}$, $\{m_{f,t+1}, \{y_{f,g,t+1}\}_{g\in\Omega^{FG}_{t+1}}, \{p^{t+1}_{f,g,t+1}\}_{g\in\Omega^{FG}_{t+1}},\}_{f\in\Omega^F}$ solves firm's problem defined in 2.

4. $T_{f,t+1}, \{s_{f,i,t+1}\}_{i\in\Omega^A}\}_{f\in\Omega^F}$ balances the demand and supply of asset in each firm.

5. $\{n^I_{f,t}, n^N_{f,t}\}_{f\in\Omega^F}$ is consistent with $\{s_{f,i,t+1}\}_{i\in\Omega^A, f\in\Omega^F}$ and $\{s^{t-1}_{i,f}\}_{i\in\Omega^A, f\in\Omega^F}$.

6. $\gamma_{1,t+1}$ and $\gamma_{2,t+1}$ satisfy 8 and 9.

7. r_{t+1} balances the demand and supply of assets in the economy.

Proposition 4. The general equilibrium defined above exists.

The proof of proposition 4 is provided in the Appendix. Note that the equilibrium may not be unique.

Now we can discuss several channels in the model that may lead to top concentration in wealth distribution. Firstly, *the portfolio effect.* Households with higher proportion of wealth in firms tend to accumulate wealth faster. They receive larger entrepreneurial income. From proposition 3, they are more likely to make new investment. Hence a smaller fraction of wealth receives the relatively low risk-free rate from the pooled fund, which raises the effective rate of return on the wealth. This effect reinforce itself over time. Secondly, *the saving rate effect.* When the risk-free rate drops to $\frac{1}{\beta}$, households start to consume a fraction of their wealth on the pooled fund. The wealthy people have a smaller fraction of wealth on the pooled fund and thus consume less proportion of their wealth, which results in a higher saving rate. This amplifies the portfolio effect. Thirdly, *the firm size effect.* Proposition 1 and 2 state that wealthier households tend to invest in larger firms while the size of larger firms tend to increase more over time. This implies that the rich may be able to invest a larger fraction of wealth in their large firms. This again reinforces the portfolio effect.

4 Data and Calibration

Nielsen barcode scanner dataset is my main data source for calibration. This dataset collects price and sales information for about 3 millions of products with a barcode by providing handheld scanners to about 55,000 households each year to scan goods they purchased and matching data from 35,000 stores in which they made purchases. These stores are from around 90 retail chains covering retail channels of food, drug, mass merchandiser, liquor and convenience. Nielsen barcode scanner dataset has several advantages for the purpose of our analysis. As discussed by Broda and Weinstein (2010) and Hottman, Redding and Weinstein (2016), a barcode is very close to the theoretical concept of a product, and this dataset covers a nearly full distribution of firms in the related product groups and almost all products produced by firms. To use Nielsen barcode scanner dataset to analyze the economy of the United States[14], I have to assume that products covered in the dataset well represent the

[14]One question that naturally arises is whether it is appropriate to think of markets covered in the dataset in a closed economy. The Census Bureau's foreign trade statistics show that food, feeds, beverages, toiletries and cosmetics, which are the main coverage of the dataset, constitute 9.8% of exports and 6.4% of imports of the United States in 2016 (I use 2016 data to avoid impacts

whole universe of goods and services in the United States.[15] Nielsen scanner dataset covers about 40% of the CPI basket of goods and consumption takes up about 60% of GDP. This implies that the dataset may represent $\frac{1}{4}$ of the economy.

I choose data in 2012 and 2013 for calibration. I remove all the products with annual sales less than 150 units from the dataset since they are likely to be the failed products in my model. The dataset then contains about 525,000 products and 24,000 firms each year.

Firstly, I calibrate the joint distribution for product draws, $F(\varphi, \delta)$. According to equation 1 the price of each product is just its appeal φ. With equation 3, I calibrate a using a common δ for all products that equals to the average of δ calibrated by Hottman, Redding and Weinstein (2016) for all products. This gives me $a = 0.79$. I then use it to recalculate δ_g for each product.

Then I compare the datasets in 2012 and 2013 to identify all the new products. They are a better representation for $F(\varphi, \delta)$ than the whole product space in one year is because the latter may be the result of firms drawing multiple times from the distribution. This gives me 2866 products. I fit them with a Gaussian copula distribution. Figure 5 shows the sample distribution of $F(\varphi, \delta)$ while Figure 6 shows the fitted distribution. Overall speaking the simulated distribution fits the sample well although it falls short of capturing the long right tail of φ.

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of changes in foreign trade policies). Moreover, exports and imports in food, feeds and beverages are \$130,555 millions and \$130,049 millions respectively. Exports and imports in toiletries and cosmetics are \$12,131 millions and \$10,891 millions respectively. These suggest that markets covered in the dataset are not seriously affected by foreign trades and that exports and imports offset each other in values in these markets

[15] One may think that those very wealthy people mainly come from hi-tech companies and that entrepreneurs producing commodities in supermarkets are underrepresented on the top of wealth distribution. However, the Forbes 400 ranking of the richest Americans shows that 46 out of 400 come from firms whose products are mainly sold in stores covered by the dataset. It is just 23% less than the Technology industry which produces 60 of them, and is more than any other industries except Finance and Investment which produces 88.

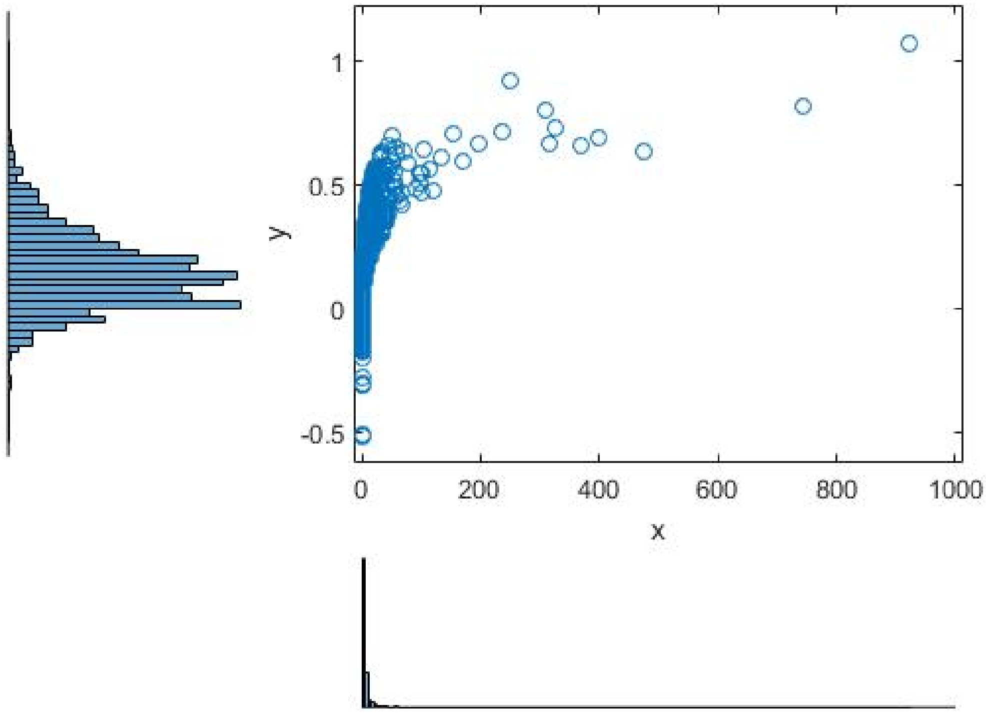

Figure 5: Sample distribution of $F(\varphi, \delta)$. The x axis is φ and the y axis is δ.

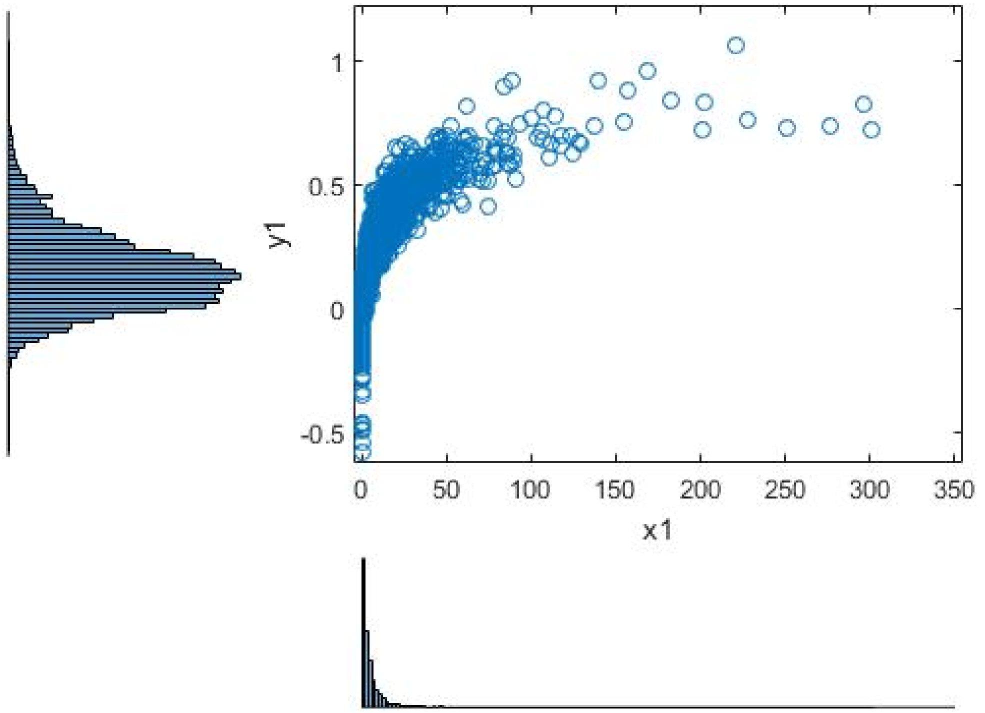

Figure 6: Fitted distribution of $F(\varphi, \delta)$. The x axis is φ and the y axis is δ.

With a and φ calibrated, I proceed to calculate the profit for each firm and again compare datasets in 2012 and 2013 to identify products without sales record in 2013. They are likely to be failed products in the model. Given H, the optimal condition for investment in quality control, m_t, allows me to use Probit regression to estimate λ. And then I estimate H by maximizing the likelihood of the sample of failed products. This gives me $H = 1.5$ and $\lambda = 0.277$.

To estimate the fraction of entrepreneurs in the population, μ, note that the dataset may represent $\frac{1}{4}$ of the economy. Also considering that the number of households in the US is about 120 millions in 2012 and multiplying the number of firms in the dataset by 4 times gives me a μ around 0.1%.

Subsistence consumption in the model is defined as $s\bar{I}$ where $\bar{I}$ is the median household income and s is a constant that measures the ratio between subsistence consumption and median household income. I estimate s with poverty thresholds for 2012 from the Census Bureau. The average size of households in 2012 is 2.5. The poverty thresholds for households of 3 members and 2 members are \$18,284 and \$14,937 respectively. The median household income in 2012 is \$50157. The estimated ratio of average poverty line to median household income is thus 0.3312, which is the estimation of s.

Finally, I set the discount factor $\beta = 1/1.02$.

5 Simulation and Results

The model is simulated with 500 entrepreneurs and 500,000 households for 102 periods. In the model one period represents one year in reality.

The bargaining power of the incumbent, α, is set at 0.1 for the simulation. The robustness of α at $\alpha = 0.5$ is tested. The Intuition of α can be seen from the premium-investment ratio,defined as investment fee over new investors' effective investment, $\frac{B}{T-B}$, in Table 2. The premium-investment ratio is not unreasonably high. Fixing $R^e = 1.15$, the ratio is no higher than 1.143% when $\alpha = 0.1$. When $\alpha = 0.5$ the ratio is no higher than 6%.

$B/(T-B)$	$\alpha = 0.1$	$\alpha = 0.5$
$r = 0.02$	1.143%	5.991%
$r = 0.1$	0.437%	2.222%

Table 2: $\frac{B}{T-B}$ with $R^e = 1.15$.

Each entrepreneur is given 1 products available to produce to start with, which means $card(\Omega_1^G) = 1$. Initial wealth is uniformly distributed, shown in Figure 7, such that the richest household is 10,000 times richer than the poorest one. The initial wealth inequality is very mild compared to data. A more-concentrated initial distribution will generate a severer wealth inequality.

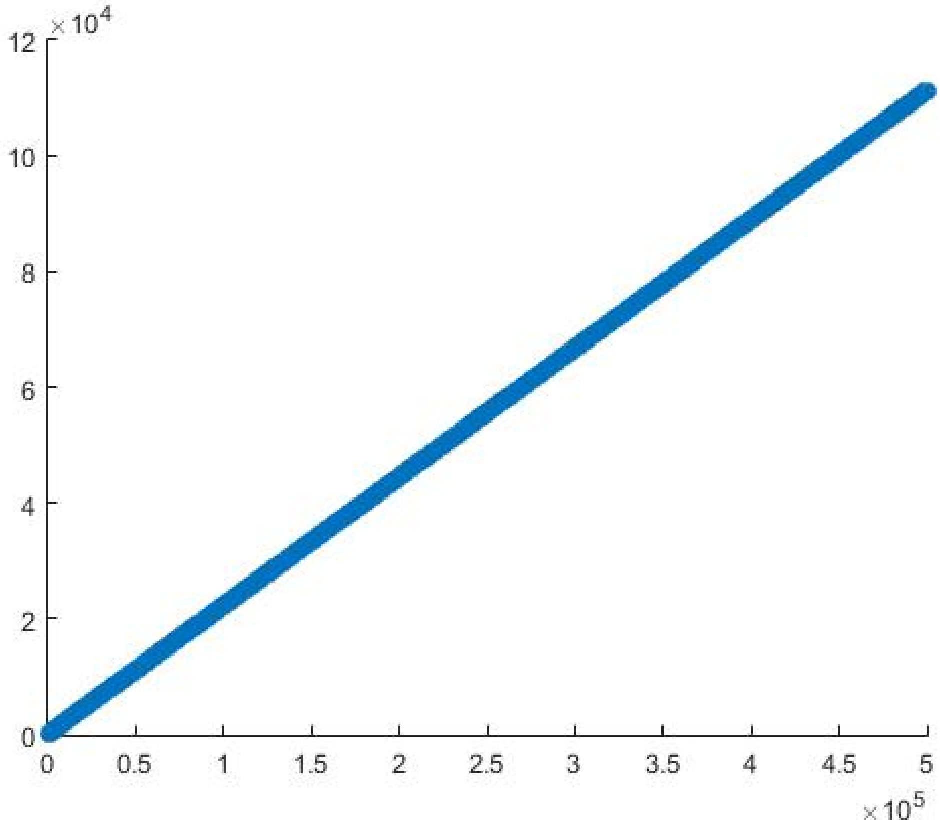

Figure 7: The initial wealth distribution (t=1). The y axis is the wealth level of each household.

5.1 Wealth distribution and its dynamics

Figure 8 is the wealth distribution after 50 periods. It shows a severe wealth inequality. The wealth of a very small number of rich households is much higher than that of the rest.

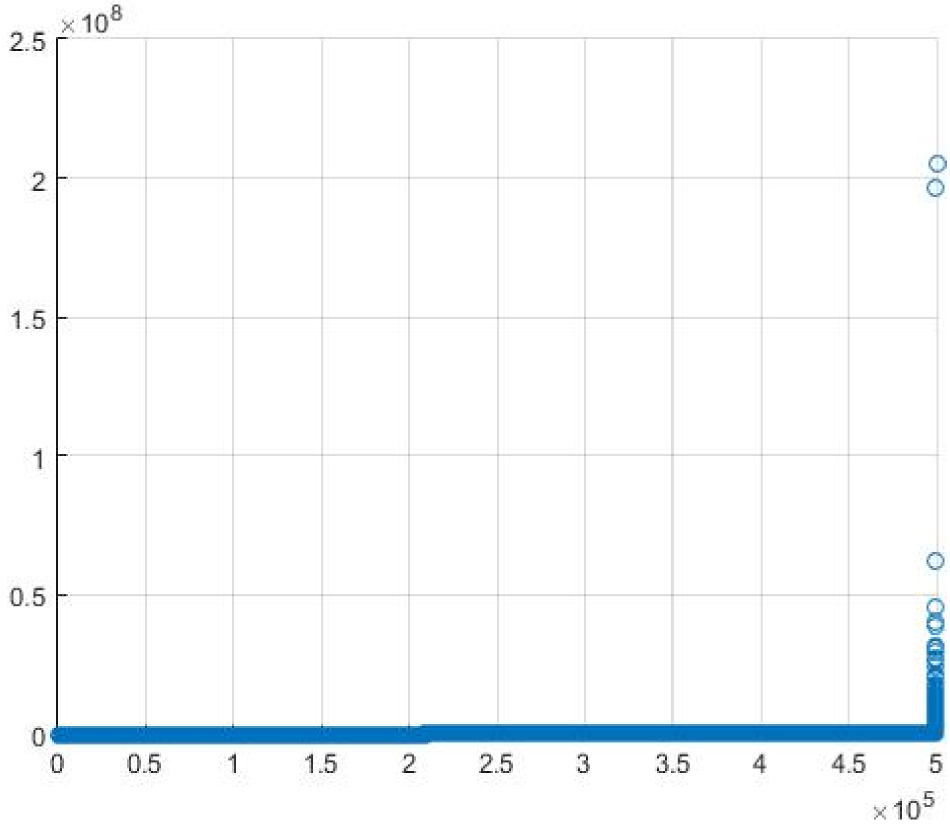

Figure 8: Wealth distribution after 50 periods (t=50). The y axis is the wealth level of each household.

Cumulative wealth distribution shows the wealth inequality and its dynamics more clearly. Figure 9 depicts the cumulative wealth distributions at $t = 1$, $t = 20$, $t = 40$, $t = 60$, $t = 80$ and $t = 100$ respectively. They have some noteworthy features.

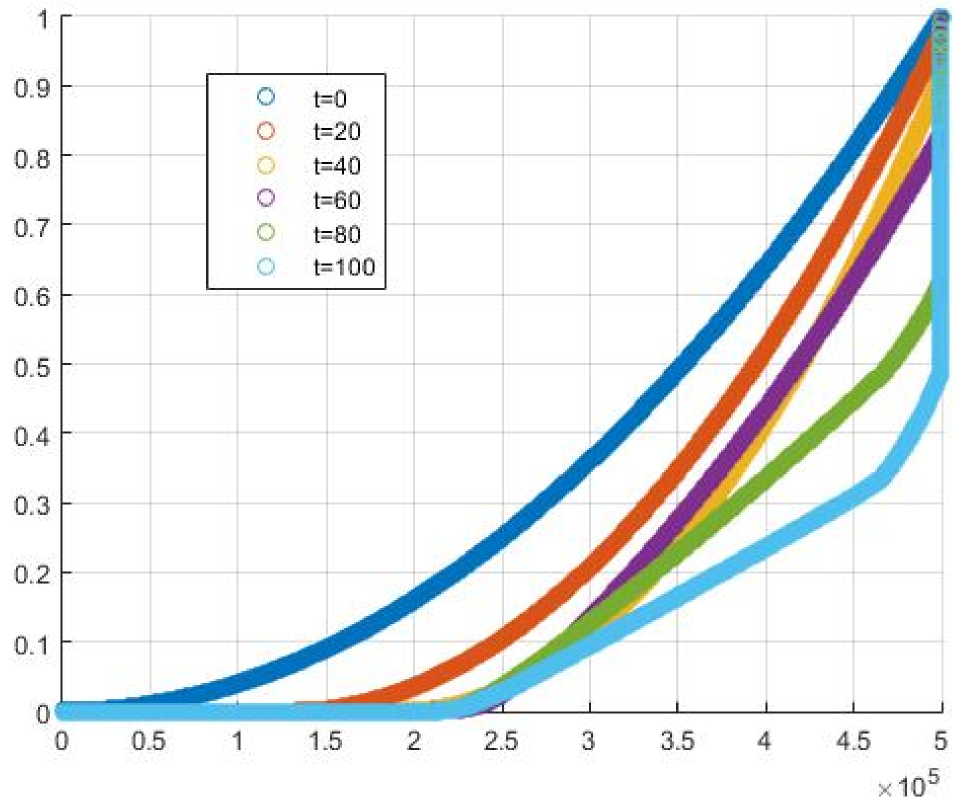

Figure 9: Cumulative wealth distributions at t-1, t=20, t=40, t=60, t=80, t=100. The y axis measures the cumulative wealth shares.

First of all, the evolution of the cumulative wealth distribution shows heavy concentration on the top. This can also be seen in Table 3. The wealth share of the top 0.01% grows from 0.02% all the way to about 25% while that of the top 0.1% grows from 0.2% to about 51%. It is also noteworthy that the increases in top 0.1%-1% is rather small, from 1.8% to 2.6%. This result is consistent with the stylized fact established in Saez and Zucman (2016) that most of the upswing in the top 1% wealth share is due to the rise in the top 0.1%. This feature in the simulation is due to entrepreneurship. As shown in Table 4, the number of entrepreneurs in the top 0.1% increases from 0 at $t = 1$ to 387 at $t = 100$, that is 77.4% of households in top 0.1%. Entrepreneurs climb up to the top because of the portfolio effect described in the previous section. If the entrepreneur's firm is not invested by any other households, she can always reinvest her entrepreneurial income in the firm without paying any investment fee as long as the her existing investment does not exceed the overall cost since she is the only incumbent and new investor at the same time. If the entrepreneur's firm is invested by other households, the capital gain received from new investors in the form of claim of equity in the firm usually sends her to the very top of the wealth distribution quickly. Either way the entrepreneur invests a larger proportion of her assets in the firm than the non-entrepreneur households do.

Since firms generally offer a higher rate of return than the pooled fund does, the effective rate of return of entrepreneurs' assets is higher. Therefore entrepreneurs accumulate wealth faster. It is consistent with the stylized fact established in Saez and Zucman (2016) that the increase in wealth inequality since 1996 is not due to rate of return differential on corporate stocks. Indeed, the model shows that households with different wealth have similar rates of return in firms. The average rate of return of firms in top 0.1%, top1%-0.1% and top 10%-1% are 1.1393, 1.1334 and 1.1411 respectively in period 100.

Another thing noteworthy in Table 3 and 4 is the increase of wealth share of the top 0.1%-0.01%. It rises from 2.98% at $t = 40$ to 26.74% at $t = 100$. This cannot be explained by the mobility of entrepreneurs alone because the top 0.1% are already overrepresented by entrepreneurs at $t = 40$ and the proportion of entrepreneurs in the top 0.1% increases only by 5%. The reason behind the explosive growth of wealth share in the top 0.1%-0.01% is due to the saving rate effect described in the previous section. Since a firm usually can only produce one additional product every period, the growth rate of firms decreases over time. For example, a firm may double its size by producing 2 products instead of one at $t = 2$ but at $t = 50$ the firm usually can only increase its products from 49 to 50, which reaches a growth rate of around 2%. As a result, the growth rate of demand for assets drops over time and the supply of asset may exceed the demand unless r drops. After $t = 50$, r equals $\frac{1}{\beta}$ for the most of times. This leads to the saving rate effect. This is also consistent with the stylized fact established in Saez and Zucman (2016) that saving rate inequality is one of major driving forces behind wealth inequality.

Wealsh share	$t = 0$	$t = 20$	$t = 40$	$t = 60$	$t = 80$	$t = 100$
top 0.01%	0.02%	2.13%	5.96%	7.11%	16.17%	24.65%
top 0.1%	0.20%	2.69%	8.84%	17.17%	38.44%	51.39%
top 0.5%	1.00%	3.74%	10.03%	18.07%	39.42%	52.59%
top 1%	1.99%	5.04%	11.50%	19.18%	40.61%	54.03%

Table 3: The wealth shares of the wealthiest x% of households at various periods.

t=0	t=1	t=2	t=3	t=4	t=5	t=10
0.0%	9.4%	11.6%	15.2%	19.0%	23.2%	44.0%
t=15	t=20	t=40	t=60	t=80	t=100	
54.0%	60.2%	72.4%	76.2%	77.2%	77.4%	

Table 4: The proportion of entrepreneurs in the top 0.1% at various periods.

Secondly, the wealth of relatively poor households is almost 0. The bottom 25% holds 0 wealth after the 17th period, the bottom 40% holds 0 after 43rd period while the wealth share of the bottom half drops from 25% to 2.8% at $t = 100$. This is because of, other than the fact that the rich accumulates wealth quickly, the subsistence requirement of consumption. The income of the poor may not be able sustain the subsistence consumption. This has small impact on the wealthy but greatly affects the bottom half. Figure 10 shows an alternative simulation at $t = 100$ without subsistence consumption requirement. The bottom half holds a significant share of wealth, around 10%.

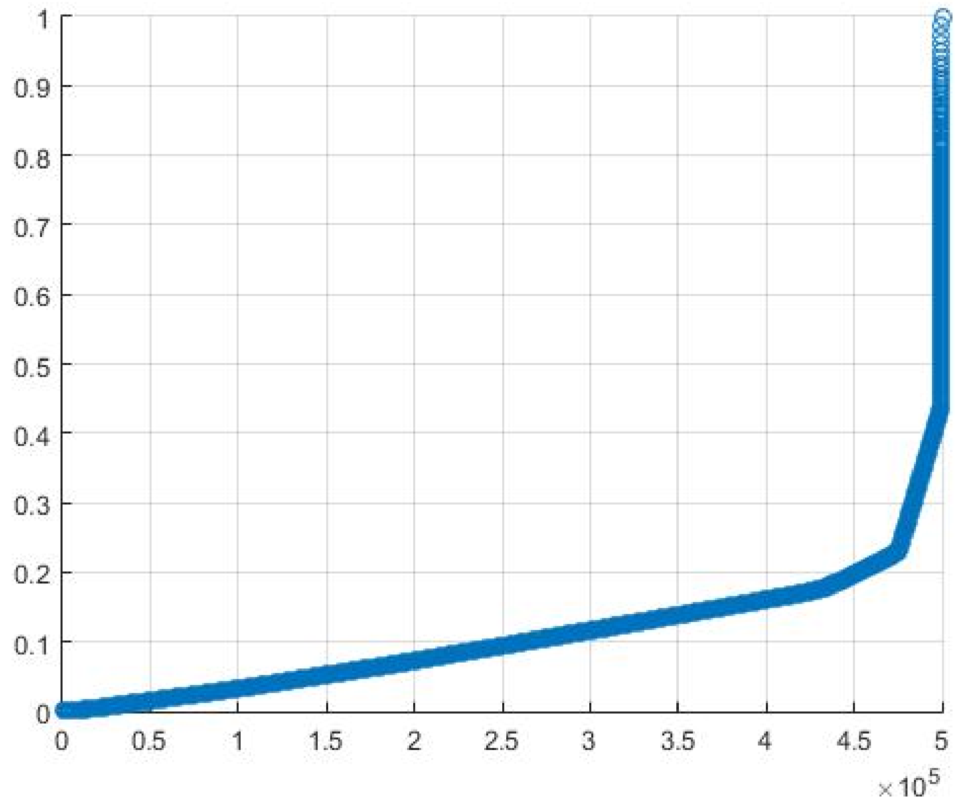

Figure 10: The cumulative wealth distribution at t=100 from simulation without subsistence consumption. The y axis measures the cumulative wealth shares.

Thirdly, after $t = 40$, more precisely after $t = 50$, wealth shares of top 15%-50% sometimes decreases instead. This is also due to the saving rate effect that results from fluctuations in r in the general equilibrium. As mentioned above, after $t = 50$ r equals $\frac{1}{\beta}$ for the most of times. Households are thus indifferent between consuming and investing in the pooled fund. They will consume until the asset market clears. Hence, for investors whose wealth does not grow fast enough to facilitate their new investment in firms, when $r = \frac{1}{\beta}$ their wealth may shrink. This is the reverse of the portfolio effect. This explains the drop in wealth shares.

Lastly, the middle class, a group of households whose wealth is significantly higher than those below them and significantly lower than households at the very top, emerges at $t = 80$ and becomes clear at $t = 100$. When r drops it has 2 opposite effects on pooled-fund-investing households' decision on switching to a new firm. A smaller r increases the investment fee but it also encourages investors to switch to a large firm by reducing their return on the pooled fund. The second effect may dominate when r is small and the investment on the pooled fund is large. If the households switch and the new firm draws new products with high rate of return, a middle class may emerge.

6 Model Fitness

In this section I fit the model with the data on wealth inequality to assess how well the model predicts the real world. All the data in this subsection is from Saez and Zucman (2016) which estimates wealth distribution with data on tax returns.

I simulate the model until it matches the wealth shares of top 0.01%, 0.1%, 0.5% and 1% in the real world in 2012 respectively. I choose year 2012 as the base year because it is the latest data I can get. It takes 72, 66, 76 and 82 periods to match the wealth shares of top 0.01%, 0.1%, 0.5% and 1%. I then retrieve the results in the previous 9 periods and compare them with the data from 2003-2012.

Table 5 shows how the model fits the data on wealth inequality in 2012. Matching wealth share of top 0.01%, the model overestimates the wealth share of top 0.1% while matching top 0.01% and underestimate top 0.01% while matching top 0.1%. This suggests that the model generates a flatter right tail from top 0.1% to top 0.01% compared to the data in 2012. The underestimation is probably due to the restriction that households can only invest in one firm. This assumption is more binding on rich entrepreneurs because they cannot increase their investment in firms by switch to larger firms. On the other hand, the model highlights a large gap between top 0.1% and top 0.5%. It overestimates the wealth share of top 0.1% while matching top 0.5% and underestimate top 0.5% while matching top 0.1%. A similar large gap is also observed between the wealth shares of top 0.5% and top 1%. It means that the model generates a steeper tail from top 1% to top 0.1%, compared to the data 2012. In other words, this model is able to produce a very heavy concentration on wealth for the top 0.1%. Note that the model does not incorporate human capital and thus may miss the impact of wage inequality. This may lead the model to underestimate the wealth share of households in top 10%-1%, where high skill workers usually populate. As for the left tail of wealth distribution, the model overestimate the wealth share of bottom 90% in all the four benchmarks. This is probably because the linear preference provides little motivation for those households to consume and results in a overstated saving rate.

	Benchmark	top 0.01%	top 0.1%	top 0.5%	top 1%
data	2012	0.1122	0.2201	0.3452	0.4182
model	top 0.01% matched	0.1134	0.2890	0.2987	0.3106
	top 0.1% matched	0.0839	0.2197	0.2287	0.2397
	top 0.5% matched	0.1271	0.3157	0.3255	0.3374
	top 1% matched	0.1679	0.3993	0.4095	0.4218

Table 5: Model fitness on wealth inequality in 2012.

The results of fitness on dynamics of wealth inequality from 2003 to 2012 are shown in Table 6, Figure 11, Figure 12, Figure 13 and Figure 14. The dynamics match the data pretty well in all the four benchmarks. Top 0.1% is the best match. The simulation predicts about 90% of the increase in wealth inequality from 2003 to 2012 and the trend is well matched. For top 0.01% the model also underestimates the increase in its wealth share and predicts 80% of the increase. This is consistent with our finding that the model tend to underestimate the wealth share at the very top. The model overestimates the increase in wealth share of top 0.5% and predicts 121% of the increases. As I said the model generates a steeper right tail of top 1% to top 0.1% than the data. This suggests that the top 0.5% may accumulate wealth at a faster rate than the data shows. The simulation also predicts 103% of the increase in the top 1% but the trend does not fit the data well. It is natural to expect that the power of the model to drop when dealing with a larger fraction of population.

	top 0.1%		top 0.01%		top 0.5%		top 1%	
	simulation	data	simulation	data	simulation	data	simulation	data
2003	15.38%	14.67%	7.56%	6.50%	22.87%	23.37%	32.40%	32.30%
2004	15.94%	15.62%	7.80%	7.02%	23.87%	24.41%	33.74%	33.54%
2005	16.59%	16.30%	8.08%	7.42%	24.96%	25.53%	35.07%	33.98%
2006	17.17%	16.77%	8.39%	7.67%	26.11%	26.71%	36.47%	34.90%
2007	17.84%	17.67%	8.75%	8.46%	27.31%	27.95%	37.88%	35.95%
2008	18.56%	18.98%	9.15%	9.17%	28.58%	29.23%	38.85%	38.13%
2009	19.37%	18.87%	9.62%	9.62%	29.87%	30.54%	39.75%	37.85%
2010	20.23%	20.71%	10.15%	10.77%	31.21%	31.88%	40.61%	39.52%
2011	21.06%	20.33%	10.73%	10.12%	32.55%	33.22%	41.44%	39.80%
2012	21.97%	22.01%	11.34%	11.22%	33.88%	34.58%	42.18%	41.82%

Table 6: Model fitness on dynamics of wealth inequality from 2003 to 2012.

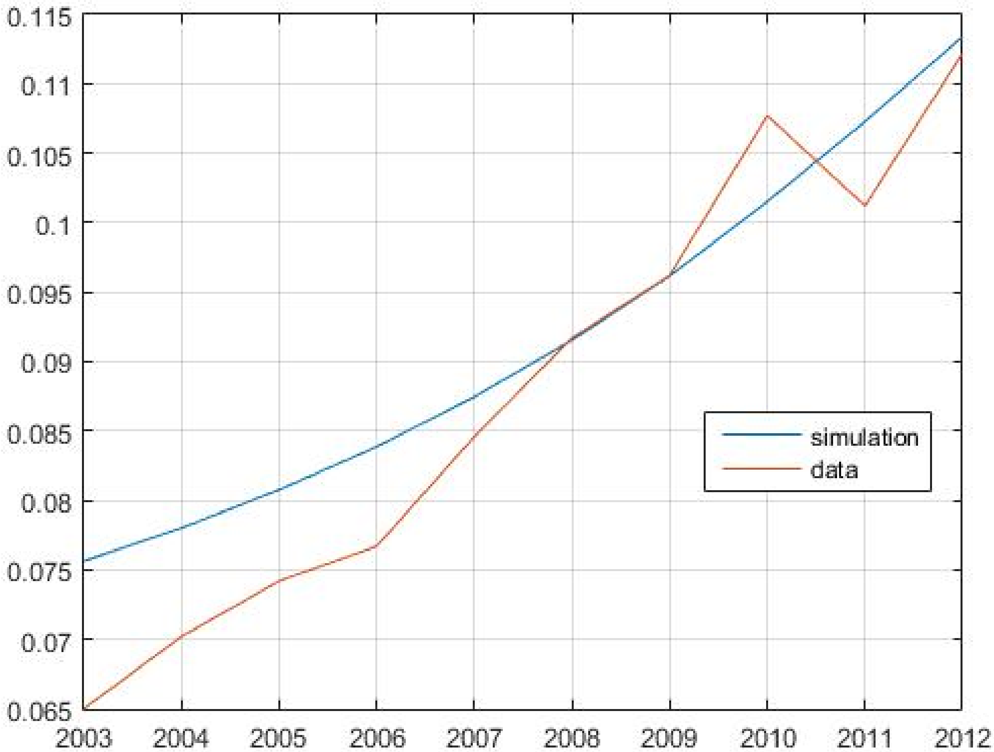

Figure 11: Top 0.01% data and simulation results (2003-2012)

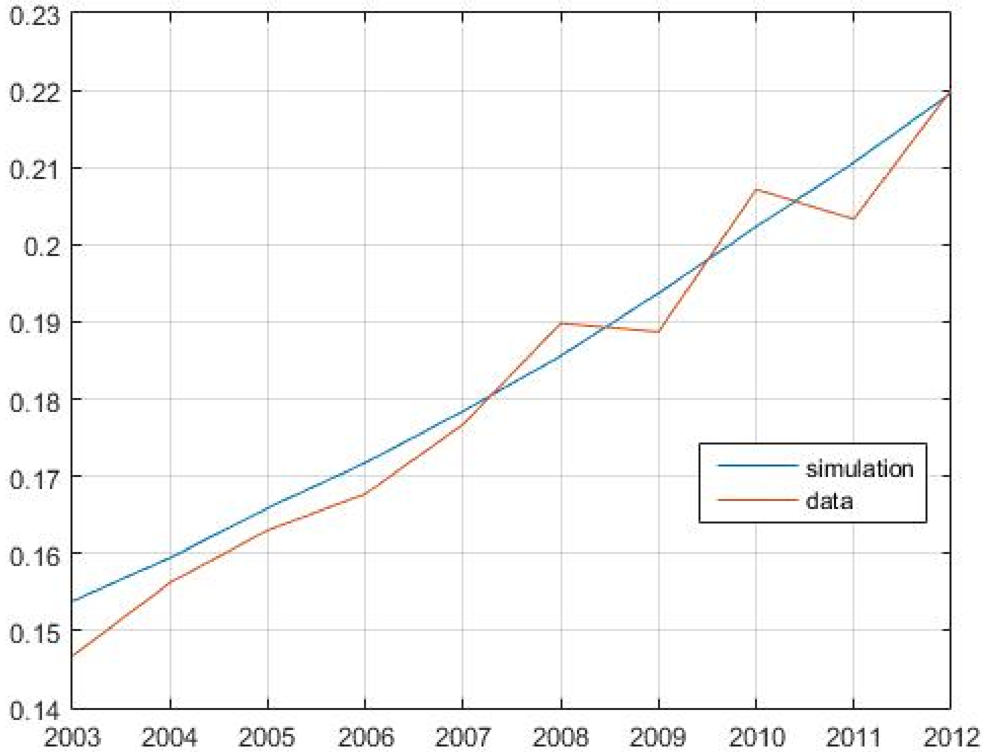

Figure 12: Top 0.1% data and simulation results (2003-2012)

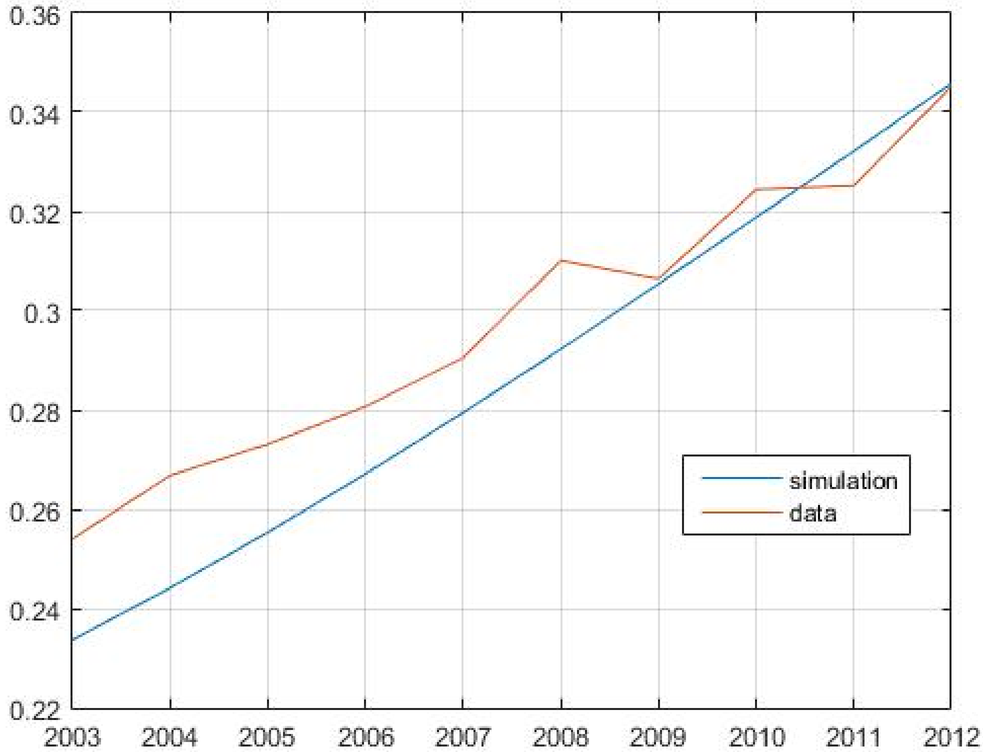

Figure 13: Top 0.5% data and simulation results (2003-2012)

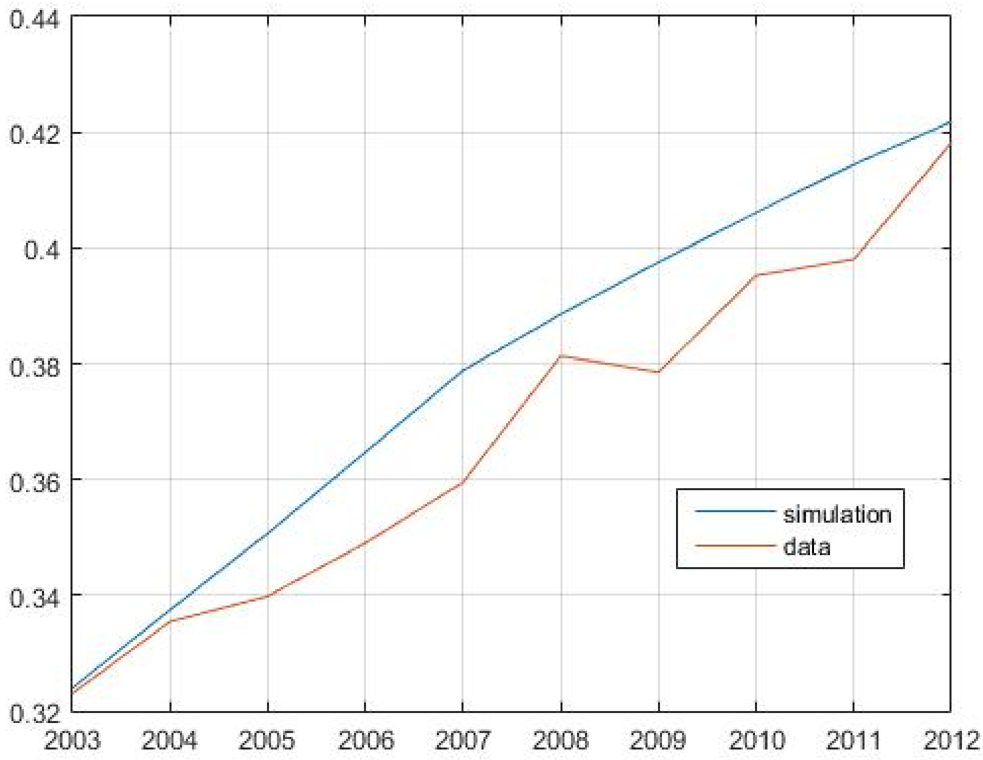

Figure 14: Top 1% data and simulation results (2003-2012)

7 Robustness

Figure 15 depicts the simulation results at $\alpha = 0.5$. The simulation still shows severe wealth inequality and the concentration on the top is even heavier than it is at $\alpha = 0.1$. This is not a surprising result. Higher α raises investment fees and discourages households from switching to new firms. This increases wealth inequality. Higher α also increases capital gains which pushes up the entrepreneur's wealth if her firm is invested by new investors.

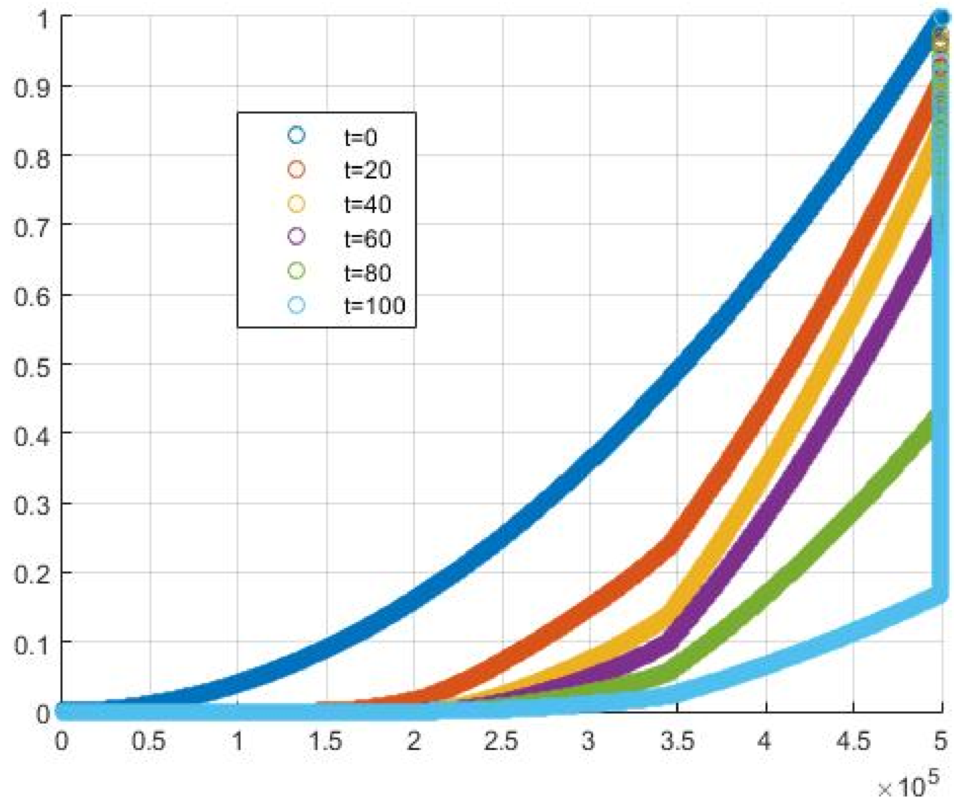

Figure 15: Robust test for $\alpha = 0.5$: cumulative wealth distributions at t-0, t=20, t=40, t=60, t=80, t=100. The y axis measures the cumulative wealth shares.

Another robustness check is performed to assess the limitation of assumption on one-period-ahead investment decision. Figure16 compares dynamics of top 0.1% wealth share under an alternative assumption of two-period-ahead investment decision with the benchmark until the simulation matches the top 0.1% wealth share in the 2012 data. The result first shows that the model with two-period-ahead investment generates wealth inequality slightly slower until the 53th period. This is because non-entrepreneur households now invest in larger firms that enable them to invest a larger amount of wealth. This generates a higher income over time. Afterwards the model with two-period-ahead investment decision quickly catch up and produces a much heavier wealth concentration. This is because although non-entrepreneur households hold more equity in firms they also have larger assets in the pooled fund. The saving rate effect thus has a larger impact on them. This result suggests that the assumption of one-period-ahead decision actually generates wealth inequality more conservatively. Figure 17 shows how the alternative simulation fit the data. It is not surprising to see that the alternative simulation overestimates the increase in the top 0.1% wealth share because it generates a severer wealth inequality. However, the trend is still well matched.

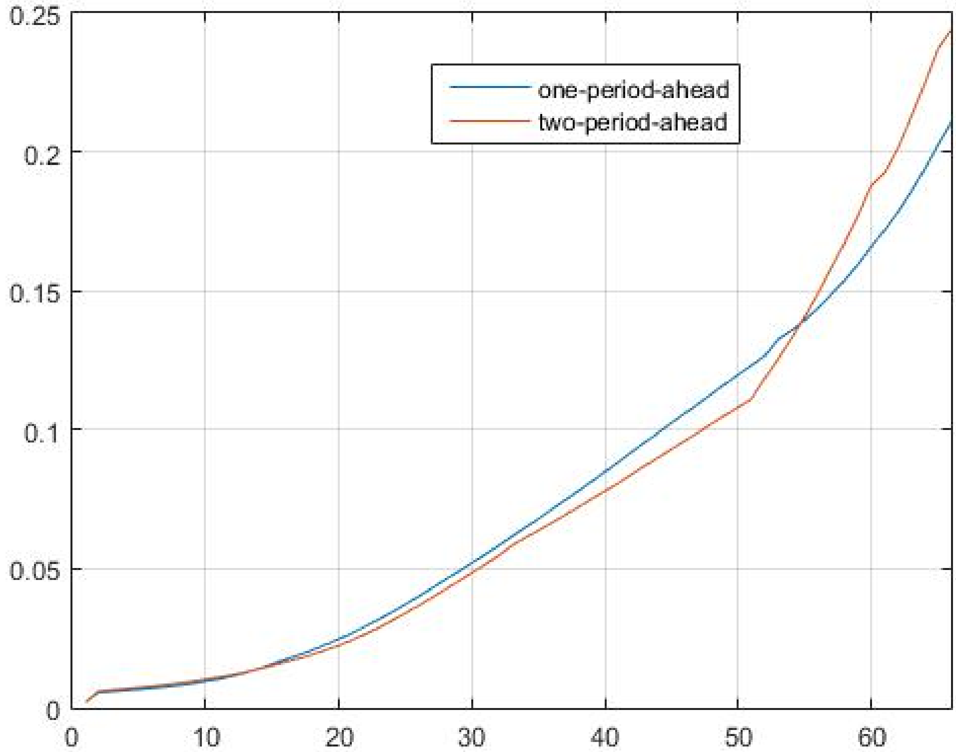

Figure 16: Robust test for one-period-ahead investment decision assumption: dynamics of wealth share of top 0.1% simulated under an alternative assumption of two-period-ahead investment decision for 66 periods. The y axis measures the wealth share of the top 0.1%.

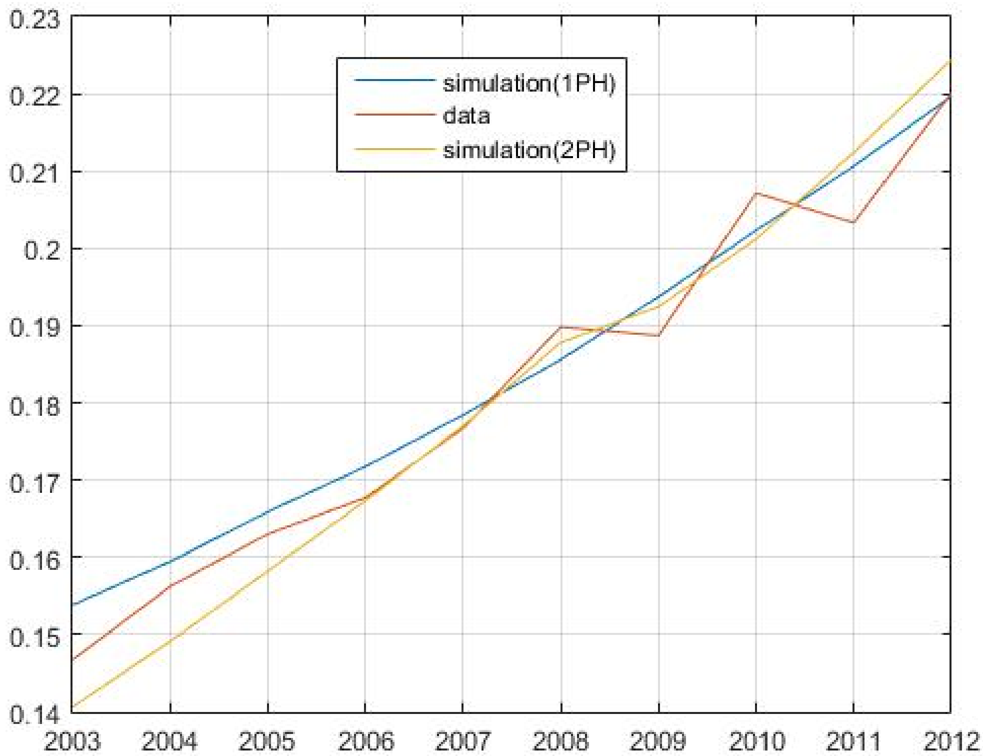

Figure 17: Robust test for one-period-ahead investment decision assumption: dynamics of wealth share of top 0.1% simulated under an alternative assumption of two-period-ahead investment decision matching the wealth share in 2012. The y axis measures the wealth share of the top 0.1%.

8 Counterfactual Exercises

In this subsection I perform counterfactual exercises for the 4 factors mentioned in subsection 4 to quantify their effects on wealth inequality. They are entrepreneurship, subsistence consumption, fluctuations in risk free rate r and access to new investment opportunities (that is, the ability of switching to other firms). Entrepreneurs tend to invest a larger fraction of their wealth in firms and benefit from the portfolio effect and saving rate effect. Subsistence level of consumption forces every households in the model to consume and reduce their wealth by the same amount. The risk free rate in the model adjusts to equate the demand with the supply of assets. Changes in r have several general equilibrium effects such as affecting capital gains and investor's choice on firms etc. The most important one among them is the saving rate effect. An indirect impact of r on wealth inequality is that it affects investor's motivation to switch to larger firms when the total investment exceeds the overall cost of the firm. Larger firms can take in more investment and increase the proportion of equity in investor's portfolio.

I in turn remove entrepreneurs from wealth distribution, set the subsistence consumption to 0, fix r at the lower bound 0.02 and forbid incumbent investors from switching to the other firms respectively to examine the effect of each factor. The results are in Table 7, Table 8, Table 9 and Table 10.[16]

Entrepreneurship is the most important factor and always facilitate the top concentration on wealth distribution. Its effect grows faster in later periods just like wealth inequality. Compared with Table 3, the effect of entrepreneurship accounts for about 100% of wealth shares of the top in wealth distribution. This confirms the importance of entrepreneurship. While entrepreneurship shapes the distribution on the top, subsistence consumption plays a big role on the left tail. Without subsistence consumption, households on the left tail have a significant share of wealth as shown in Figure 10. However its impact on the top is limited. Its boost on wealth inequality is comparable to that of entrepreneurship up to $t = 42$. After that its effect is reduced and sometimes drops to negative as in Table 8, 9 and 10. This is because the relatively poor households hold more assets in the pooled fund when there is no subsistence consumption. Their wealth is thus affected more when the saving rate effect sets in, which exacerbates inequality.

Fluctuations in r have negative effects, which means fixing r at its lower bound increases wealth inequality instead. This is due to the portfolio effect. Entrepreneurs usually have less proportions of their assets on the pooled fund and are thus less affected by the low risk free rate. It is noteworthy that the effect of fluctuations in r is mitigated after around $t = 72$ in all the four top percentiles. This is because in the benchmark simulation the risk free rate frequently drops to its lower bound in the later periods. As a result, fixing r to $\frac{1}{\beta}$ does not further generate large deviation from the benchmark during those periods. However, fluctuations in r still plays a significant role at the end of simulation. It offsets about $\frac{2}{5}$ to $\frac{1}{2}$ of the effect of entrepreneurship for cases of top 0.1%, 0.5% and 1%.

The effect of switching to larger firms only sets in after $t = 50$. This is because the lower the risk free rate is the more motivated investors are to switch to larger firms and r drops to the lower bound only after $t = 50$. After that the ability to switch to large firms shows a big and negative effect on wealth inequality. It allows non-entrepreneurs to adjust their portfolio, facilitate the formation of a middle

[16] One caveat to the interpretation of the results is that these four factors are not independent between each other. For example, the fluctuation of r and the access to other firms are correlated as mentioned above. The sum of effects of the four factors may thus be negative. As a result, the contribution of entrepreneurship may turn out negative. This not mean that entrepreneurship reduces inequality.

class and thus helps reduce wealth inequality. This effect does not fade away like fluctuation in r but heightens instead. It offsets about $\frac{1}{2}$ to $\frac{3}{5}$ of the effect of entrepreneurship for all the four percentiles at the end of simulation. The sum of these two negative effects can sometimes dominate that of entrepreneurship.

The model also allows us to study the effect of mobility on wealth distribution. Table 11 shows wealth shares of households at the top in the initial wealth distribution. It is noteworthy that the effect of mobility is highly correlated with the effect of entrepreneurship. This is because most of the mobility is caused by entrepreneurs.

	Entrepreneurship		Subsistence		r fluctuation		Firm switch		
t	factual	contri.	factual	contri.	factual	contri.	factual	contri.	sum
2	0.38%	96.96%	0.02%	5.92%	-0.01%	-2.88%	0.00%	0.00%	0.39%
12	0.95%	78.15%	0.42%	34.47%	-0.15%	-12.62%	0.00%	0.00%	1.21%
22	2.47%	138.34%	1.52%	84.89%	-2.20%	-123.23%	0.00%	0.00%	1.79%
32	4.50%	131.02%	2.88%	83.97%	-3.95%	-115.00%	0.00%	0.00%	3.43%
42	6.26%	119.74%	4.08%	78.00%	-5.11%	-97.73%	0.00%	0.00%	5.23%
52	7.42%	198.60%	3.57%	95.43%	-6.99%	-186.97%	-0.26%	-7.07%	3.74%
62	7.36%	-163.76%	0.89%	-19.77%	-10.19%	226.89%	-2.54%	56.63%	-4.49%
72	11.31%	1175.80%	2.18%	226.37%	-9.02%	-938.16%	-3.50%	-364.02%	0.96%
82	16.75%	150.45%	4.08%	36.66%	-6.03%	-54.16%	-3.67%	-32.95%	11.14%
92	20.93%	176.53%	2.95%	24.91%	-3.74%	-31.53%	-8.29%	-69.91%	11.86%
102	25.20%	263.44%	1.92%	20.07%	-2.34%	-24.49%	-15.21%	-159.02%	9.57%

Table 7: Counterfactual results for top 0.01%. The 'factual' column shows how much the wealth share in conterfactual simulation is smaller than it is in the benchmark. The 'sum' column is the sum of the 4 'factual's. The 'contri.' column measures the contribution of the factor, defined as $\frac{factual}{sum}$.

	Entrepreneurship		Subsistence		r fluctuation		Firm switch		
t	factual	contri.	factual	contri.	factual	contri.	factual	contri.	sum
2	0.38%	94.82%	0.03%	7.13%	-0.01%	-1.95%	0.00%	0.00%	0.40%
12	1.02%	76.76%	0.48%	35.89%	-0.17%	-12.65%	0.00%	0.00%	1.33%
22	2.90%	116.34%	1.74%	69.83%	-2.15%	-86.17%	0.00%	0.00%	2.49%
32	5.85%	98.84%	3.57%	60.40%	-3.50%	-59.24%	0.00%	0.00%	5.91%
42	9.22%	90.71%	5.56%	54.69%	-4.61%	-45.40%	0.00%	0.00%	10.16%
52	12.92%	157.97%	3.87%	47.38%	-8.27%	-101.13%	-0.35%	-4.22%	8.18%
62	18.30%	-877.47%	-0.65%	31.16%	-15.43%	739.98%	-4.30%	206.33%	-2.09%
72	28.55%	-238150.13%	-0.15%	1275.80%	-19.29%	160874.60%	-9.12%	76099.74%	-0.01%
82	39.51%	374.10%	2.29%	21.72%	-19.72%	-186.71%	-11.52%	-109.11%	10.56%
92	45.80%	1045.74%	-1.50%	-34.29%	-20.19%	-460.84%	-19.74%	-450.61%	4.38%
102	51.99%	1672.42%	-3.49%	-112.12%	-18.92%	-608.78%	-26.47%	-851.52%	3.11%

Table 8: Counterfactual results for top 0.1%. The 'factual' column shows how much the wealth share in conterfactual simulation is smaller than it is in the benchmark. The 'sum' column is the sum of the 4 'factual's. The 'contri.' column measures the contribution of the factor, defined as $\frac{factual}{sum}$.

	Entrepreneurship		Subsistence		r fluctuation		Firm switch		
t	factual	contri.	factual	contri.	factual	contri.	factual	contri.	sum
2	0.38%	85.86%	0.05%	11.84%	0.01%	2.30%	0.00%	0.00%	0.44%
12	1.01%	57.47%	0.69%	39.12%	0.06%	3.41%	0.00%	0.00%	1.76%
22	2.87%	90.73%	2.09%	66.22%	-1.80%	-56.95%	0.00%	0.00%	3.16%
32	5.77%	86.23%	4.02%	60.08%	-3.10%	-46.32%	0.00%	0.00%	6.70%
42	9.09%	83.15%	6.06%	55.38%	-4.21%	-38.53%	0.00%	0.00%	10.94%
52	12.74%	149.78%	4.07%	47.81%	-7.93%	-93.15%	-0.38%	-4.44%	8.51%
62	18.11%	-614.44%	-0.96%	32.61%	-15.57%	528.27%	-4.53%	153.56%	-2.95%
72	28.16%	-1517.54%	-1.23%	66.07%	-19.78%	1066.08%	-9.01%	485.38%	-1.86%
82	38.85%	515.59%	1.12%	14.93%	-21.24%	-281.96%	-11.19%	-148.55%	7.53%
92	44.77%	10242.39%	-2.29%	-523.97%	-23.05%	-5272.24%	-19.00%	-4346.18%	0.44%
102	50.67%	-2839.07%	-4.16%	233.06%	-22.74%	1273.94%	-25.56%	1432.08%	-1.78%

Table 9: Counterfactual results for top 0.5%. The 'factual' column shows how much the wealth share in conterfactual simulation is smaller than it is in the benchmark. The 'sum' column is the sum of the 4 'factual's. The 'contri.' column measures the contribution of the factor, defined as $\frac{factual}{sum}$.

	Entrepreneurship		Subsistence		r fluctuation		Firm switch		
t	factual	contri.	factual	contri.	factual	contri.	factual	contri.	sum
2	0.37%	76.70%	0.08%	16.66%	0.03%	6.63%	0.00%	0.00%	0.49%
12	1.00%	43.60%	0.95%	41.45%	0.34%	14.95%	0.00%	0.00%	2.30%
22	2.83%	70.93%	2.53%	63.43%	-1.37%	-34.36%	0.00%	0.00%	3.99%
32	5.68%	74.19%	4.58%	59.78%	-2.60%	-33.97%	0.00%	0.00%	7.66%
42	8.94%	75.18%	6.67%	56.10%	-3.72%	-31.28%	0.00%	0.00%	11.89%
52	12.53%	139.34%	4.30%	47.84%	-7.42%	-82.53%	-0.42%	-4.65%	8.99%
62	17.87%	-486.03%	-1.35%	36.80%	-15.40%	418.71%	-4.80%	130.52%	-3.68%
72	27.69%	-801.58%	-2.60%	75.18%	-19.66%	569.12%	-8.89%	257.29%	-3.45%
82	38.04%	749.82%	-0.37%	-7.29%	-21.79%	-429.45%	-10.81%	-213.08%	5.07%
92	43.53%	-1546.75%	-3.30%	117.18%	-24.92%	885.54%	-18.12%	644.04%	-2.81%
102	49.09%	-883.65%	-4.87%	87.63%	-25.30%	455.42%	-24.48%	440.60%	-5.56%

Table 10: Counterfactual results for top 1%. The 'factual' column shows how much the wealth share in conterfactual simulation is smaller than it is in the benchmark. The 'sum' column is the sum of the 4 'factual's. The 'contri.' column measures the contribution of the factor, defined as $\frac{factual}{sum}$.

t	top 0.01%	top 0.1%	top 0.5%	top 1%
2	0.38%	0.38%	0.38%	0.38%
12	0.95%	1.03%	1.03%	1.03%
22	2.47%	2.91%	2.91%	2.91%
32	4.50%	5.86%	5.86%	5.87%
42	6.26%	9.25%	9.25%	9.25%
52	7.42%	12.96%	12.96%	12.96%
62	7.36%	18.34%	18.35%	18.35%
72	11.32%	28.75%	29.13%	29.57%
82	16.78%	39.82%	40.39%	41.05%
92	20.98%	46.29%	47.16%	48.19%
102	25.26%	52.56%	53.48%	54.57%

Table 11: Counterfactual results when mobility is forbidden. Top x% refers to the top x% households in the initial wealth distribution.

9 Conclusion

In this paper I develop a dynamic general equilibrium model with a novel mechanism of investment between heterogeneous households and firms to study how firm heterogeneity affects wealth distribution through entrepreneurial income and capital gains. Theoretical analysis wealthy households may accumulate wealth faster because a larger fraction of their wealth is invested in firms and low risk free rate affects them less.

The model is calibrated with Nielsen barcode dataset. Simulation results show that the model generates very large wealth shares for households on the top of wealth distribution but may underestimate the wealth share for the top 0.01% and overestimate the bottom 90%. The simulation can match the dynamics of top wealth shares well for 2003-2012. The results also match some stylized facts established in Saez and Zucman 2016: 1. the upswing in the top 1% wealth share is due to the rise in the top 0.1% and that 2. the increase in wealth inequality is not due to rate of return differential on corporate stocks but rather saving rate inequality.

The counterfactual exercises highlight the importance of entrepreneurship, risk free rate, access to new firms and subsistence consumption. Entrepreneurship is the most important factor and accounts for most of the top concentration of wealth distribution. A low risk free rate directly concentrates wealth on the top but also motive investors to move to firms that better fit them and thus reduce inequality. Subsistence consumption exhausts the wealth of households on the left tail but barely affects the top.

The model has several interesting extensions. Introducing distortionary taxes allow it to perform analysis on tax policy. Making the product distribution to vary over time enables the model to work on a longer time horizon and study emerging industries. Manipulating parameters on product appeal and production curvature allows it to analyze the impact of crises.

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